Complex Renewable Energy Networks

(SoSe 2017, FIAS & Goethe-Universität Frankfurt)

HomeWork Sheet II

To be prepared for the exercise session on Wednesday, 17.05.2017.

Problem II.1 (analytical optimal mix). Figure 1 shows approximations to the seasonal variations of wind and solar power generation $W(t)$ and $S(t)$ and load $L(t)$:

$$W(t) = 1 + A_W \cos \omega t$$
$$S(t) = 1 - A_S \cos \omega t$$
$$L(t) = 1 + A_L \cos \omega t$$

The time series are normalized to

$$\langle W \rangle = \langle S \rangle = \langle L \rangle := \frac{1}{T} \int_0^T L(t) \mathrm{d}t = 1 \,,$$

and the constants have the values

$$\omega = \frac{2\pi}{T} \qquad T = 1\,\text{year}$$
$$A_W = 0.4 \qquad A_S = 0.75 \qquad A_L = 0.1$$

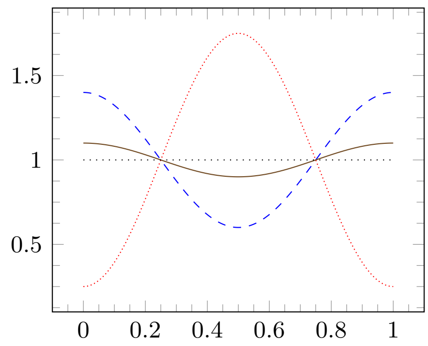

Figure 1: Seasonal variations of wind and solar power generation $W(t)$ - - - and $S(t)$ ········, and load $L(t)$ —— around the mean 1 ········.

(a) What is the seasonal optimal mix α, which minimizes

$$\langle\, [\alpha W(\cdot) + (1-\alpha) S(\cdot) - L(\cdot)]^2 \,\rangle = \frac{1}{T} \int_0^T [\alpha W(t) + (1-\alpha)S(t) - L(t)]^2 \,\mathrm{d}t,$$

(b) How does the optimal mix change if we replace $A_L \to -A_L$?

(c) A constant conventional power source $C(t) = 1 - \gamma$ is introduced; The mismatch then becomes

$$\Delta(t) = \gamma \, [\alpha W(t) + (1-\alpha) S(t)] + C(t) - L(t). \tag{1}$$

Analogously to (a), find the optimal mix α as a function of $0 \leq \gamma \leq 1$, which minimizes $\langle\, \Delta^2 \,\rangle$.

Problem II.2 (network theory basics). Consider the simple network shown in Figure 2. Calculate in Python or by hand:

(a) Compile the *nodes list* and the *edge list* (while graph-theoretically both lists are unordered sets, let's agree on an ordering now which can serve as basis for the matrices in exercises (c), (e) and (f): we sort everything in ascending numerical order, i.e. node 1 before node 2 and edge $(1, 2)$ before edge $(1, 4)$ before edge $(2, 3)$).

(b) Determine the *order* and the *size* of the network.

(c) Compute the *adjacency matrix* A and check that it is symmetric.

(d) Find the *degree* k_n of each node n and compute the *average degree* of the network.

(e) Determine the *incidence matrix* K by assuming the links are always directed from smaller-numbered node to larger-numbered node, i.e. from node 2 to node 3, instead of from 3 to 2.

(f) Compute the *Laplacian* L of the network using k_n and A. Rememember that the Laplacian can also be computed as $L = KK^T$ and check that the two definitions agree.

(g) Find the *diameter* of the network by simply looking at Figure 2.

Problem II.3 (linear power flow). If you map the nodes to countries like 0 = DK, 1 = DE, 2 = CH, 3 = IT, 4 = AT and 5 = CZ, the network in Figure 2 represents a small part of the European electricity network (albeit very simplified).

On the course home page[1], you can find the *power imbalance* time series for the six countries for January 2017 in hourly MW in the file `imbalance.csv`. They have been derived from the Physical Flows as published by ENTSO-E[2].

The linear power flow is given by

$$p_i = \sum_j \tilde{L}_{i,j}\theta_j \ , \qquad f_l = \frac{1}{x_l}\sum_i K_{i,l}\theta_i \ , \tag{2}$$

where the weighted Laplacian is $\tilde{L}_{i,j} = \sum_l K_{i,l}\frac{1}{x_l}K_{j,l}$. For simplicity, we assume identity reactance on all links $x_l = 1$.

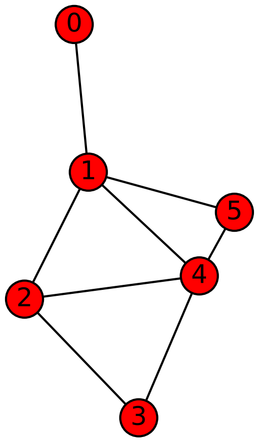

Figure 2: Simple Network.

(a) Compute the *voltage angles* θ_j and *flows* f_l for the first hour in the dataset with the convention of $\theta_0 = 0$; i.e. the slack bus is at node 0 (hint: linear equation systems are solved efficiently using `numpy.linalg.solve`).

(b) Determine the average flow on each link in January 2017 and draw it as a directed network on a sheet of paper. Is it a tree?

[1] https://nworbmot.org/courses/complex_renewable_energy_networks/

[2] https://transparency.entsoe.eu/transmission-domain/physicalFlow/show