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Electricity Markets
3. Exercise Sheet Solutions
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Exercise 1: Feed-In Tariffs and Investment in Renewable Generators

1. Average output: 35.67%
2. Annual revenue: 0.3567×200 MW$\times$ €70/MWh$\times$ 8760 h = €43.7 million
3. Annual payment for the loan: 20000 kW$\times$ €1100/kW$\times$annuity = €23.3 million
4. Yearly profit: revenue - costs = €20.4 million
5. Annual payment with interest rate 20%: €44.2 million
6. Rate of return (20% own capital, interest rate 10%): investment €176 million, annuity €18.66 million, rate of return 10.5%

Exercise 2: Utilization Factors, Variable and Fixed Costs

Solve $10Q + 15 = 25Q + 5$, i.e. $Q = 2/3$

Exercise 3: Feasible Network Transactions

PTDF matrix:

$$H = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} 0 & -\frac{4}{5} & -\frac{2}{5} \\ 0 & -\frac{1}{5} & -\frac{3}{5} \\ 0 & \frac{1}{5} & -\frac{2}{5} \end{pmatrix}$$

Set 1 (all in MW): $Z_1 = -100$, $Z_2 = 200$, $Z_3 = -100$
Set 2 (all in MW): $Z_1 = 400$, $Z_2 = 400$, $Z_3 = -800$
Set 3 (all in MW): $Z_1 = -100$, $Z_3 = -300$, $Z_3 = 400$

	$F_{1\to2}$ (MW)	$F_{1\to3}$ (MW)	$F_{2\to3}$ (MW)	Feasible?
Set 1	-120	20	80	Yes
Set 2	0	400	400	No
Set 3	80	-180	-220	Yes

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Exercise 4: Two-bus power system

1. Price λ_i, production Q_i^S, flow F.
   (a) $\lambda_A = 80$ €/MWh, $\lambda_B = 35$ €/MWh, $Q_A^S = 2000$ MW, $Q_B^S = 1000$ MW, $F = 0$
   (b) $\lambda_A = 53$ €/MWh, $\lambda_B = 53$ €/MWh, $Q_A^S = 1100$ MW, $Q_B^S = 1900$ MW, $F = -900$ MW
   (c) $\lambda_A = 65$ €/MWh, $\lambda_B = 65$ €/MWh, $Q_A^S = 1500$ MW, $Q_B^S = 1500$ MW, $F = -500$ MW
   (d) $\lambda_A = 57$ €/MWh, $\lambda_B = 57$ €/MWh, $Q_A^S = 900$ MW, $Q_B^S = 2100$ MW, $F = -1100$ MW
   (e) $\lambda_A = 62$ €/MWh, $\lambda_B = 47$ €/MWh, $Q_A^S = 1400$ MW, $Q_B^S = 1600$ MW, $F = -600$ MW

2. Generator revenues R_i, generator costs C_i, generator profits P_i, consumer payments E_i. Find the generator profits by substracting the costs from the revenue. Costs are given by integrating the marginal cost, i.e. $C_A = 20Q + 0.015Q^2$ and $C_B = 15Q + 0.01Q^2$. The generator

Case	a	b	c	d	e
E_A (€)	160000	106000	130000	114000	124000
E_B (€)	35000	53000	65000	57000	47000
R_A (€)	160000	58300	97500	51300	86800
R_B (€)	35000	100700	97500	119700	75200
C_A (€)	100000	40150	63750	30150	57400
C_B (€)	25000	64600	45000	75600	49600
P_A (€)	60000	18150	33750	21150	29400
P_B (€)	10000	36100	52500	44100	25600

   at B and the consumers at A benefit from the line (price increases at B, decreases at B)

3. Congestion surplus 9000 €:

$$(E_A + E_B) - (R_A + R_B) = |F| \times (\lambda_A - \lambda_B)$$

Congestion surplus is equal to zero when the flow $F = 0$, or when it is equal to the unconstrained value $F = -900$ MW (then $\lambda_A = \lambda_B$).

Exercise 5: Another three-bus system

This question was moved to Sheet 4 and the solution can be found there.