Complex Renewable Energy Networks
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Optimisation Energy System Operation: Network

Generators at a single node

Last time we saw that if the demand is inelastic and fixed, welfare maximisation is equivalent to a generation cost minimisation problem:

$$\min_{\{g_s\}} \sum_s o_s g_s$$

such that:

$$\begin{aligned} \sum_s g_s - d = 0 \qquad &\leftrightarrow \qquad \lambda \\ g_s \leq G_s \qquad &\leftrightarrow \qquad \bar{\mu}_s \\ -g_s \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_s \end{aligned}$$

Several generators at different nodes in a network

Now let's suppose we have several nodes i with different loads and different generators, with flows f_ℓ in the network lines and voltage angles θ_i at the nodes (we use the linear power flow approximation).

Now we have additional optimisation variables f_ℓ and θ_i AND additional constraints:

$$\min_{\{g_{i,s}\},\{f_\ell\},\{\theta_i\}} \sum_{i,s} o_{i,s} g_{i,s}$$

such that demand is met either by generation or by the network at each node i

$$\sum_s g_{i,s} - d_i = \sum_\ell K_{i\ell} f_\ell \qquad \leftrightarrow \qquad \lambda_i$$

and generator constraints are satisified

$$g_{i,s} \leq G_{i,s} \qquad \leftrightarrow \qquad \bar{\mu}_{i,s}$$
$$-g_{i,s} \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{i,s}$$

Several generators at different nodes in a network

In addition we have constraints on the line flows.

First, they have to satisfy Kirchoff's Voltage Law (KVL) around each closed cycle c, which we guarantee by fixing

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

In addition the flows cannot overload the thermal limits, $|f_\ell| \leq F_\ell$

$$\begin{aligned} f_\ell \leq F_\ell \qquad & \leftrightarrow \qquad \bar{\mu}_\ell \\ -f_\ell \leq F_\ell \qquad & \leftrightarrow \qquad \underline{\mu}_\ell \end{aligned}$$

Simplest example: two nodes connected by a single line

At node 1 we have demand of $d_1 = 100$ MW and a generator with costs $o_1 = 10$ €/MWh and a capacity of $G_1 = 300$ MW.

At node 2 we have demand of $d_2 = 100$ MW and a generator with costs $o_1 = 20$ €/MWh and a capacity of $G_2 = 300$ MW.

What happens if the capacity of the line connecting them is $F_\ell = 0$?

What about $F_\ell = 50$ MW?

What about $F_\ell = \infty$?

Congestion rent

In this example we saw that the sum of what consumers pay does not always equal the sum of generator revenue.

In fact if we take the balance constraint and sum it weighted by the market price at each node we find

$$\sum_i \lambda_i^* d_i - \sum_i \lambda_i^* \sum_s g_{i,s}^* = -\sum_i \lambda_i^* \sum_\ell K_{i\ell} f_\ell^*$$

The quantity for each ℓ

$$-f_\ell^* \sum_i K_{i\ell} \lambda_i^* = f_\ell (\lambda_{\text{end}}^* - \lambda_{\text{start}}^*)$$

is called the congestion rent and is the money the network operator receives for transferring power from a low price node (start) to a high price node (end), 'buy it low, sell it high'.

It is zero if: a) the flow is zero or b) the price difference is zero.

Storage Optimisation

Storage equations

Now, like the network case where we add different nodes i with different loads, for storage we have to consider different time periods t.

Label conventional generators by s, storage by r and now minimise

$$\min_{\{g_{i,s,t}\},\{g_{i,r,t,\mathrm{store}}\},\{g_{i,r,t,\mathrm{dispatch}}\},\{f_{\ell,t}\},\{\theta_{i,t}\}} \left[\sum_{i,s,t} o_{i,s} g_{i,s,t} + \sum_{i,r,t} o_{i,r,\mathrm{store}}\, g_{i,r,t,\mathrm{store}} + \sum_{i,r,t} o_{i,r,\mathrm{dispatch}}\, g_{i,r,t,\mathrm{dispatch}}\right]$$

The power balance constraints are now (cf. Lecture 4) for each node i and time t that the demand is met either by generation, storage or network flows:

$$\sum_s g_{i,s,t} + \sum_r (g_{i,r,t,\mathrm{dispatch}} - g_{i,r,t,\mathrm{store}}) - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t} \quad \leftrightarrow \quad \lambda_{i,t}$$

Storage equations

We have constraints on normal generators

$$0 \leq g_{i,s,t} \leq G_{i,s}$$

and on the storage

$$0 \leq g_{i,r,t,\mathrm{dispatch}} \leq G_{i,r,\mathrm{dispatch}}$$
$$0 \leq g_{i,r,t,\mathrm{store}} \leq G_{i,r,\mathrm{store}}$$

The energy level of the storage is given by

$$e_{i,r,t} = \eta_0 e_{i,r,t-1} + \eta_1 g_{i,r,t,\mathrm{store}} - \eta_2^{-1} g_{i,r,t,\mathrm{dispatch}}$$

and limited by

$$0 \leq e_{i,r,t} \leq E_{i,r}$$

Storage equations

Finally for the flows we repeat the constraints for each time t.

We have KVL for the flows, therefore

$$f_{\ell,t} = \frac{1}{x_\ell} \sum_i K_{i,\ell} \theta_{i,t} \qquad \leftrightarrow \qquad \lambda_{\ell,t}$$

and in addition the flows cannot overload the thermal limits, $|f_{\ell,t}| \leq F_\ell$

$$\begin{aligned} f_{\ell,t} &\leq F_\ell \qquad &\leftrightarrow \qquad &\bar{\mu}_{\ell,t} \\ -f_{\ell,t} &\leq F_\ell \qquad &\leftrightarrow \qquad &\underline{\mu}_{\ell,t} \end{aligned}$$

Idea of storage

Storage does 'buy it low, sell it high' arbitrage, like network, but in time rather than space, i.e. between cheap times (e.g. with lots of zero-marginal-cost renewables) and expensive times (e.g. with high demand, low renewables and expensive conventional generators).

Investment Optimisation: Generation

Investment optimisation

Now we also optimise investment in the capacities of generators, storage and network lines, to maximise long-run efficiency.

We will promote the capacities $G_{i,s}$, $G_{i,r,*}$, $E_{i,r}$ and F_ℓ to optimisation variables.

For generation investment, we want to answer the following questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find price and load duration curves very useful.

Definition of long-run efficiency

Up until now we have considered short-run equilibria that ensure short-run efficiency (static), i.e. they make the best use of presently available productive resources.

Long-run efficiency (dynamic) requires in addition the optimal investment in productive capacity.

Concretely: given a set of options, costs and constraints for different generators (nuclear/gas/wind/solar) what is the optimal generation portfolio for maximising long-run welfare?

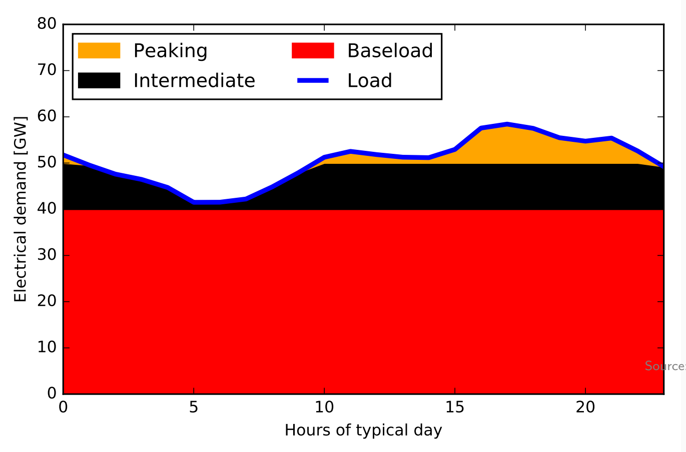
From an indivdual generators' perspective: how best should I invest in extra capacity?

We will show again that with perfect competition and no barriers to entry, the system-optimal situation can be reached by individuals following their own profit.

Baseload versus Peaking Plant

Load (= Electrical Demand) is low during night; in Northern Europe in the winter, the peak is in the evening.

To meet this load profile, cheap baseload generation runs the whole time; more expensive peaking plant covers the difference.



Source: Tom Brown

System-optimal generator capacities and dispatch

Suppose we have generators labelled by s at a single node with marginal costs o_s arising from each unit of production $g_{s,t}$ and capital costs c_s that arise from fixed costs regardless of the rate of production (such as the investment in building capacity G_s). For a variety of demand values d_t in representative situation t we optimise the total system costs

$$\min_{\{g_{s,t}\},\{G_s\}} \left[\sum_s c_s G_s + \sum_{s,t} o_s g_{s,t} \right]$$

such that

$$\begin{aligned} \sum_s g_{s,t} = d_t \qquad & \leftrightarrow \qquad \lambda_t \\ -g_{s,t} \leq 0 \qquad & \leftrightarrow \qquad \underline{\mu}_{s,t} \\ g_{s,t} - G_s \leq 0 \qquad & \leftrightarrow \qquad \bar{\mu}_{s,t} \end{aligned}$$

We will also allow load-shedding with a 'dummy' generator $s = S$, $o_S = V$ (Value of Lost Load), $c_S = 0$ (the capacity to she load doesn't cost anything, so can be as big as d_t if necessary).

System-optimal generator capacities and dispatch

Stationarity gives us for each s and t:

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = o_s - \lambda_t^* - \bar{\mu}_{s,t}^* + \underline{\mu}_{s,t}^*$$

and for each s:

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = c_s + \sum_t \bar{\mu}_{s,t}^*$$

and from complementarity we get

$$\begin{aligned} \bar{\mu}_{s,t}^*(g_{s,t}^* - G_s^*) &= 0 \\ \underline{\mu}_{s,t}^* g_{s,t}^* &= 0 \end{aligned}$$

and dual feasibility (for minimisation) $\bar{\mu}_{s,t}^*, \underline{\mu}_{s,t}^* \leq 0$.

System-optimal generator capacities and dispatch

The solution for the dispatch $g^*_{s,t}$ is exactly the same as without capacity optimisation. For each t, find m such that $\sum_{s=1}^{m-1} G_s < d_t < \sum_{s=1}^{m} G_s$.

For $s < m$ we have $g^*_{s,t} = G^*_s$, $\underline{\mu}^*_{s,t} = 0$, $\bar{\mu}^*_{s,t} = o_s - \lambda^*_t \leq 0$.

For $s = m$ we have $g^*_{m,t} = d_t - \sum_{s=1}^{m-1} G^*_s$ to cover what's left of the demand. Since $0 < g^*_{m,t} < G_m$ we have $\underline{\mu}^*_{m,t} = \bar{\mu}^*_{m,t} = 0$ and therefore $\lambda^*_t = o_m$.

For $s > m$ we have $g^*_{s,t} = 0$, $\underline{\mu}^*_{s,t} = \lambda^*_t - o_s \leq 0$, $\bar{\mu}^*_{s,t} = 0$.

What about the G^*_s?

System-optimal generator capacities and dispatch

The G_s^* are determined implicitly based on the interactions between costs and prices.

From stationarity we had the relation

$$c_s = -\sum_t \bar{\mu}^*_{s,t}$$

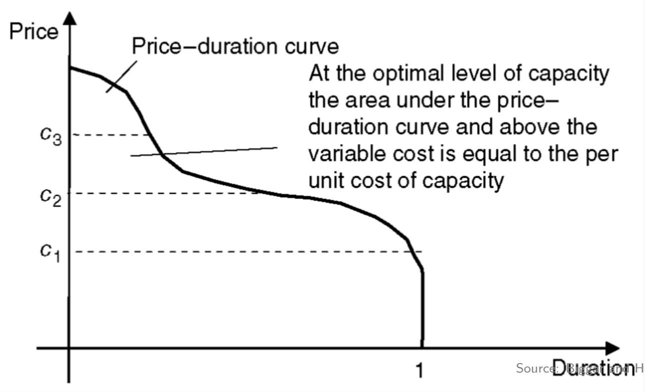
The $\bar{\mu}^*_{s,t}$ were only non-zero with $\lambda_t^* > o_s$ so we can re-write this as

$$c_s = \sum_{t|\lambda_t^* > o_s} (\lambda_t^* - o_s)$$

'Increase capacity until marginal increase in profit equals the cost of extra capacity.'

Multiple price duration

The optimal mix of generation is where, for each generation type, the area under the price–duration curve and above the variable cost of that generation type is equal to the fixed cost of adding capacity of that generation type. (In the graphic c_s is o_s in our notation.)

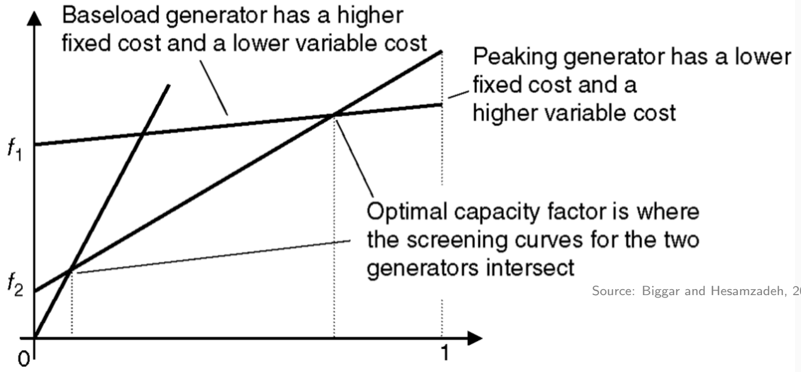


Source: Biggar and Hesamzadeh, 2014

Screening curve

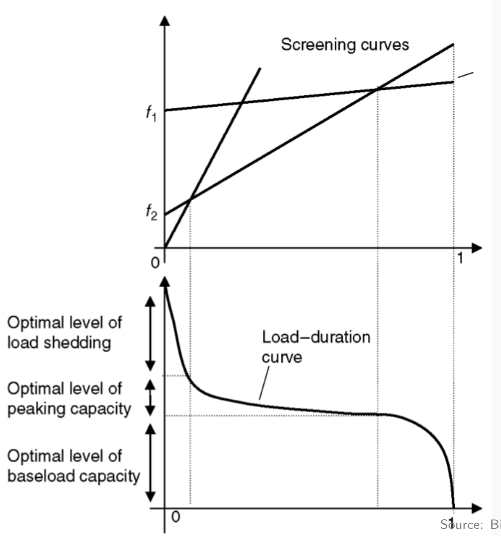
The costs as a function of the capacity factors can be drawn together as a screening curve (more expensive options are *screened* from the optimal inner polygon).

The intersection points determine the optimal capacity factors and hence, using the load duration curve, the optimal capacities of each generator type. (In the graphic f_s is c_s in our notation.)



Source: Biggar and Hesamzadeh, 2014

Screening curve versus Load duration



Source: Biggar and Hesamzadeh, 2014

Individual generator optimising capacity and dispatch

Suppose a generator has marginal costs o arising from each unit of production g_t and capital costs c that arise from fixed costs regardless of the rate of production (such as the investment in building capacity G).

For a variety of representative situation t there is a market price λ_t over which the generator has no influence.

The generator will try and choose their capacity G and dispatch g_t to maximise their long-run profit:

$$\max_{\{g_t\},G} \left[\sum_t \lambda_t g_t - \sum_t o g_t - cG \right]$$

such that

$$\begin{aligned} -g_t &\leq 0 &\qquad \leftrightarrow \qquad& \underline{\mu}_t \\ g_t - G &\leq 0 &\qquad \leftrightarrow \qquad& \bar{\mu}_t \end{aligned}$$

Compare to before: we've added G as an optimisation variable (not a constant) and added costs for the capacity c to the objective function.

Individual generator optimising capacity and dispatch

From KKT we get

$$0 = \frac{\partial \mathcal{L}}{\partial g_t} = \lambda_t - o + \underline{\mu}_t - \bar{\mu}_t$$

$$0 = \frac{\partial \mathcal{L}}{\partial G} = -c + \sum_t \bar{\mu}_t$$

This now splits into two cases:

1. $\lambda_t < o$: The market price is lower than the operating costs, so $\bar{\mu}_t = 0$, $\underline{\mu}_t = o - \lambda_t$ and $g_t = 0$.
2. $\lambda_t \geq o$: The market price is higher than the operating costs, so $\underline{\mu}_t = 0$, $\bar{\mu}_t = \lambda_t - o$ and $g_t = G$.

The relation between c, o and λ_t is given by

$$c = \sum_t \bar{\mu}_t = \sum_{t|\lambda_t > o} (\lambda_t - o)$$

Investment Optimisation: Transmission

Investment optimisation: transmission

As before, our approach to the question of "What is the optimal amount of transmission" is determined by the most efficient long-term solution, i.e. the infrastructure investement that maximising social welfare over the long-run.

Promote F_ℓ to an optimisation variable with capital cost c_ℓ.

In brief: Exactly as with generation dispatch and investment, we continue to invest in transmission until the marginal benefit of extra transmission (i.e. extra congestion rent for extra capacity) is equal to the marginal cost of extra transmission. This determines the optimal investment level.