T. Brown
M. Schäfer

Exercise 1: Feed-In Tariffs and Investment in Renewable Generators

The government of Syldavia has decided to encourage renewable power by guaranteeing to buy the output of wind power plants at a fixed price of € 70/MWh, a so-called Feed-In Tariff (FIT).
The company Syldavia Go Green is considering taking a loan to buy a 200 MW wind plant. The plant has an expected lifetime of 30 years and an investment cost of € 1100/kW. Based on an assessment of the plant site, the output of the plant will be:

Output as a fraction of capacity [%]	Hours per year
100	1700
75	1200
50	850
25	400
0	4610

The bank offers a loan for the entire project over the lifetime of the plant with an interest rate of 10%. The annualised payment for the loan is given by the annuity formula

$$a(C,n,r) = C\frac{r(1+r)^n}{(1+r)^n - 1}$$

where C is the loan amount, r is the per unit interest rate (i.e. 10% becomes 0.1) and n is the lifetime of the loan.

1. What is the average output of the plant over the year as a fraction of its capacity?
2. What is the annual revenue of the plant from the Feed-In Tariff?
3. What is the annual payment for the loan?
4. What is the yearly profit that the company makes from the plant?
5. How does the calculation change if the interest rate is 20%?

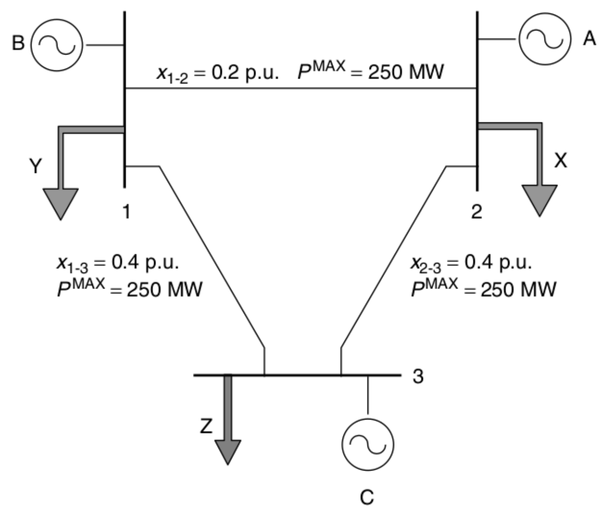
6. *Non-exam bonus question: In a more realistic scenario, the company uses its own capital to pay for 20% of the initial capital cost of the project. What is the company's rate of return on this investment, given the bank loan for the remaining 80% at an interest rate of 10%?

Exercise 2: Utilization Factors, Variable and Fixed Costs

Generator A has variable costs of € 10/MWh and fixed costs of € 15/MW/h.
Generator B has variable costs of € 25/MWh and fixed costs of € 5/MW/h.
Above which utilisation factor does generator A become cheaper to run than generator B?

Exercise 3: Feasible Network Transactions

Consider the following three-node power system:



The Power Transfer Distribution Factor (PTDF) of the network is given by

$$H = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 0 & -\frac{4}{5} & -\frac{2}{5} \\ 0 & -\frac{1}{5} & -\frac{3}{5} \\ 0 & \frac{1}{5} & -\frac{2}{5} \end{pmatrix}$$

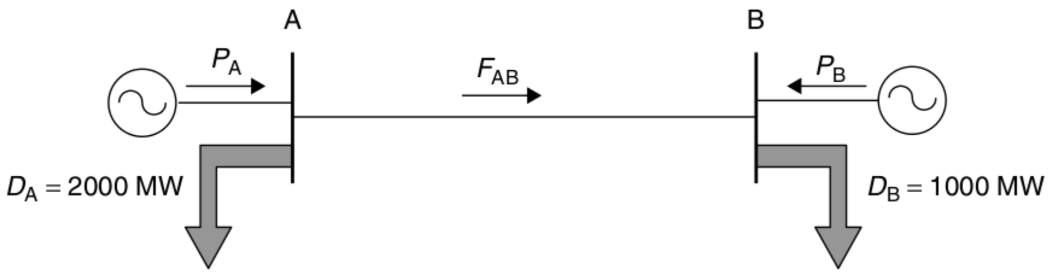
1. *Non-exam bonus question: Use Kirchoff's Voltage Law, described in Kirschen & Strbac 6.2.2.1, to derive the PTDF, using node 1 as the reference node.

2. Assuming that the only limitations imposed by the network are imposed by the thermal capacity of the transmission lines and that the reactive power flows are negligible, check that the following sets of transactions are simultaneously feasible:

	Seller	Buyer	Amount
Set 1	B	X	200
	A	Z	400
	C	Y	300
Set 2	B	Z	600
	A	X	300
	A	Y	200
	A	Z	200
Set 3	C	X	1000
	X	Y	400
	B	C	300
	A	C	200
	A	Z	100

Exercise 4: Two-bus power system

Consider the two-bus power system ("bus" is another word for "node"):



The marginal cost of production of the generators connected to buses A and B are given respectively by the following expressions:

$$MC_A = 20 + 0.03P_A \qquad \$/MWh$$
$$MC_B = 15 + 0.02P_B \qquad \$/MWh$$

Assume that the demand D_* is constant and insensitive to price, that energy is sold at its marginal cost of production and that there are no limits on the output of the generators.

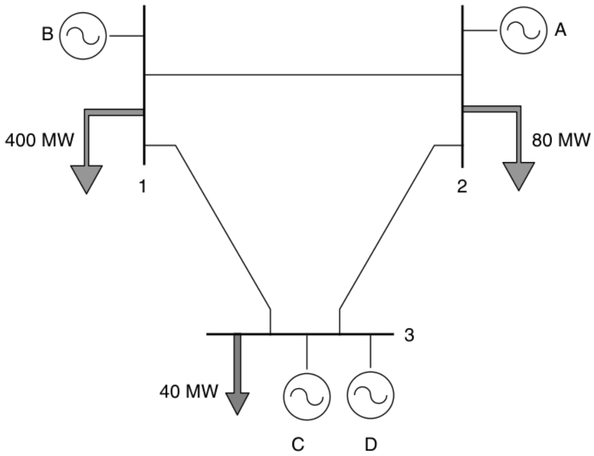
1. Calculate the price of electricity at each bus, the production of each generator, the flow on the line, and the value of any KKT multipliers for the following cases:
   (a) The line between buses A and B is disconnected.
   (b) The line between buses A and B is in service and has an unlimited capacity.
   (c) The line between buses A and B is in service and has an unlimited capacity, but the maximum output of Generator B is 1500 MW.
   (d) The line between buses A and B is in service and has an unlimited capacity, but the maximum output of Generator A is 900 MW. The output of Generator B is unlimited.
   (e) The line between buses A and B is in service but its capacity is limited to 600 MW. The output of the generators is unlimited.
2. Calculate the generator revenues, generator profits, consumer payments and consumer net surplus for all the cases considered in the above problem. Who benefits from the line connecting these two buses?
3. Calculate the congestion surplus for case (e). Check your answer using the results of Problem 2. For what values of the flow on the line between buses A and B is the congestion surplus equal to zero?

T. Brown
M. Schäfer

Electricity Markets
3. Exercise Sheet
23.05.2016

Exercise 5: Another three-bus system

Consider the three-bus system:



The generators have the following data:

Generator	Capacity [MW]	Marginal Cost [$/MWh]
A	150	12
B	200	15
C	150	10
D	400	8

1. Calculate the unconstrained economic dispatch and the market clearing price of the three-node system.

2. *Bonus non-exam question: Given the branch data:

Branch	Reactance [p.u.]	Capacity [MW]
1-2	0.2	250
1-3	0.3	250
2-3	0.3	250

confirm that, using bus 1 as the reference bus, the PTDF is:

$$H = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{8} \\ 0 & \frac{1}{4} & \frac{5}{8} \\ 0 & -\frac{1}{4} & \frac{3}{8} \end{pmatrix}$$

T. Brown
M. Schäfer

3. Calculate the flow that would result if the generating units were dispatch for the unconstrained case in Problem 1. Identify all the violations of security constraints.

4. Determine two ways of removing the constraint violations that you identified by redispatching generating units. Which redispatch is preferable?

5. Calculate the nodal prices for the three-bus system when the generating units have been optimally redispatched to relieve the constraint violations. Calculate the merchandising surplus and show that it is equal to the sum of the surpluses of each line.

6. Suppose now that the capacity of branch 1-2 is reduced to 140 MW while the capacity of the other lines remains unchanged. Calculate the optimal dispatch and the nodal prices for these conditions. [Hint: the optimal solution involves a redispatch of generating units at all three buses.]