Energy Economics - Exhaustible Ressources

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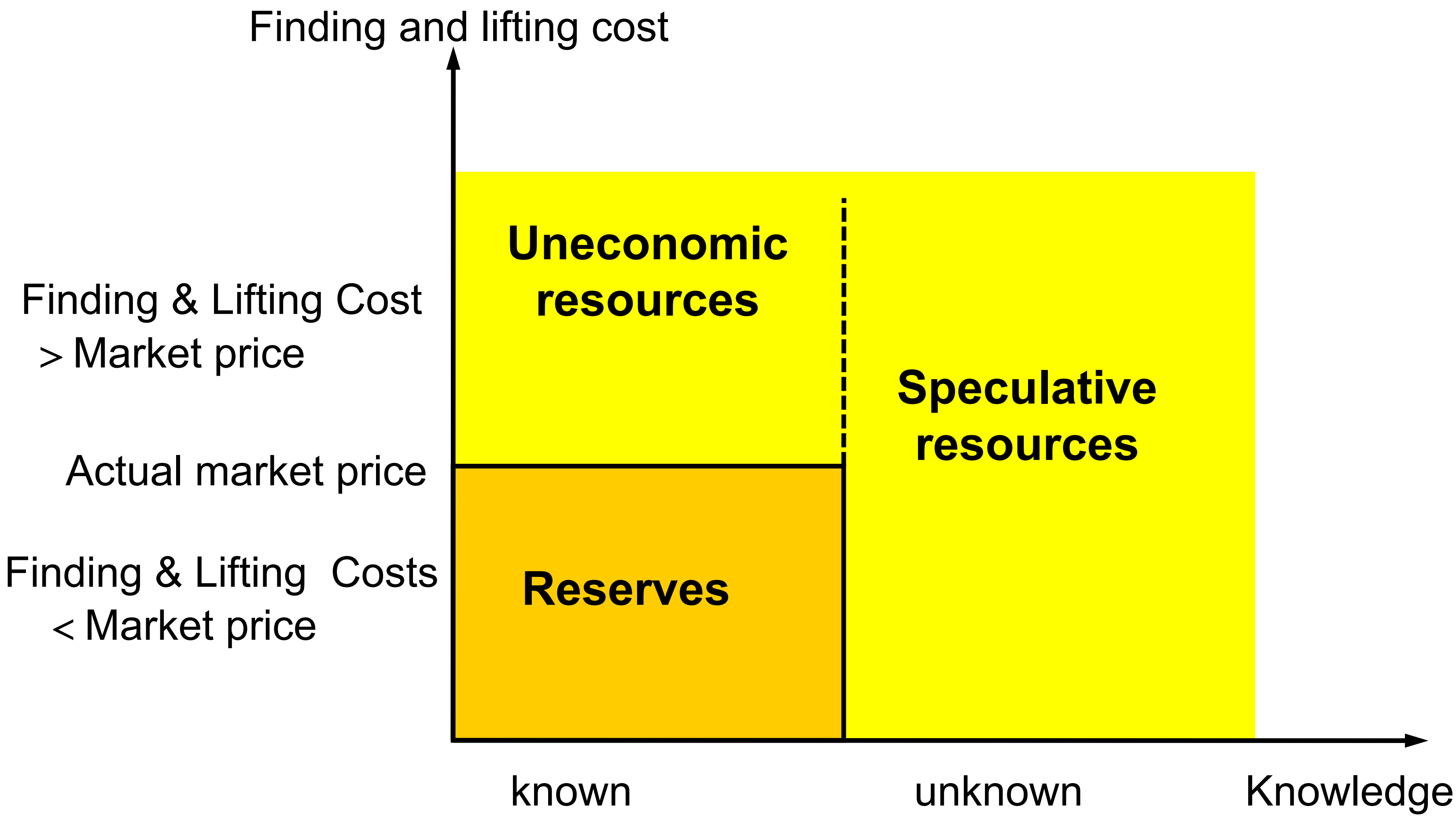
Brief theoretical summary of the topic: Exhaustible Resources

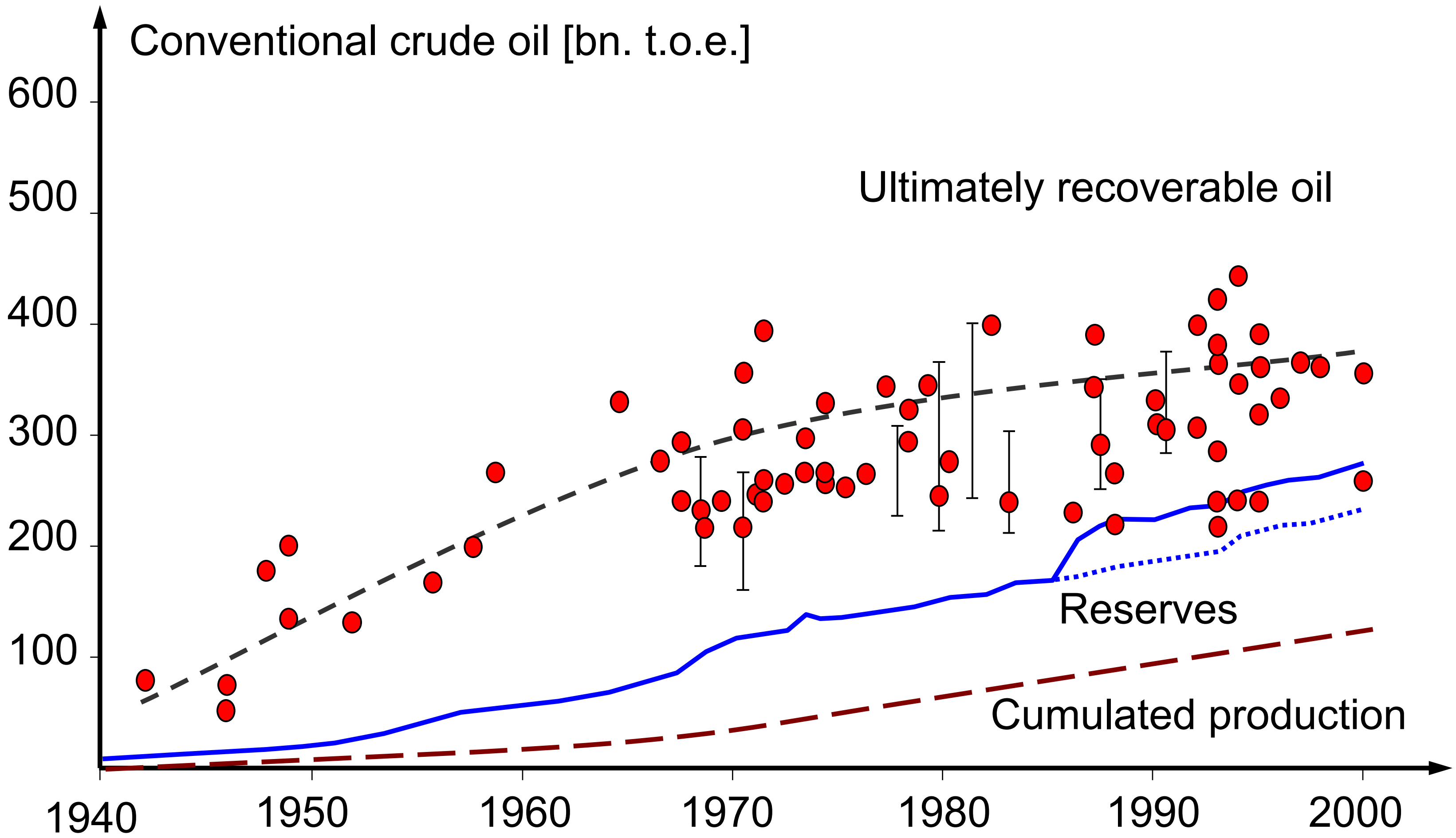
- Resources and Reserves
- Resource Extraction: Hotelling's Rule
- The Green Paradox

Resources and Reserves



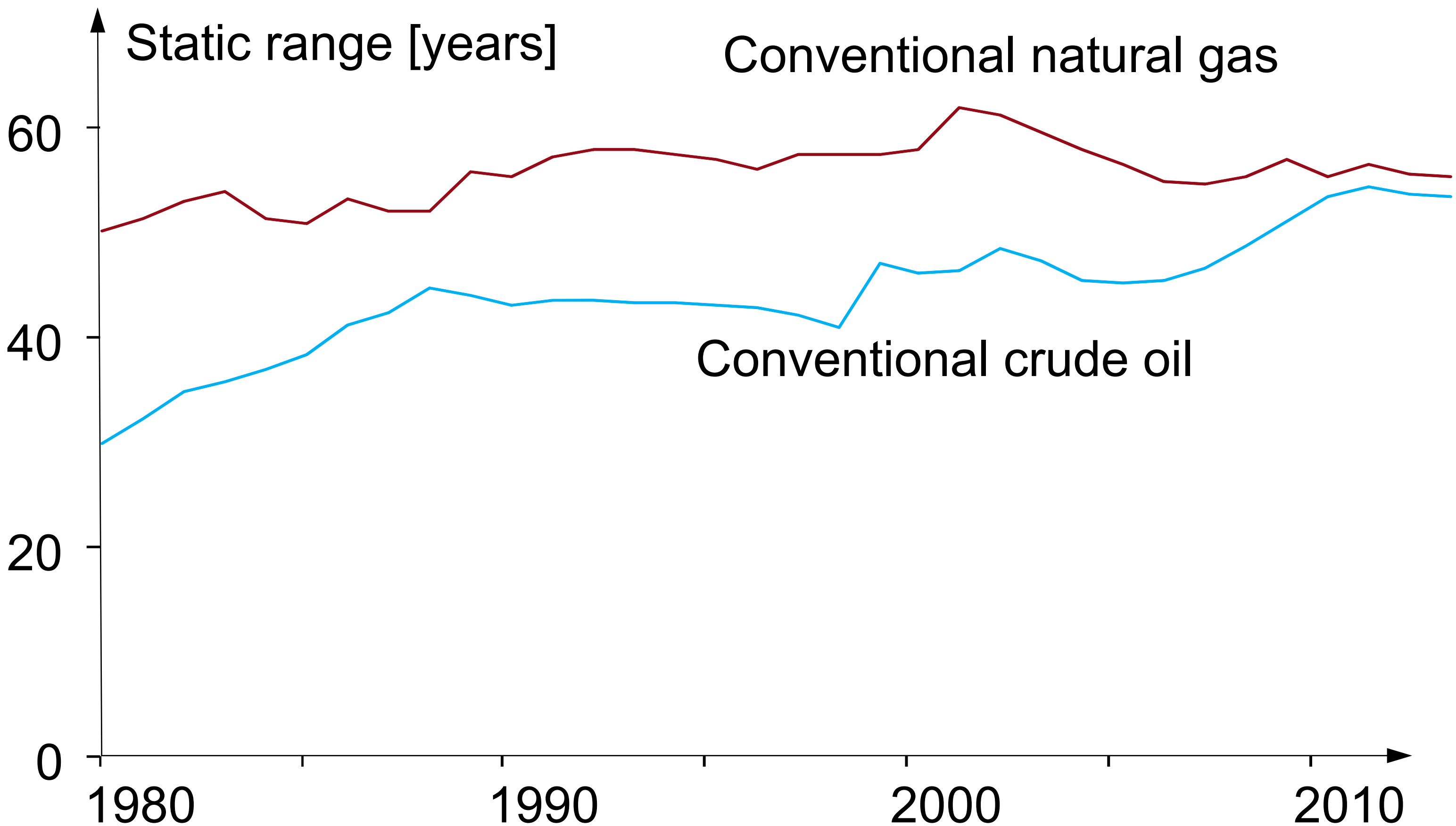
Estimated Ultimative Recovery

[Source: BGR 2004]



Static Range of Conventional Oil and Gas

[90% probability; Source BP Statistical Review of World Energy]



Global Potential of Renewable Energies

[Source: IIASA 2012]

	Theoretical potential [EJ/yr]	Technical potential [EJ/yr]	Used potential 2005 [EJ]
Biomass, solid waste	2’200	160–270	46.3
Hydro	200	50–60	11.7
Geothermal	1’500	810–1’545	2.3
Wind	110’000	1’250–2’250	1.3
Solar	3’900’000	62’000–280’000	0.5
Ocean	1’000’000	3’240–10’500	–
Share of renewables			13%

The data are energy-inputs, not outputs. Considering technology-specific conversion factors greatly reduces the output potentials.
Source: GEA, 2012: *Global Energy Assessment - Toward a Sustainable Future*, Cambridge University Press, Technical Summary (www.iiasa.ac.at/web/home/research/Flagship-Projects/Global-Energy-Assessment/Home-GEA.en.html)

Resource Extraction: Hotelling's Rule

- Basic assumptions
  - Perfectly Competitive Markets
  - Resource Owners:
    - Profit Maximizing Behavior
    - Constant marginal extraction costs *c*
    - Perfect information about the finite resource stock *S*

Basic Decision Problem of the Resource Owner:

- The market Price p_t cannot be influenced by the resource owner ("price taker"), therefore she just adjusts the extraction rate R_t in each period t
- Profit, Π_t, in each period follows as:

$$\Pi_t = p_t R_t - c R_t$$

- To extract, or not to extract?

Resource Extraction: Hotelling's Rule

- If the profit in the next period, Π_{t+1}, is greater than the Profit in the current period times the discount factor (Π_t*(1+i)), **we do not extract**

$$\Pi_{t+1} = p_{t+1}R_{t+1} - cR_{t+1} > \Pi_t(1+i)$$

- If the profit in the next period, Π_{t+1}, is less than the Profit in the current period times the discount factor (Πt*(1+i)), we extract (and put the profit on a bank to earn interest Π_t *(1+i))

$$\Pi_{t+1} = p_{t+1}R_{t+1} - cR_{t+1} < \Pi_t(1+i)$$

- If all resource owners behave profit maximizing, they adjust their extraction rates until:

$$\Pi_{t+1} = p_{t+1}R_{t+1} - cR_{t+1} = \Pi_t(1+i)$$

Resource Extraction: Hotelling's Rule

- The resource owners maximize the Net Present Values of profits by adjusting the extraction rates each period:

$$NPV = \sum_{t=0}^{T} \Pi_t \cdot (1+i)^{-t} = \sum_{t=0}^{T} (p_t R_t - c R_t)(1+i)^{-t} \rightarrow \max!$$

- Extraction is constrained by the available resource stock, S and hence:

$$\sum_{t=0}^{T} R_t = S$$

- With Lagrange-Multiplier, λ>0, we introduce the constraint into the objective function:

$$L = \sum_{t=0}^{T} (p_t R_t - c R_t)(1+i)^{-t} - \lambda \left(\sum_{t=0}^{T} R_t - S \right) \rightarrow \max!$$

- The first order optimality conditions are:

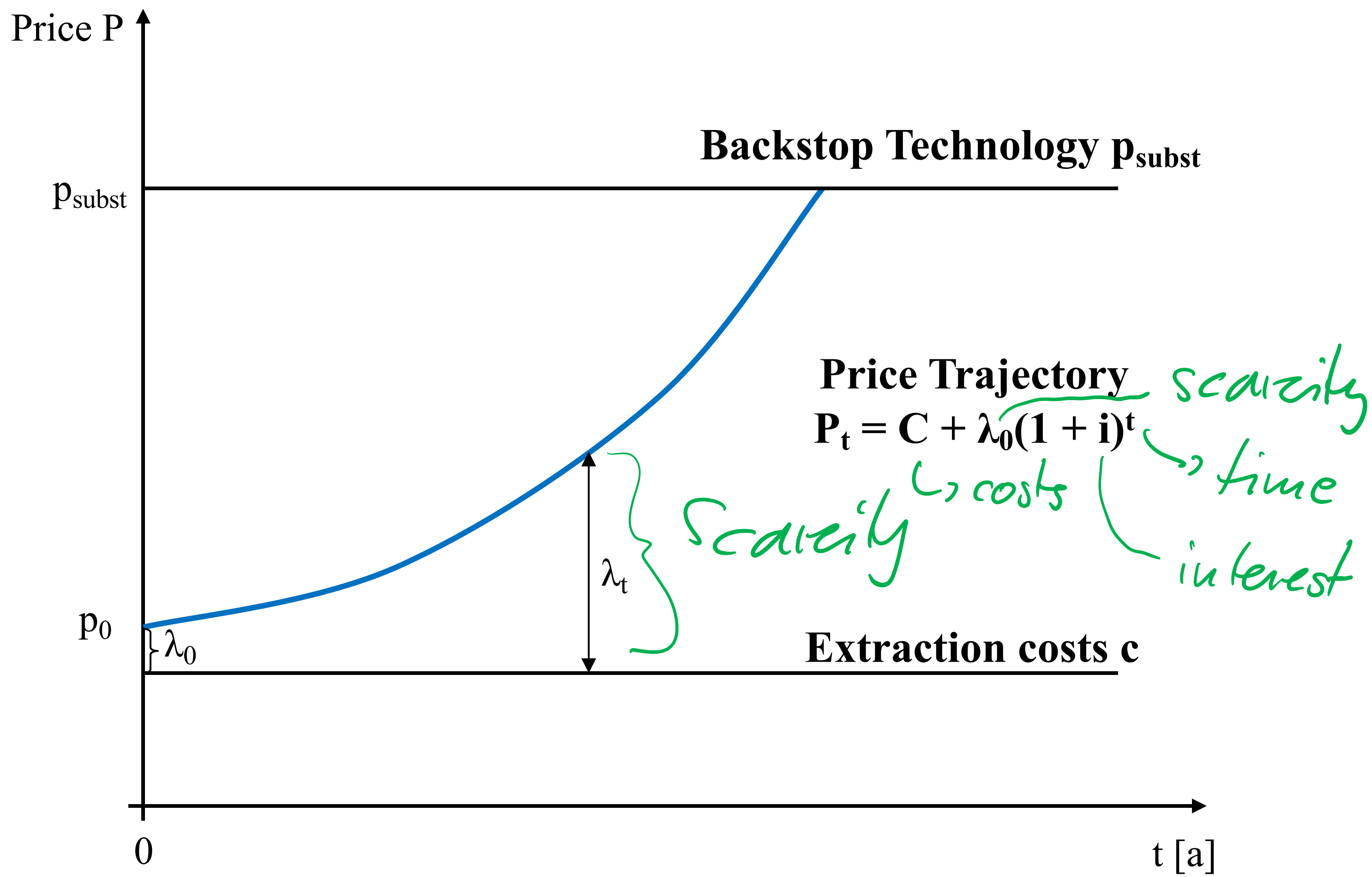
$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_{t=0}^{T} R_t = S \qquad (1) \qquad \frac{\partial L}{\partial R_t} = (p_t - c)(1+i)^{-t} - \lambda = 0 \qquad (2)$$

$$\boxed{\Rightarrow p_t = c + \lambda (1+i)^{t} \quad \text{Hotelling Rule}}$$

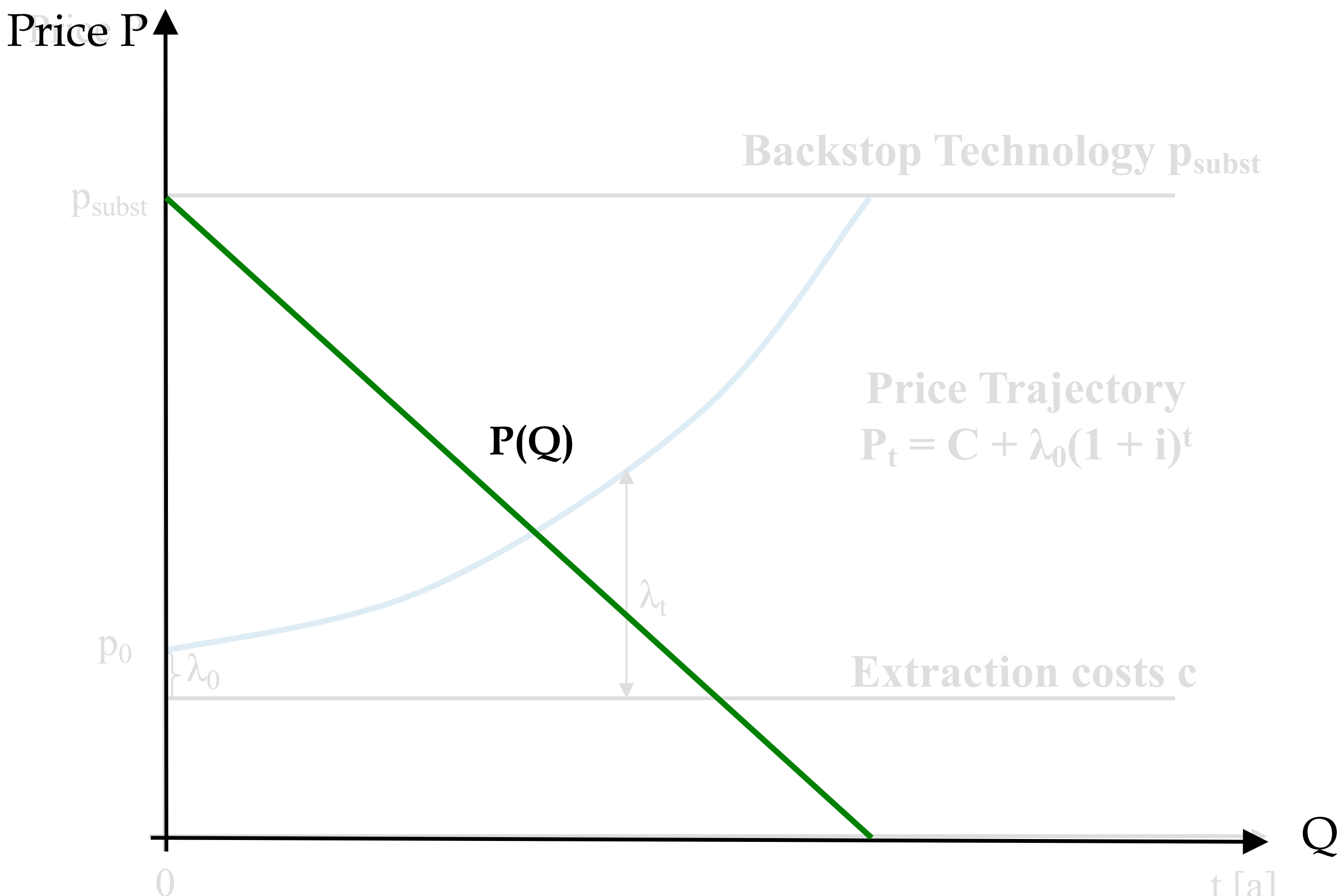
Resource Extraction: Hotelling's Rule: Price Formation

- Scarcity rent (*Knappheitsrente*) λ_t: $\lambda_t = p_t - c =$ *value of reserve*
- Decision criterion in $t = 0$: with the capital market interest rate i) $\lambda_1 < \lambda_0 (1+i)$ or $\lambda_1 > \lambda_0 (1+i)$
- Equilibrium (indifference condition): $\lambda_1 = \lambda_0 (1+i)$
- Trajectory of the resource rent: $\lambda_t = \lambda_0 (1+i)^t$
- Price trajectory under constant extraction cost c: $p_t = c + \lambda_t = c + \lambda_0 (1+i)^t$
- Resource rent at exhaustion $T > t$: with the backstop technology price p_{subst} $\lambda_T = p_{subst} - c$
- Optimal trajectory for $t < T$: $\lambda_t = \lambda_T (1+i)^{t-T} = (p_{subst}-c) (1+i)^{t-T}$

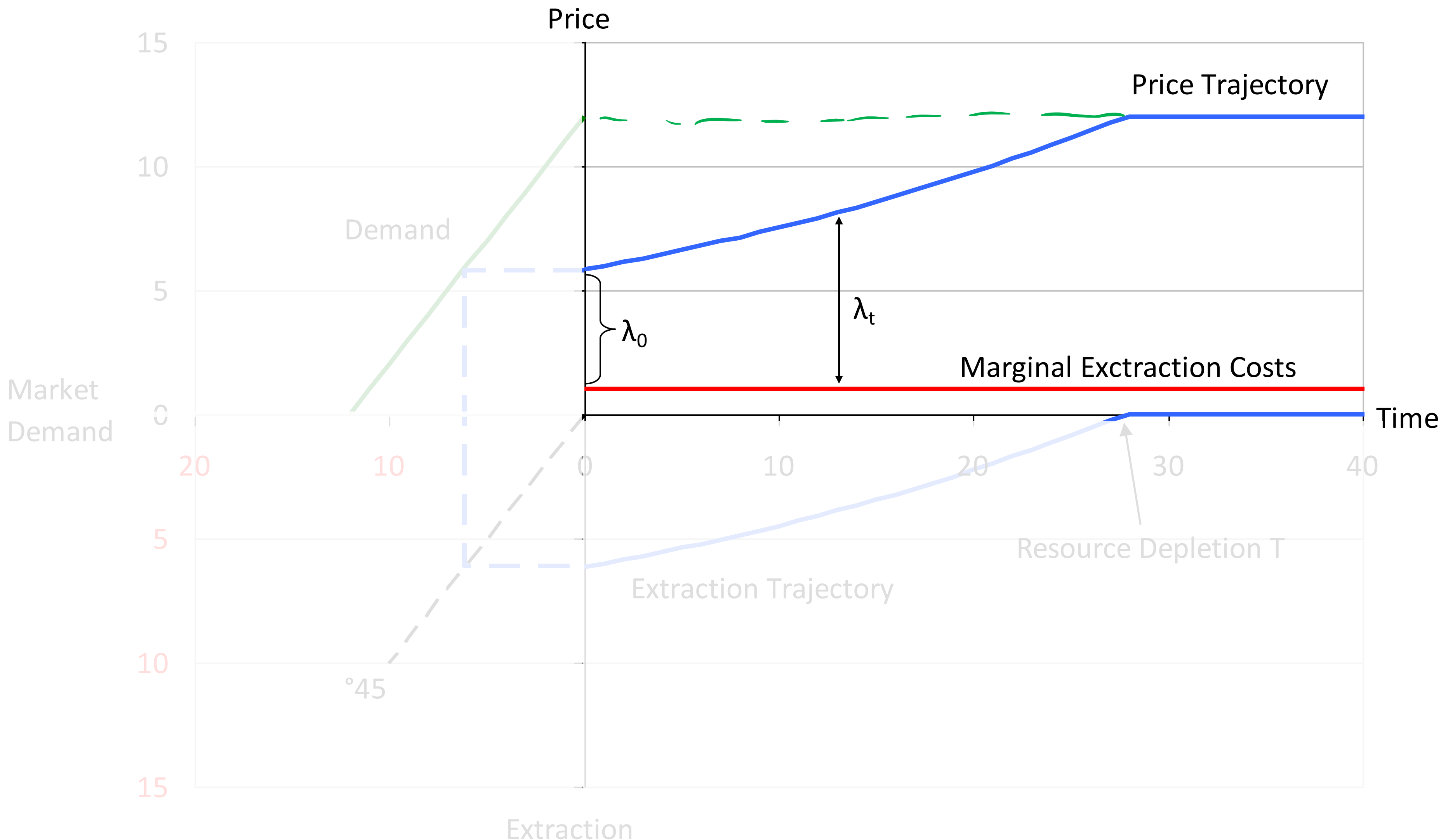
Hotelling Price Trajectory



Hotelling Price Trajectory: Demand P(Q)



Task 2) Hotelling‘s Rule



Task 1) Resources

a) The current market price for oil is 60 US$/bbl. The marginal extraction costs for oil (MC_{Oil}) are given by the following function: $MC_{Oil}(x) = \frac{1}{30} \cdot x + 10$, where $MC_{Oil}(x)$ are the extraction costs in [US$/bbl] and x is the amount of oil extracted in billion barrels. The entire stock of oil is 2 000 billion barrels. Given this information, how much of the stock of oil can exactly be classified as resource and how much as reserve? Please state reasons for your answer.

- resource = entire amount of oil $\Rightarrow 2000 \cdot 10^9$ bbl
- reserve = attainable at current market price (with current technology)

$$P = MC$$

$$60 \frac{\$}{bbl} = \left(\frac{1}{30} \cdot x + 10\right) \frac{\$}{bbl}$$

$$x = 1500 \cdot 10^9 \text{ bbl} \quad \text{reserves}$$

Task 1) Resources

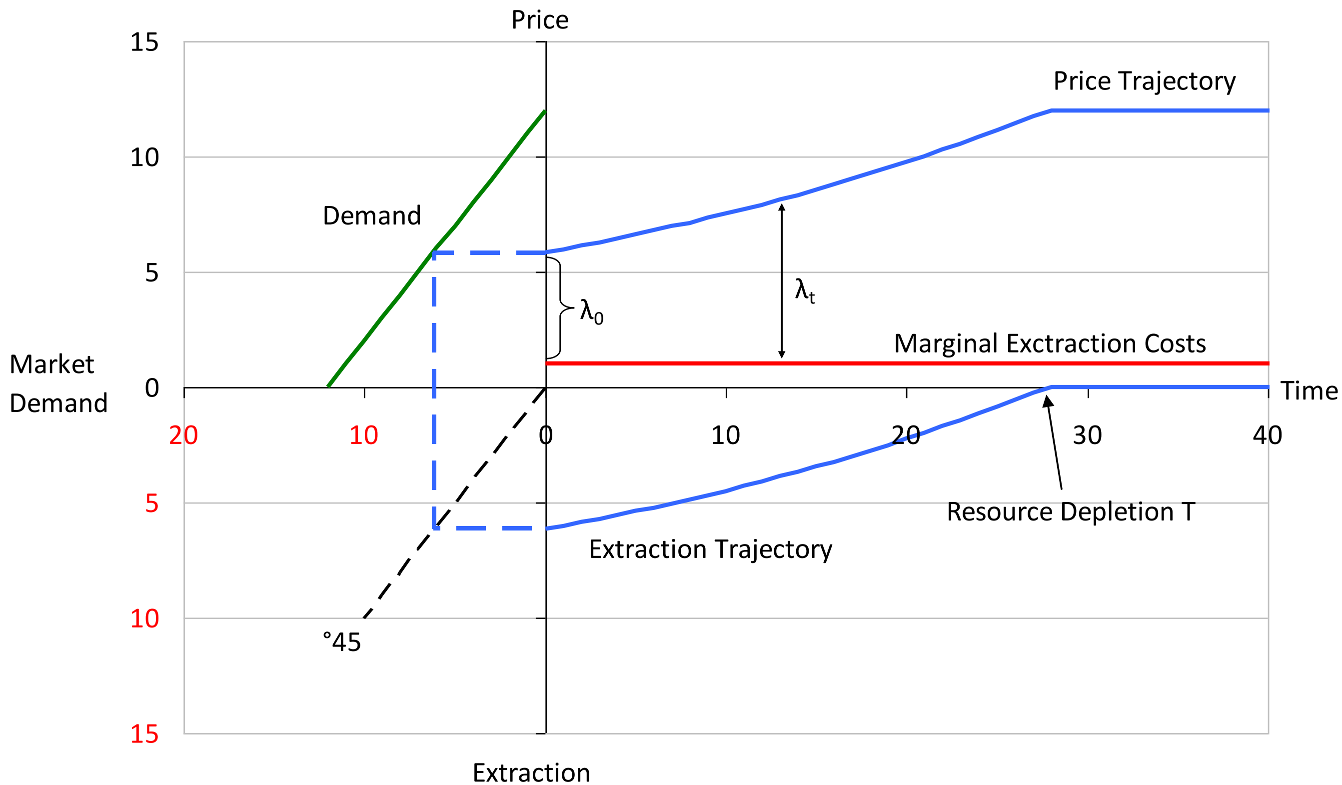
b) According to the BP Statistical Review of World Energy the annual oil consumption in 2015 was 34 billion barrels, whereas the proven oil reserves amounted to 1700 billion barrels in the same year. Please calculate the static range of the oil reserves based on the data given.

$$\text{range} = \frac{\text{Stock}}{\text{annual extraction}} = \frac{1700 \cdot \cancel{10^9} \, \cancel{bbl}}{34 \cdot \cancel{10^9} \, \frac{\cancel{bbl}}{a}}$$

$$= 50 \, a$$

Task 2) Hotelling‘s Rule

Consider the graphical illustration of the Hotelling’s rule below and answer the following questions:



a) What is the impact of increasing / decreasing interest rates?
b) What is the impact of increasing / decreasing marginal extraction costs?
c) What is the impact of increased demand?
d) What is the impact of a lowered backstop price? (Assumption: demand function does not change.)

Task 2) Hotelling's Rule

Consider the graphical illustration of the Hotelling's rule below and answer the following questions:

a) What is the impact of increasing / decreasing interest rates?

- increase in i (constant c, P_{subst}, S) leads to decreasing P_0, λ_0 and the depletion time

$P_t = c + \lambda (1+i)^t$ → increasing interest rate i

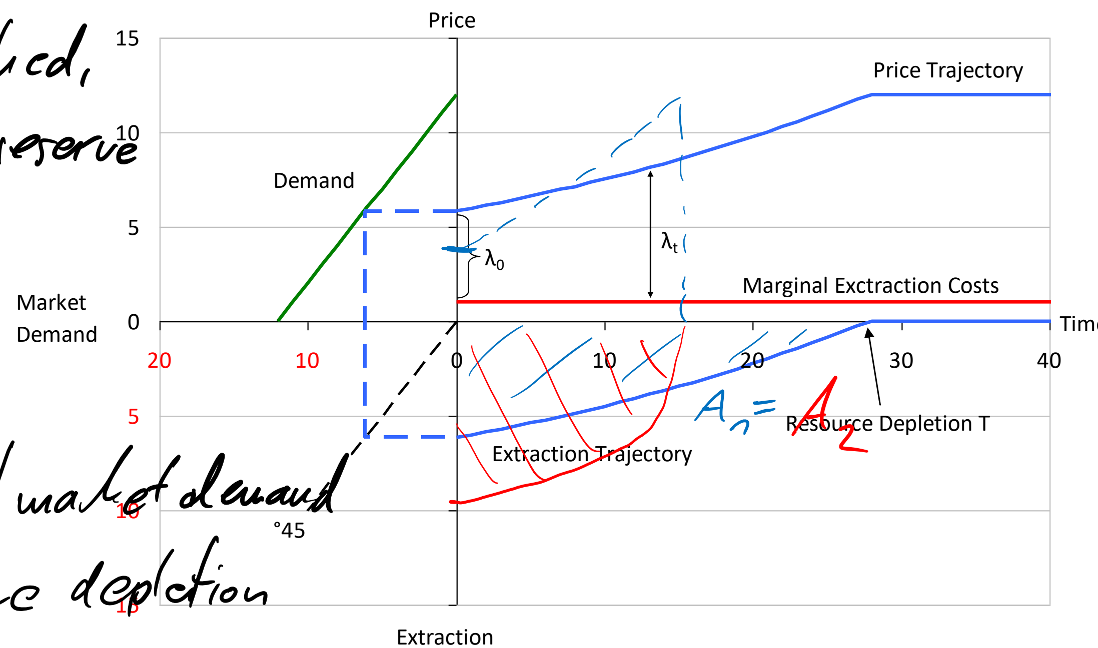
⇒ higher i ⇒ faster growing p ⇒ steeper slope

- when backstop price is reached, demand drops to zero and reserve is (not) depleted

⇒ initial price must be lower

↳ increasing extraction rate / market demand

↳ earlier reserve depletion



Task 2) Hotelling's Rule

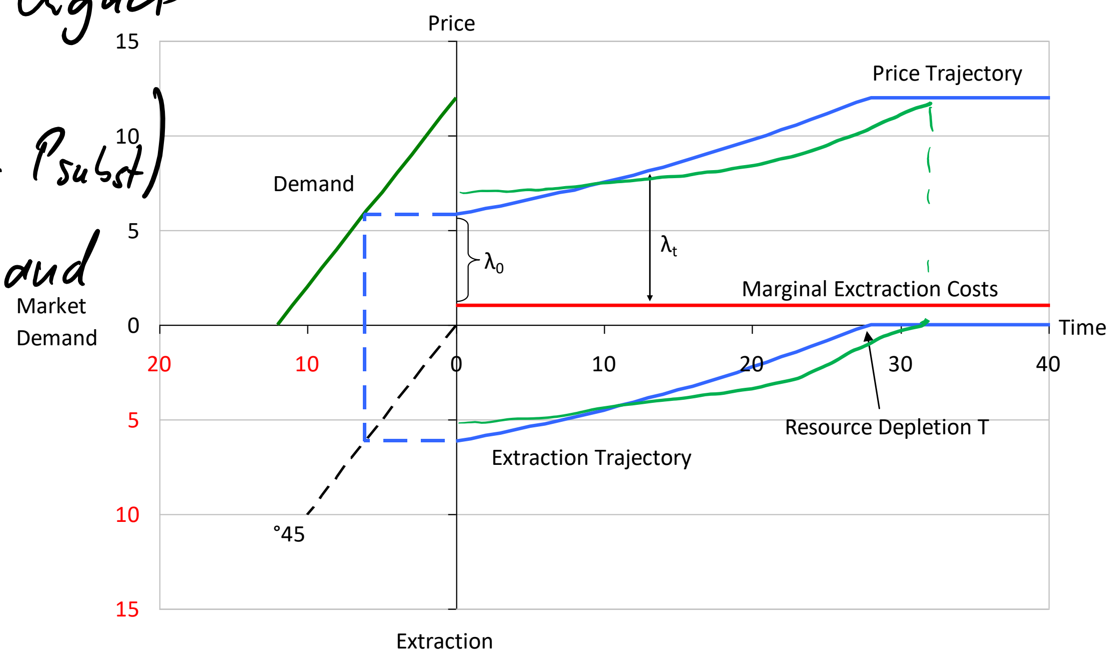
Consider the graphical illustration of the Hotelling's rule below and answer the following questions:

b) What is the impact of increasing / decreasing marginal extraction costs?

- $p_t = c \cdot \lambda (1+i)^t$ => increasing c
- with increasing extraction costs, initial scarcity rent decreases ($\lambda_0 = p_0 - c$)
  - → flatter price curve / slower increase
  - → stock is depleted before backstop is reached / demand drops to 0

↳ initial price has to be higher

- increasing c (constant i, p_{subst}) leads to decreasing λ_0 and increasing p_0 & T

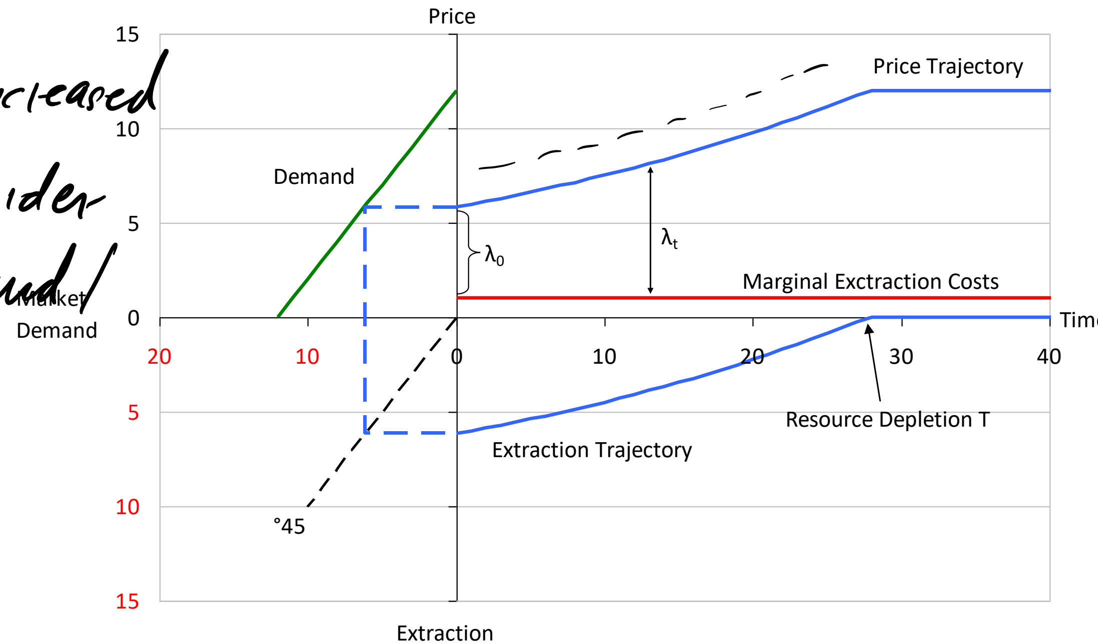


Task 2) Hotelling‘s Rule

Consider the graphical illustration of the Hotelling's rule below and answer the following questions:

c) What is the impact of increased demand?

- increased demand (at constant prices) would lead to an increased extraction
  ↳ resource depletion before we actually reach the maximum (backstop) price
- initial price path cannot be maintained
- initial price with an increased demand is higher in order to dampen the demand / extraction



Task 2) Hotelling's Rule

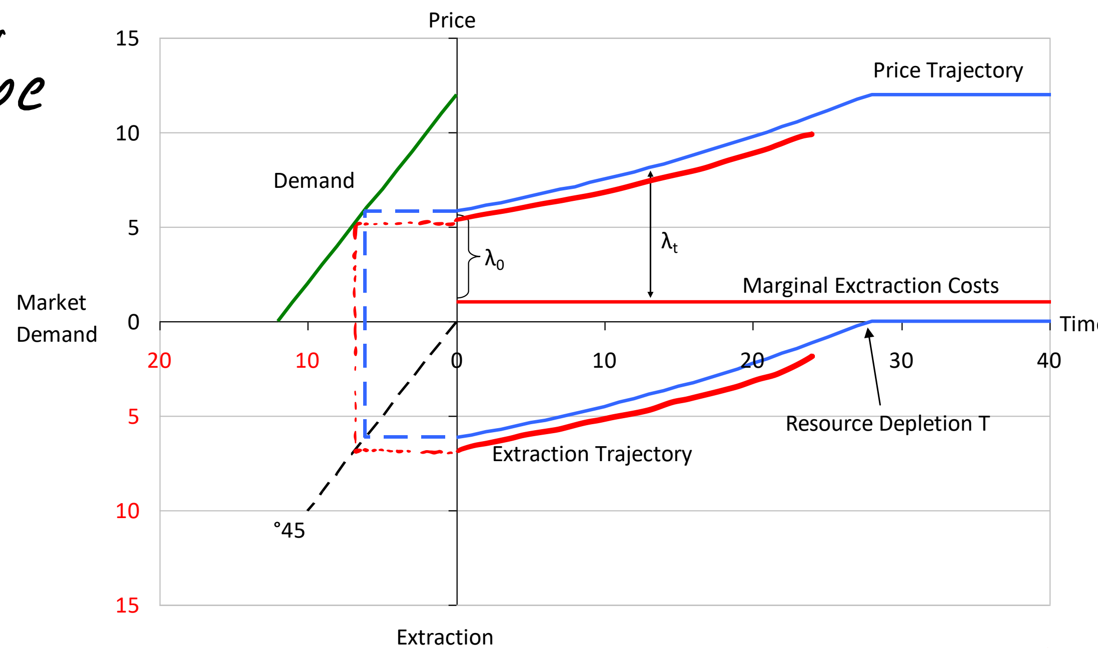
Consider the graphical illustration of the Hotelling's rule below and answer the following questions:

d) What is the impact of a lowered backstop price? (Assumption: demand function does not change.)

⇒ How can the backstop price drop suddenly? → subsidies → technological advancement

- lower backstop price at the original path
  → backstop price reached earlier
    ↳ demand is zero before resource is depleted

⇒ initial price has to be lower



Task 3) Hotelling‘s Rule

The marginal extraction cost c of an exhaustible resource is given by 1 $/unit. The demand function for this resource is given by $Q_t(p_t) = 12 - p_t$, where Q_t is the demand (in tons) and p_t is the price for the resource in period t. The general interest rate in the market is 5 %.

a) What is the backstop price in this example?
b) According to Hotelling’s rule, what will be the scarcity rent (Hotelling rent, resource rent) at the time the backstop price is reached?
c) Assume time to depletion is T = 30 years. What would be the price of the resource in t = 0?
d) Assume the scarcity rent in t = 0 is given by 3.7 $/unit. How long does is take until the resource is depleted?

Task 3) Hotelling's Rule

The marginal extraction cost c of an exhaustible resource is given by 1 $/unit. The demand function for this resource is given by $Q_t(p_t) = 12 - p_t$, where Q_t is the demand (in tons) and p_t is the price for the resource in period t. The general interest rate in the market is 5 %.

a) What is the backstop price in this example?

at backstop price the demand is zero

$$Q_t(p_t) = 0 \overset{!}{=} 12 - p_t$$

$$p_{subst} = 12 \frac{\$}{unit}$$

Task 3) Hotelling‘s Rule

The marginal extraction cost c of an exhaustible resource is given by 1 $/unit. The demand function for this resource is given by $Q_t(p_t) = 12 - p_t$, where Q_t is the demand (in tons) and p_t is the price for the resource in period t. The general interest rate in the market is 5 %.

b) According to Hotelling’s rule, what will be the scarcity rent (Hotelling rent, resource rent) at the time the backstop price is reached?

$$P_t = c + \lambda_t \qquad \lambda_t = P_t - c$$

$$\lambda_t \text{(resource depletion)} = P_{subst} - c$$

$$\lambda_t = (12 - 1)\frac{\$}{unit} = 11\frac{\$}{unit}$$

Task 3) Hotelling's Rule

The marginal extraction cost c of an exhaustible resource is given by 1 $/unit. The demand function for this resource is given by $Q_t(p_t) = 12 - p_t$, where Q_t is the demand (in tons) and p_t is the price for the resource in period t. The general interest rate in the market is 5 %.

c) Assume time to depletion is T = 30 years. What would be the price of the resource in t = 0?

$$P_0 = c + \lambda_0$$

$$\lambda_t = \lambda_0 (1+i)^t$$

$$\lambda_0 = \frac{\lambda_t}{(1+i)^t}$$

$$P_0 = c + \frac{\lambda_t}{(1+i)^t} = 1 + \frac{11}{(1{,}05)^{30}} = 3{,}55 \frac{\$}{\text{unit}}$$

Task 3) Hotelling's Rule

The marginal extraction cost c of an exhaustible resource is given by 1 \$/unit. The demand function for this resource is given by $Q_t(p_t) = 12 - p_t$, where Q_t is the demand (in tons) and p_t is the price for the resource in period t. The general interest rate in the market is 5 %.

d) Assume the scarcity rent in t = 0 is given by 3.7 \$/unit. How long does is take until the resource is depleted?

$$\lambda_0 = 3{,}7 \frac{\$}{\text{unit}}$$

$$P_t = \underline{P_{subst}} = 12 \frac{\$}{\text{unit}} = c + \lambda_t = \underline{c + \lambda_0 (1+i)^T}$$

$$T = \frac{\ln\left(\frac{P_{subst} - c}{\lambda_0}\right)}{\ln(1+i)} = \frac{\ln\left(\frac{12-1}{3{,}7}\right)}{\ln(1{,}05)}$$

$$T = 22{,}3\,a$$

Task 4)

You are the owner of two oil fields of which you can only recover one at a time due to financial bottlenecks. The current market interest rate is 8 percent. It is expected that oil prices will rise only moderately by 1 $/bbl. per year / per time period

[US$/bbl]	Field A	Field B
Current price	25	25
Recovering costs	10	20

a) At which field would you start to recover?

b) You get a different market analysis which forecasts an increase in prices of 2 $/bbl. Does this analysis change your decision?

Give reasons for your answers.

[US$/bbl]	Field A (extract)	Field B
Current price	25	25
Recovering costs	10	20

Task 4)

You are the owner of two oil fields of which you can only recover one at a time due to financial bottlenecks. The current market interest rate is 8 percent. It is expected that oil prices will rise only moderately by 1 $/bbl.

a) At which field would you start to recover?

1 Ressource ? Reserve ?

MP > MC

2 extract or not extract

$$\pi_{t+1} \overset{?}{<>} \pi_t (1+i)$$

we assume: $R_t = R_{t+1}$

Field A

$$p_{t+1} \cdot R_{t+1} - c_{t+1} \cdot R_{t+1} \overset{?}{<>} (p_t \cdot R_t - c_t \cdot R_t)(1+i)$$

$$p_{t+1} - c_{t+1} \overset{?}{<>} (p_t - c_t)(1+i)$$

$$(25+1) - 10 \overset{?}{<>} (25-10)(1+1{,}08)$$

$$16 < 16{,}2 \Rightarrow \text{extract!}$$

profit tomorrow < profit today + interest

[US$/bbl]	Field A	Field B
Current price	25	25
Recovering costs	10	20

Task 4)

Field B

$$(25+1)-20 \overset{?}{<>} (25-20)\cdot 1{,}08$$

$$6 > 5{,}4 \Rightarrow \text{not extract!}$$

[US$/bbl]	Field A	Field B
Current price	25	25
Recovering costs	10	20

Task 4)

You are the owner of two oil fields of which you can only recover one at a time due to financial bottlenecks. The current market interest rate is 8 percent. It is expected that oil prices will rise only moderately by 1 $/bbl.

b) You get a different market analysis which forecasts an increase in prices of 2 $/bbl. Does this analysis change your decision?

	A	B
• scarcity rent$_t$	$25 - 10 = 15$	$25 - 20 = 5$
• scarcity rent$_{t+1}$	$27 - 10 = 17$	$27 - 20 = 7$
• % scarcity rent	$\frac{17 - 15}{15} = 13{,}3\,\%$	$\frac{7 - 5}{5} = 40\,\%$

$\Rightarrow$ scarcity rent increase is bigger than the interest rate in both cases

$\rightarrow$ no extract from either field