Electricity Markets
T. Brown 2. Exercise Sheet
M. Schäfer 25.04.2016

Exercise 1: Investment versus marginal costs (*)

You would like to buy a lightbulb to light a room in your house for 4 hours a day. Two lightbulbs are on the market, each with a lifetime of 5 years and the same light output. Bulb A has an electricity consumption of 15 W and Bulb B has an electricity consumption of 25 W. The electricity price is €0.3/kWh and is assumed to be constant. Assume a discount rate of 0% for simplicity.

1. For each lightbulb, calculate how much the electricity will cost for your lighting needs over the 5 year lifetimes of the bulbs.
2. How much cheaper than Bulb A does Bulb B have to be before the total costs of buying and operating the lightbulb over 5 years are lower with Bulb B?

Exercise 2: Shadow prices of limits on consumption

Consider Exercise 4 from Exercise Sheet 1, where an industrial company has utility function

$$U(q) = 70q - 3q^2 [€/h] \quad , \quad q_{min} = 2 \leq q \leq q_{max} = 10,$$

where q is the demand in MW and q_{min}, q_{max} are the minimum and maximum demand.

1. Suppose the company is maximising its net surplus for a given electricity price π, i.e. it maximises $\max_q [U(q) - \pi q]$. If the price is $\pi = 5$ €/MWh, what is the optimal demand q^*? What is the value of the KKT multiplier μ_{max} for the constraint $q \leq q_{max} = 10$ at this optimal solution? What is the value of μ_{min} for $q \geq q_{min} = 2$?
2. Suppose now the electricity price is $\pi = 60$ €/MWh. What are the optimal demand q^*, μ_{max} and μ_{min} now?

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Exercise 3: Revenue, profit and consumer surplus

Consider the example of bids and offers in an electricity market from slides 28 to 31 from Lecture 2 on 18.04.2016 (taken from the book Kirschen and Strbac pages 56-58).

1. Compute the revenue and profit of each generating company and the net surplus of each consuming company.
2. If consumer company "Orange" withdraws its offers from the market, how do the market price and the profits/surpluses of generators and consumers change?

Exercise 4: Generator constraints, transmission constraints and investment

Two generators are connected to the grid by a single transmission line (with no electrical demand on their side of the transmission line). A single company owns both the generators and the transmission line. Generator 1 has a linear cost curve $C(q) = 5q$ [€/h] and a capacity of 300 MW and Generator 2 has a linear cost curve $C(q) = 10q$ [€/h] and a capacity of 900 MW. The transmission line has a capacity of 1000 MW.
Suppose the demand in the grid is always high enough to absorb the generation from the two generators and that the market price of electricity π is never below 15 €/MWh and averages 20 €/MWh.

1. Determine the full set of equations (objective function and constraints) for the generators to optimise their dispatch to maximise total economic welfare.
2. What is the optimal dispatch?
3. What are the values of the KKT multipliers for all the constraints in terms of π?
4. A new turbo-boosting technology can increase the capacity of Generator 1 from 300 MW to 350 MW. At what annualised capital cost would it be efficient to invest in this new technology?
5. A new high temperature conductor technology can increase the capacity of the transmission line by 200 MW. At what annualised capital cost would it be efficient to invest in this new technology?