III.1 (a)

No losses $\Rightarrow \langle L_N \rangle = \langle g_{N,w} g_{N,w}(t) \rangle$

$= A_{L,N} = g_{N,w} \, Cf_W \Rightarrow g_{N,w} = \frac{A_{LN}}{Cf_W} = \frac{20 \, GW}{0,3}$

$g_{S,s} = \frac{A_{LS}}{Cf_S} = \frac{30 \, GW}{0,12}$

III.1 (b)

Norden:

$g^N_{s,\text{store dispatch}} = \max\left(\mp g^N_s(t)\right)$

$= \max\left(\mp\left[\overbrace{L_N(t) - g_{N,w}\, g_{N,w}(t)}^{m(t)}\right]\right)$

$= \max\left(\mp\left[L_N(t) - \frac{A_{L,N}}{Cf_W} Cf_W (1 + A_W \sin \omega_N t)\right]\right)$

$= \max\left(\pm A_{L,N} A_W \sin \omega_N t\right) = A_{L,N} A_W = 0,3 \cdot 20\,GW = 1\,GW$

South:

$g^S_{s,\text{store dispatch}} = \max\left(\mp g^S_s(t)\right)$

$= A_{L,S} A_S = 30\,GW$

III.1 (c)

North:

$e^N_s(t) = \int_0^t -g^N_s(t')\,dt'$

$= \int_0^t A_{L,N} A_W \sin \omega_N t' \, dt'$

$= A_{L,N} A_W \left. \left(\frac{-\cos \omega_N t'}{\omega_N}\right)\right|_0^t = A_{L,N} A_W \frac{1-\cos \omega_N t}{\omega_N}$

$E^N_s = \max_t e^N_s(t) = \frac{2 A_{L,N} A_W}{\omega_N} = \frac{2 \cdot 20\,GW \cdot 0,3}{2\pi} \, 7\,d \simeq 1\,TWh$

South:

$E^S_s = \frac{2 A_{L,S} A_S}{\omega_S} = \frac{2 \cdot 30\,GW}{2\pi} \, 24\,h \simeq 230\,GWh$

III.1 (d)

North: $P^N_W = \frac{20\,GW}{0{,}3} \cdot 1200\,€/kW = 80 \cdot 10^9\,€$

Hydrogen

$P^N_{H} = 750\,€/kW \cdot \mathcal{S}^N_{s,st/disp} + 10\,€/kWh \cdot E^N_s$

$= 13{,}5 \cdot 10^9\,€ + 10 \cdot 10^9\,€$

$P^N_{H_2} = 23{,}5 \cdot 10^9\,€$

Battery

$P^N_B = 300\,€/kW \cdot \mathcal{S}^N_{s,st/disp} + 200\,€/kWh \cdot E^N_s$

$= 5{,}4 \cdot 10^9\,€ + 200 \cdot 10^9\,€$

$= 205 \cdot 10^9\,€$

$\underbrace{100 \cdot 10^9\,€}_{\text{No losses}} \le \underbrace{P^N_{W+H_2}}_{\text{Actual Price}} \le \underbrace{\frac{100 \cdot 10^9\,€}{0{,}75 \cdot 0{,}58}}_{\text{Losses overhead worst case}} = 230 \cdot 10^9\,€$

$205 \cdot 10^9\,€ \le P^N_{W+B} \le \frac{205 \cdot 10^9\,€}{0{,}82} = 250 \cdot 10^9\,€$

South: $P^S_S = \frac{30\,GW}{0{,}12} \cdot 600\,€/kW = 150 \cdot 10^9\,€$

Hydrogen

$P^S_{H_2} = 750\,€/kW \cdot \mathcal{S}^S_{s,st/disp} + 10\,€/kWh \cdot E^S_s$

$= 22{,}5 \cdot 10^9\,€ + 2{,}3 \cdot 10^9\,€$

Battery

$P^S_B = 300\,€/kW \cdot \mathcal{S}^S_{s,st/disp} + 200\,€/kWh \cdot E^S_s$

$= 9 \cdot 10^9\,€ + 46 \cdot 10^9\,€$

$\underbrace{175 \cdot 10^9\,€}_{\text{No losses}} \le P^S_{S+H_2} \le 400 \cdot 10^9\,€$

$205 \cdot 10^9\,€ \le P^S_{S+B} \le 250 \cdot 10^9\,€$

Without taking losses into account:

$\tilde{P}^N_{W+H_2} = 100 \cdot 10^9\,€ / 20\,GW \approx 5 \cdot 10^9/GW$

$\tilde{P}^S_{S+H_2} = 175 \cdot 10^9\,€ / 30\,GW \approx 6 \cdot 10^9/GW$

(With heuristic losses treatment, i.e. assume real price is directly the average of loss-less and overblown loss overhead

$P^{N,\text{heuristic}}_{W+H_2} \approx 165 \cdot 10^9\,€ / 20\,GW = 8\tfrac{1}{4} \cdot 10^9\,€/GW$

$P^{S,\text{heuristic}}_{S+B} \approx 230 \cdot 10^9\,€ / 30\,GW \approx 7\tfrac{2}{3} \cdot 10^9\,€/GW$)

III.1 (c) $\tilde{p}^N_{W+H_2} < \tilde{p}^S_{S+H_2}$ $\Rightarrow$ Energy exports from North to South

$E^N > E^S$ $\quad E^N + E^S = 50\,GW$

$$P_{tot} = E^N \cdot \tilde{p}^N_{W+H_2} + E^S \tilde{p}^S_{S+H_2} + (E^N - E^S) \cdot 200\,€/kW$$

$$(E_N - ~~20~~ A_{kW}) \cdot 200\,€/kW$$

$$= E^N \cdot \tilde{p}^N_{W+H_2} + (50\,GW - E^N)\, \tilde{p}^S_{S+H_2} + (2E^N - 50\,GW)\, 200\,€/kW$$

$$~~\frac{dP_{tot}}{dE^N} = \tilde{p}^N_{W+H_2} - \tilde{p}^S_{S+H_2} + 400\,€/kW~~$$

$$= E^N \left(\tilde{p}^N_{W+H_2} - \tilde{p}^S_{S+H_2} + 400\,€/kW\right) + 50\,GW \left(\tilde{p}^S_{S+H_2} - 200\,€/kW\right)$$

$$\min P_{tot} = 30\,GW\left((5 - 6 + 0{,}4) \cdot 10^9\,€/GW\right) + 50\,GW\,(6 - 0{,}2) \cdot 10^9\,€/GW$$

$$= -0{,}6 \cdot 30 \cdot 10^9\,€ + 50 \cdot 5{,}8 \cdot 10^9\,€ = 272 \cdot 10^9\,€$$

Aller Strom aus Wind!