T. Brown
M. Schäfer

Electricity Markets
2. Exercise Sheet Solutions
25.04.2016

Exercise 1: Investment versus marginal costs (*)

1. Number of hours of lighting over 5 years: (4h/d)*(5a)*(365d/a) = 7300h.

   For Bulb A cost is (€0.3/kWh) * (0.015 W) * 7300 h = €32.85.

   For Bulb B cost is (€0.3/kWh) * (0.025 W) * 7300 h = €54.75.

2. Bulb B must be at least €21.90 cheaper than Bulb A (i.e. the difference in operating costs over 5 years) before the overall costs of Bulb B are lower. So buying expensive lightbulbs with high efficiency often makes sense...

   NB: With a non-zero discount rate, the difference is smaller, since we have to price in the lost revenue from investing our capital elsewhere - but such considerations are overkill for such small investment decisions.

Exercise 2: Shadow prices of limits on consumption

We convert the exercise to an optimisation problem with objective

$$\max_q U(q) - \pi q \tag{2.1}$$

with constraints

$$q \leq q_{max} \qquad \leftrightarrow \qquad \mu_{max} \tag{2.2}$$

$$-q \leq -q_{min} \qquad \leftrightarrow \qquad \mu_{min} \tag{2.3}$$

From stationarity we get:

$$\begin{aligned} 0 &= \frac{\partial}{\partial q}\left(U(q) - \pi q\right) - \mu_{max}\frac{\partial}{\partial q}(q - q_{max}) - \mu_{min}\frac{\partial}{\partial q}(-q + q_{min}) \\ &= U'(q) - \pi - \mu_{max} + \mu_{min} \end{aligned} \tag{2.4}$$

1. The marginal utility curve is $U'(q) = 70 - 6q$ [€/Mwh]. At $\pi = 5$, the demand would be determined by $5 = 70 - 6q$, i.e. $q = 65/6 = 10.8333$, which is above the consumption limit $q_{max} = 10$. Therefore the optimal demand is $q^* = 10$, the upper limit is binding $\mu_{max} \geq 0$ and the lower limit is non-binding $\mu_{min} = 0$.

   To determine the value of μ_{max} we use (2.4) to get $\mu_{max} = U'(q^*) - \pi = U'(10) - 5 = 5$.

2. At $\pi = 60$, the demand would be determined by $60 = 70 - 6q$, i.e. $q = 10/6 = 1.667$, which is below the consumption limit $q_{min} = 2$. Therefore the optimal demand is $q^* = 2$, the upper limit is non-binding $\mu_{max} = 0$ and the lower limit is binding $\mu_{min} \geq 0$.

   To determine the value of μ_{min} we use (2.4) to get $\mu_{min} = \pi - U'(q^*) = 60 - U'(2) = 2$.

Exercise 3: Revenue, profit and consumer surplus

1. The system marginal price is \$ 16/MWh, so for the generators Notice

Company	Production [MWh]	Costs [$]	Revenue [$]	Profit [$]
Red 1	200	2400	3200	800
Red 2	50	750	800	50
Blue	100	1300	1600	300
Green	100	1600	1600	0
Total	450	6050	7200	1150

   that Green makes no profit.

   For the consumers

Company	Consumption [MWh]	Utility [$]	Expense [$]	Net Surplus [$]
Orange	200	5000	3200	1800
Yellow	100	2300	1600	700
Purple	150	3300	2400	900
Total	450	10600	7200	3400

2. If consumer company “Orange” withdraws its offers from the market, the market will clear at a lower price of \$ 13/MWh.

   The supply and demand meets in a line between 250 MWh and 300 MWh, which makes the final result somewhat ambiguous.

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Exercise 4: Generator constraints, transmission constraints and investment

Note that it is important in this example that the same company owns both the generators and the transmission line; if an independent TSO owned the transmission line, he could take the congestion revenue for himself.

1. If we label the dispatch of Generator 1 by q_1 and of Generator 2 by q_2, then the objective function is to maximise total profit

$$\max_{q_1,q_2} [\pi(q_1+q_2) - C_1(q_1) - C_2(q_2)] = \max_{q_1,q_2} [\pi(q_1+q_2) - 5q_1 - 10q_2] \tag{4.1}$$

The constraints are

$$q_1 \leq \hat{q}_1 \quad \leftrightarrow \quad \bar{\mu}_1 \tag{4.2}$$
$$-q_1 \leq 0 \quad \leftrightarrow \quad \underline{\mu}_1 \tag{4.3}$$
$$q_2 \leq \hat{q}_2 \quad \leftrightarrow \quad \bar{\mu}_2 \tag{4.4}$$
$$-q_2 \leq 0 \quad \leftrightarrow \quad \underline{\mu}_2 \tag{4.5}$$
$$q_1 + q_2 \leq K \leftrightarrow \quad \mu_T \tag{4.6}$$

Where the first four constraints come from generation, where $\hat{q}_1 = 300$ MW and $\hat{q}_1 = 900$ MW and the final constraint comes from the transmission, where $K = 1000$ MW is the capacity of the export transmission line.

2. Since the market price is always higher than the marginal price of the generators, they will both run as high as possible given the constraints. Since Generator 1 is cheaper than Generator 2, it will max-out its capacity first, so that $q_1^* = \hat{q}_1 = 300$ MW. Generator 2 will output as much as it can given the transmission constraint, so that $q_2^* = 700$ MW.

3. From stationarity we have for q_1 the non-zero terms:

$$\begin{aligned} 0 &= \frac{\partial}{\partial q_1}(\pi(q_1+q_2) - 5q_1 - 10q_2) - \bar{\mu}_1\frac{\partial}{\partial q_1}(q_1 - \hat{q}_1) - \underline{m}_1\frac{\partial}{\partial q_1}(-q_1) - \mu_T\frac{\partial}{\partial q_1}(q_1+q_2-K) \\ &= \pi - 5 - \bar{\mu}_1 + \underline{\mu}_1 - \mu_T \end{aligned} \tag{4.7}$$

For q_2 we have

$$\begin{aligned} 0 &= \frac{\partial}{\partial q_2}(\pi(q_1+q_2) - 5q_1 - 10q_2) - \bar{\mu}_2\frac{\partial}{\partial q_2}(q_2 - \hat{q}_2) - \underline{m}_2\frac{\partial}{\partial q_2}(-q_2) - \mu_T\frac{\partial}{\partial q_2}(q_1+q_2-K) \\ &= \pi - 10 - \bar{\mu}_2 + \underline{\mu}_2 - \mu_T \end{aligned} \tag{4.8}$$

At the optimal point we can see that $\underline{\mu}_1$, $\bar{\mu}_2$ and $\underline{\mu}_2$ are non-binding, so these are zero. To solve for μ_T and $\bar{\mu}_1$ we have two equations:

$$\begin{aligned} 0 &= \pi - 5 - \bar{\mu}_1 - \mu_T \\ 0 &= \pi - 10 - \mu_T \end{aligned} \tag{4.9}$$

Therefore

$$\mu_T = \pi - 10 \tag{4.10}$$

$$\bar{\mu}_1 = 5 \tag{4.11}$$

4. The value of $\bar{\mu}_1$ gives us the increase in profit for a small increase in $\hat{q}_1$. We want to understand a large increase in $\hat{q}_1$ of 50 MW, therefore we have to integrate over $\bar{\mu}_1$ as a function of $\hat{q}_1$, since the value of $\bar{\mu}_1$ may change as $\hat{q}_1$ changes. The total increase in profitability for expanding $\hat{q}_1$ from 300 MW to 350 MW is then

$$\int_{300}^{350} \bar{\mu}_1(\hat{q}_1) d\hat{q}_1 \tag{4.12}$$

Because of the linearity of the problem, $\bar{\mu}_1$ is actually constant as we expand $\hat{q}_1$ in the region from 300 MW to 350 MW. The extra profit would be per year: 5 €/MWh * 50 MW * 8760h/a = €2.19 million/a. At or below this annualised capital cost, it would be worth investing.

5. Here μ_T changes as K is expanded, so we have to integrate:

$$\int_{1000}^{1200} \mu_T(K) dK \tag{4.13}$$

Since μ_T is constant as we expand K from 1000 MW to 1200 MW, the extra profit would be per year: (average(π)-10) €/MWh * 200 MW * 8760h/a = €17.52 million/a. At or below this annualised capital cost, it would be worth investing.

NB: An extension beyond 1200 MW would not bring anything, because the generator constraints would be then binding.