Complex Renewable Energy Networks
Summer Semester 2017, Lecture 4

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Credit Points

This course has 3 Credit Points (CPs).

To obtain the credit points, you should turn up to at least 12 lectures (out of 14) and 5 tutorials (out of 7). Exceptions can be made if you started the course late or you have personal extenuating circumstances.

Benotung

To get a Note at the end, there will be a mündliche Prüfung (oral exam) in the week after the end of the semester, probably on one of July 25th/26th/27th 2017.

Loose ends from last time

Rotating field in a three-phase induction motor

A brilliant insight (credited to Tesla, but the history is complicated) was that with three-phase power, you can place your wires spaced at $2\pi/3$ to create a rotating magnetic field

`https://www.youtube.com/watch?v=LtJoJBUSe28`

which can then induce a current in a rotor cage, which then experiences a torque thanks to the magnetic field: this is the principle of the induction motor.

It would not be possible to create such a rotating field with a single-phase or two-phase system.

Computing the Linear Power Flow

Framing the load flow problem

Suppose we have N nodes labelled by i, and L edges labelled by ℓ forming a directed graph G.

Suppose at each node we have a power imbalance p_i ($p_i > 0$ means its generating more than it consumes and $p_i < 0$ means it is consuming more than it).

Since we cannot create or destroy energy (and we're ignoring losses):

$$\sum_i p_i = 0$$

Question: How do the flows f_ℓ in the network relate to the nodal power imbalances?

Answer: According to the impedances (generalisation of resistance for oscillating voltage/current) and the corresponding voltages.

Kirchhoff's Current Law (KCL)

KCL says (in this linear setting) that the nodal power imbalance at node i is equal to the sum of direct flows arriving at the node. This can be expressed compactly with the incidence matrix

$$p_i = \sum_{\ell} K_{i\ell} f_\ell \qquad \forall i$$

Kirchhoff's Voltage Law (KVL)

KVL says that the sum of voltage differences across edges for any closed cycle must add up to zero.

If the voltage at any node is given by θ_i (this is infact the voltage angle - more next week) then the voltage difference across edge ℓ is

$$\sum_i K_{i\ell}\theta_i$$

And Kirchhoff's law can be expressed using the cycle matrix encoding of independent cycles

$$\sum_\ell C_{\ell c} \sum_i K_{i\ell}\theta_i = 0 \qquad \forall c$$

[Automatic, since we already said KC = 0.]

Kirchhoff's Voltage Law (KVL)

If we express the flow on each line in terms of the voltage angle (a relative of $V = IR$) then for a line ℓ with reactance x_ℓ

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

KVL now becomes

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad\qquad \forall c$$

Solving the equations

If we combine

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

with Kirchhoff's Current Law we get

$$p_i = \sum_\ell K_{i\ell} f_\ell = \sum_\ell K_{i\ell} \frac{1}{x_\ell} \sum_j K_{j\ell} \theta_j$$

This is a weighted Laplacian. If we write $B_{k\ell}$ for the diagonal matrix with $B_{\ell\ell} = \frac{1}{x_\ell}$ then

$$L = KBK^t$$

and we get a discrete Poisson equation for the θ_i sourced by the p_i

$$p_i = \sum_j L_{ij} \theta_j$$

We can solve this for the θ_i and thus find the flows.

Solving the equations

Given p_i at every node, we want to find the flows f_ℓ. We have the equations

$$p_i = \sum_j L_{ij}\theta_j$$
$$f_\ell = \frac{1}{x_\ell}\sum_i K_{i\ell}\theta_i$$

Basic idea: invert L to get θ_i in terms of p_i

$$\theta_i = \sum_k (L^{-1})_{ik} p_k$$

then insert to get the flows as a linear function of the power injections p_i

$$f_\ell = \frac{1}{x_\ell}\sum_{i,k} K_{i\ell}(L^{-1})_{ik} p_k = \sum_k \mathrm{PTDF}_{\ell k} p_k$$

Inverting Laplacian L

There is one small catch: L is not invertible since we saw last time it has (for a connected network) one zero eigenvalue, with eigenvector $(1, 1, \ldots 1)$, since by construction $\sum_j L_{ij} = 0$.

This is related to a gauge freedom to add a constant to all voltage angles

$$\theta_i \to \theta_i + c$$

(corresponding to the zero eigenvector of L) which does not affect physical quantities:

$$p_i = \sum_j L_{ij}(\theta_j + c) = \sum_j L_{ij}(\theta_j)$$

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i + c) = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i)$$

Typically we choose a slack or reference bus such that $\theta_0 = 0$.

Inverting Laplacian L

Two solutions:

1. Set $\theta_0 = 0$, invert the lower-right $(N-1) \times (N-1)$ part of L to find the remaining $\{p_i\}_{i=1,\dots N-1}$ in terms of the $\{\theta_i\}_{i=1,\dots N-1}$, then derive p_0 from $\sum_i p_i = 0$.

2. Use the Moore-Penrose pseudo-inverse.

Write L in terms of its basis of orthonormal eigenvectors

$$L = \sum_n |\Phi_n\rangle \, \lambda_n \, \langle\Phi_n|$$

then the Moore-Penrose pseudo-inverse is:

$$L^\dagger = \sum_{n|\lambda_n \neq 0} \frac{|\Phi_n\rangle \langle\Phi_n|}{\lambda_n}$$

Check:

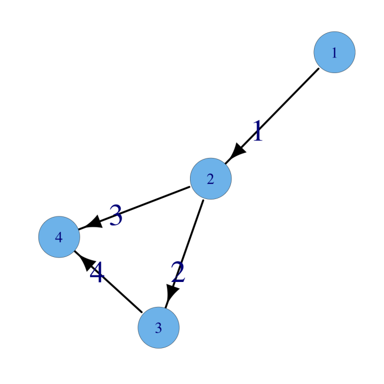
$$L^\dagger L = \sum_{n|\lambda_n \neq 0} |\Phi_n\rangle \langle\Phi_n| = I - |\Phi_0\rangle \langle\Phi_0|$$

4-node example

$$\mathbf{K}_{i\ell} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$\mathbf{L}_{ij} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$

$$\mathbf{PTDF}_{\ell i} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -2/3 & -1/3 \\ 0 & 0 & -1/3 & -2/3 \\ 0 & 0 & 1/3 & -1/3 \end{pmatrix}$$

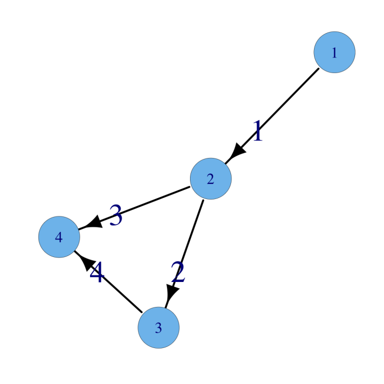


PTDF as sensitivity

Can also 'experimentally' determine the Power Transfer Distribution Factors (PTDF) by choosing a slack bus (in this case bus 1).

Each column (labelled by i) is then the resulting line flows if we have a simple power transfer from bus i to the slack $p_i = 1$ and $p_1 = -1$.

$$\mathbf{PTDF}_{\ell i} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -2/3 & -1/3 \\ 0 & 0 & -1/3 & -2/3 \\ 0 & 0 & 1/3 & -1/3 \end{pmatrix}$$



Consequences of limiting power transfers

Thermal limits

You cannot pass infinite current through a transmission line.

As it warms, it sags, then it will hit a building/tree and cause a short-circuit. [Or you may get voltage instability.]

Typically each line has a well-defined thermal limit on the amount of current that can flow through it, which translates to a limit on the active power in the linear approximation.

$$|f_\ell| \leq F_\ell$$

These limits may prevent the transfer of power.

Adjusting generator dispatch to avoid overloading

To avoid overloading the power lines, we must adjust our generator output (or the demand) so that the power imbalances do not overload the network.

We will now generalise and adjust our notation.

From lecture 2 we had for a single node:

$$-p_t = m_t - b_t + c_t = d_t - Ww_t - Ss_t - b_t + c_t = 0$$

where p_t was the nodal power balance, m_t was the mismatch (load d_t minus wind Ww_t and solar Ss_t), b_t was the backup power and c_t was curtailment.

We generalised this to multiple nodes labelled by i

$$-p_{i,t} = m_{i,t} - b_{i,t} + c_{i,t} = d_{i,t} - W_i w_{i,t} - S_i s_{i,t} - b_{i,t} + c_{i,t}$$

where now we don't enforce $p_{i,t} = 0$ but $\sum_i p_{i,t} = 0$ for all t.

Adjusting generator dispatch to avoid overloading

Now we write the dispatch of all generators at node i (wind, solar, backup) labelled by technology s as $g_{i,s,t}$ (i labels node, s technology and t time) so that we have a relation between load $d_{i,t}$, generation $g_{i,s,t}$ and network flows $f_{\ell,t}$

$$p_{i,t} = \sum_s g_{i,s,t} - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t}$$

Where s runs over the wind, solar and backup capacity generators (e.g. hydro or natural gas) at the node.

A dispatchable generator's $g_{i,s,t}$ output can be controlled within the limits of its power capacity $G_{i,s}$

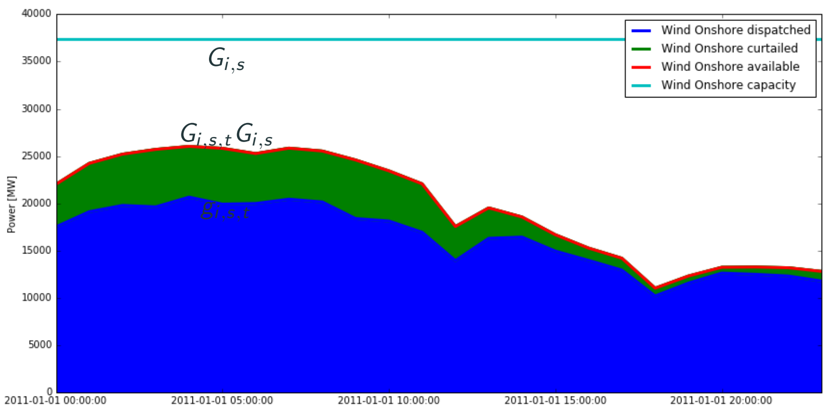
$$0 \leq g_{i,s,t} \leq G_{i,s}$$

Variable generation constraints

For a renewable generator we have time series of availability $0 \leq G_{i,s,t} \leq 1$ (the s_t and w_t before; W and S are the capacity $G_{i,s}$):

$$0 \leq g_{i,s,t} \leq G_{i,s,t} G_{i,s} \leq G_{i,s}$$

Curtailment corresponds to the case where $g_{i,s,t} < G_{i,s,t} G_{i,s}$:

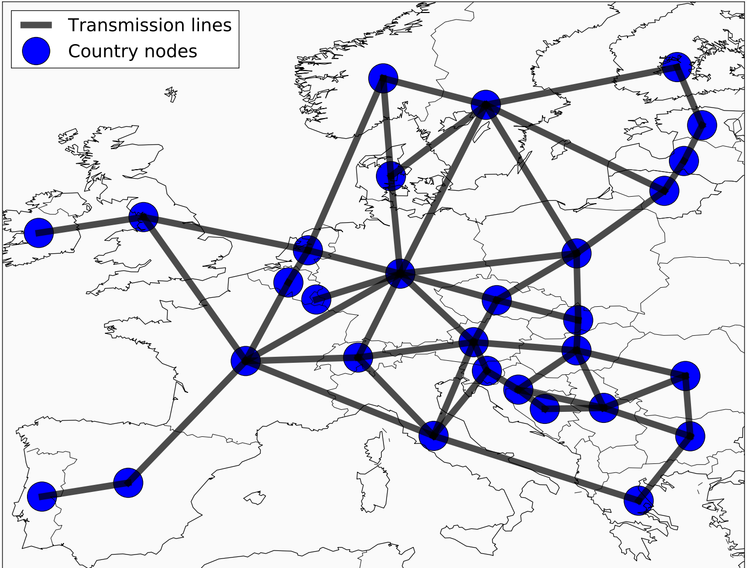


Germany curtailment example

See https://pypsa.org/examples/scigrid-lopf-then-pf.html.

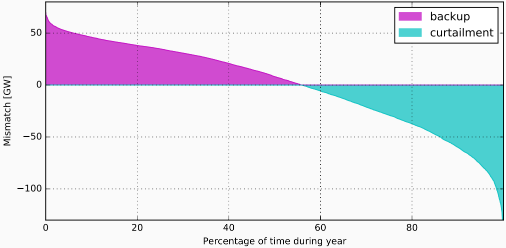
European transmission versus backup energy

Consider backup energy in a simplified European grid:

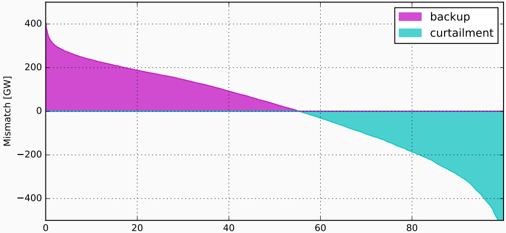


DE versus EU backup energy from last time

Germany needed backup generation for 31% of total load:

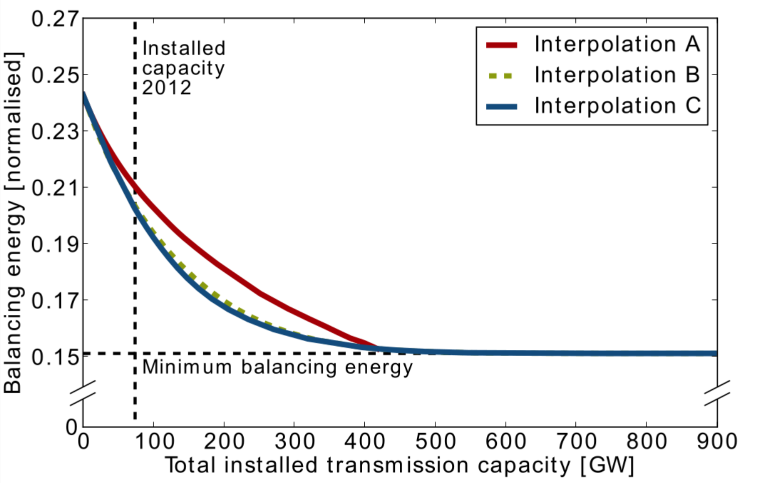


Europe needed Backup generation for only 24% of the total load:



European transmission versus backup energy

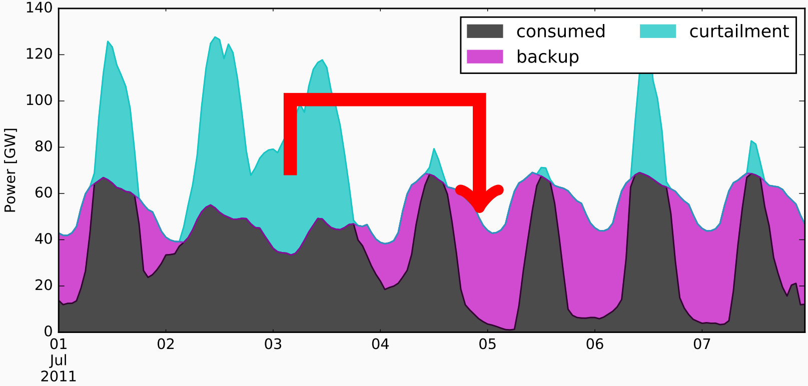
Transmission needs across a fully renewable European power system by Rodriguez, Becker, Andresen, Heide, Greiner, Renewable Energy, 2014



Principles of electricity storage

Basic idea of storage

Networks were used to shift power imbalances between different places, i.e. in space. Electricity storage can shift power in time.



Storage consistency

Storage units, such as batteries or hydrogen storage, can both dispatch power within a certain capacity:

$$0 \leq g_{i,s,t,\mathrm{dispatch}} \leq G_{i,s,\mathrm{dispatch}}$$

and consume power to store energy:

$$0 \leq g_{i,s,t,\mathrm{store}} \leq G_{i,s,\mathrm{store}}$$

The total power can then be written:

$$g_{i,s,t} = g_{i,s,t,\mathrm{dispatch}} - g_{i,s,t,\mathrm{store}}$$

There is also a limit on the total energy $e_{i,s,t}$ at each time

$$0 \leq e_{i,s,t} = -\int^{t} g_{i,s,t'}\, dt' \leq E_{i,s}$$

where $E_{i,s}$ is the energy capacity (in MWh). Or in iterative form

$$0 \leq e_{i,s,t} = e_{i,s,t-1} + g_{i,s,t,\mathrm{store}} - g_{i,s,t,\mathrm{dispatch}} \leq E_{i,s}$$

Continuous example

Consider a single node with a constant demand

$$d(t) = D$$

and a renewable wind generator with a capacity $G = 2D$ and an availability time series

$$G(t) = \frac{1}{2}\left(1 + \sin(\omega t)\right)$$

so that

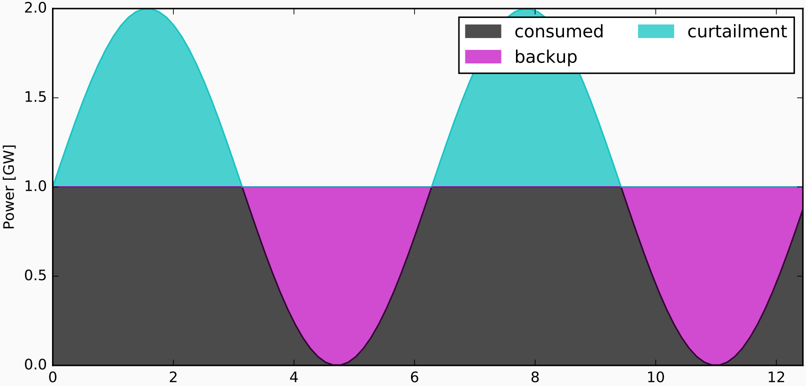
$$\langle G(t) G \rangle = D$$

Mismatch

Our mismatch is now

$$m(t) = d(t) - GG(t) = -D\sin(\omega t)$$

For $D = 1, \omega = 1$:

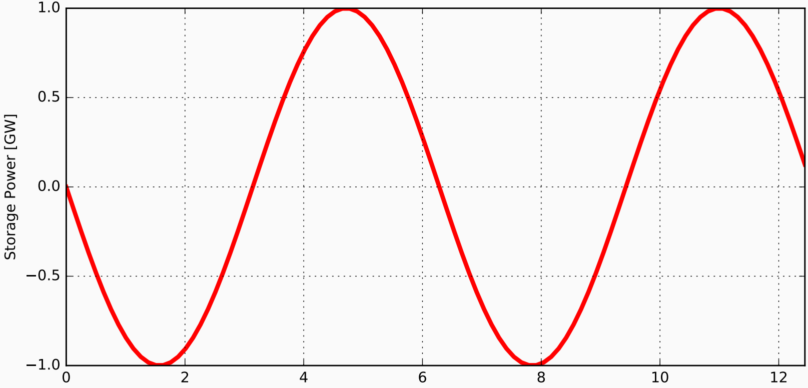


Storage Power

To balance this, we need a storage unit with a power profile to match the mismatch

$$g_s(t) = m(t) = -D\sin(\omega t)$$

This will have power capacities $G_{s,\text{store}} = G_{s,\text{dispatch}} = D$.

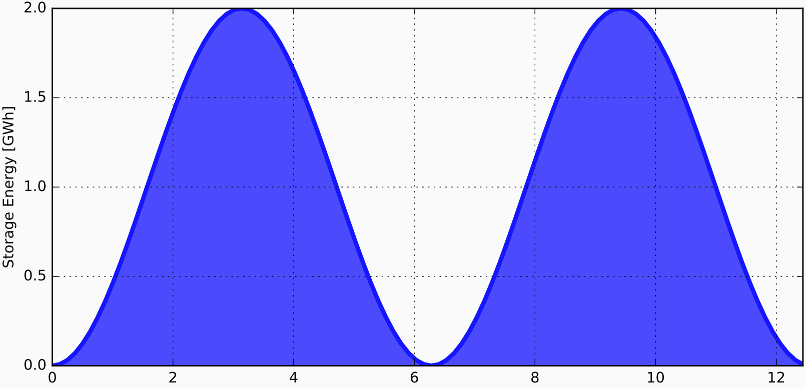


Storage Energy

How much energy capacity E_s do we need? The energy profile is:

$$e_s(t) = \int_0^t (-g_s(t'))dt' = D \int_0^t \sin(\omega t')dt' = \frac{D}{\omega}\left[1 - \cos(\omega t)\right]$$

so $E_s = \max_t\{e_s(t)\} = \frac{2D}{\omega}$. Faster oscillations $\Rightarrow$ less energy capacity.



Efficiency losses

There are a few extra details to add now. First, no renewable has a perfectly regular sinusoidal profile.

Second, the iterative integration equation for the storage energy

$$e_{i,s,t} = e_{i,s,t-1} + g_{i,s,t,\text{store}} - g_{i,s,t,\text{dispatch}}$$

needs to be amended for efficiency losses η

$$e_{i,s,t} = \eta_0 e_{i,s,t-1} + \eta_1 g_{i,s,t,\text{store}} - \eta_2^{-1} g_{i,s,t,\text{dispatch}}$$

Different storage units have different parameters

We can relate the power capacity G_s to the energy capacity E_s with the maximum number of hours the storage unit can be charged at full power before the energy capacity is full, $E_s = \text{max-hours} * G_s$.

	Battery	Hydrogen	Pumped-Hydro	Water Tank
η_0	0	0	0	depends on size
η_1	0.9	0.75	0.9	0.9
η_2	0.9	0.58	0.9	0.9
max-hours	2-10	weeks	4-10	depends on size
euro per kW [G_s]	300	300+450	depends	low
euro per kWh [E_s]	200	10	depends	low

Parameters are roughly based on Budischak et al, 2012 with projections for 2030.