T. Brown
M. Schäfer

Electricity Markets
4. Exercise Sheet Solutions
06.06.2016

Exercise 1: Another three-bus system (copied from 3. Exercise Sheet

This question is based on Kirschen & Strbac Ex. 6.6.

1. The total load is 520 MW, so for the unconstrained system we dispatch the cheapest generators first, resulting in $P_D = 400$ MW and $P_C = 120$ MW.

2. The Power Transfer Distribution Factor (PTDF) tells you how much power flows along each line for a transfer of power from a node to the reference node.

   Node 1 is the reference node, so a transfer of 1 MW from the reference node to itself has no effect on the flows; hence the first row of H is zero.

   If 1 MW flows from Node 2 to the reference Node 1, it has two paths: a flow of x MW on the path $2 \to 1$ with reactance 0.2 and a flow of $1-x$ MW on the path $2 \to 3 \to 1$ with total reactance $0.3+0.3 = 0.6$. By Kirchoff's Voltage Law the voltage drop along both paths must be the same, so $0.2x = 0.6(1-x)$, i.e. $x = \frac{3}{4}$. x and $(1-x)$ and the correct flow direction gives the second column in H.

   If 1 MW flows from Node 3 to the reference Node 1, it has two paths: a flow of x MW on the path $3 \to 1$ with reactance 0.3 and a flow of $1-x$ MW on the path $3 \to 2 \to 1$ with total reactance $0.3+0.2 = 0.5$. By Kirchoff's Voltage Law the voltage drop along both paths must be the same, so $0.3x = 0.5(1-x)$, i.e. $x = \frac{5}{8}$. x and $(1-x)$ and the correct flow direction gives the second column in H.

   With the more standard sign convention we get:

$$H = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} 0 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & -\frac{1}{4} & -\frac{5}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix}$$

3. To obtain the flows, we multiply H with the net power injections Z_i at each bus (generation minus load), i.e. the flow on each line ℓ is given by $F_\ell = \sum_i H_{\ell i} Z_i$. The net power injections are $Z_1 = -400$, $Z_2 = -80$, $Z_3 = -40 + 400 + 120 = 480$. You can check $\sum_i Z_i = 0$.

For the unconstrained flow case the flows are thus:

$$\begin{matrix} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{matrix} \begin{pmatrix} 0 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & -\frac{1}{4} & -\frac{5}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix} \begin{pmatrix} -400 \\ -80 \\ 480 \end{pmatrix} = \begin{matrix} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{matrix} \begin{pmatrix} -120 \\ -280 \\ -200 \end{pmatrix}$$

All lines have a thermal limit of 250 MW, so line $1 \to 3$ is in violation of the security constraints.

4. There are two options for redispatch to deload line $1 \to 3$: a) Shift power from Generator C to B or b) Shift power from Generator C to A.

   We need to work out how much power to shift in each case.

   a) Suppose we shift x MW from C to B. The resulting flows are:

$$\begin{matrix} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{matrix} \begin{pmatrix} 0 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & -\frac{1}{4} & -\frac{5}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix} \begin{pmatrix} -400+x \\ -80 \\ 480-x \end{pmatrix} = \begin{matrix} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{matrix} \begin{pmatrix} -120+\frac{3}{8}x \\ -280+\frac{5}{8}x \\ -200+\frac{3}{8}x \end{pmatrix}$$

   If Line $1 \to 3$ is to be secure we need to satisfy

$$-250 = -280 + \frac{5}{8}x \tag{1.1}$$

   Therefore $x = 48$ MW. Shifting this amount from Generator C to B will increase costs by \$(15-10)*48 = \$240.

   b) Suppose we shift x MW from C to A. The resulting flows are:

$$\begin{matrix} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{matrix} \begin{pmatrix} 0 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & -\frac{1}{4} & -\frac{5}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix} \begin{pmatrix} -400 \\ -80+x \\ 480-x \end{pmatrix} = \begin{matrix} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{matrix} \begin{pmatrix} -120-\frac{3}{8}x \\ -280+\frac{3}{8}x \\ -200+\frac{5}{8}x \end{pmatrix}$$

   If Line $1 \to 3$ is to be secure we need to satisfy

$$-250 = -280 + \frac{3}{8}x \tag{1.2}$$

   Therefore $x = 80$ MW. Shifting this amount from Generator C to A will increase costs by \$(12-10)*80 = \$160.

   Therefore redispatch option b) is preferable, because it costs less.

5. We have an inelastic load and plenty of generator capacity, so the nodal price is determined by the cost of supplying an extra small load at each node.

   At node 3 any small additional load can be supplied locally by the generator with the globally cheapest available power, i.e. Generator C, so the nodal price at node 3 is $\lambda_3 =$\$10/MWh.

   At node 2 the situation is less clear: we can either supply a small additional load from generator A, which costs \$12/MWh, but we could also try to supply it from Generator C at node 3, which is cheaper. However if Generator C increases its dispatch, line $1 \to 3$ would become overloaded, also forcing Generator B to increase its dispatch to compensate. Let's see how much the combination of B and C would cost: suppose the load at node 2 increases by ϵ, x of which is covered by Generator C and $\epsilon - x$ is covered by Generator B. The flows are then:

$$\begin{pmatrix} 0 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & -\frac{1}{4} & -\frac{5}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix} \begin{pmatrix} -400 + \epsilon - x \\ -\epsilon \\ 400 + x \end{pmatrix} = \begin{pmatrix} -150 + \frac{3}{4}\epsilon - \frac{3}{8}x \\ -250 + \frac{1}{4}\epsilon - \frac{5}{8}x \\ -150 - \frac{1}{4}\epsilon - \frac{3}{8}x \end{pmatrix}$$

   For line $1 \to 3$ to remain secure, we require

$$-250 = -250 + \frac{1}{4}\epsilon - \frac{5}{8}x \tag{1.3}$$

   i.e. $x = \frac{2}{5}\epsilon$. In this case the cost is $\left(\frac{2}{5}10 + \frac{3}{5}15\right)$ \$/MWh = 13 \$/MWh, so it's actually cheaper to use Generator A to locally supply the load and then $\lambda_2 = 12$ \$/MWh.

   For node 1 there is a similar consideration. It's clear that Generator B will be more expensive than some combination of A and C which doesn't overload line $1 \to 3$, so let's again split the coverage of an extra load ϵ at node 1 between A and C. The flows are then

$$\begin{pmatrix} 0 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & -\frac{1}{4} & -\frac{5}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix} \begin{pmatrix} -400 - \epsilon \\ \epsilon - x \\ 400 + x \end{pmatrix} = \begin{pmatrix} -150 - \frac{3}{4}\epsilon + \frac{3}{8}x \\ -250 - \frac{1}{4}\epsilon - \frac{3}{8}x \\ -150 + \frac{1}{4}\epsilon - \frac{5}{8}x \end{pmatrix}$$

   To avoid overloading line $1 \to 3$ we need

$$0 = -\frac{1}{4}\epsilon - \frac{3}{8}x$$

i.e. $x = -\frac{2}{3}\epsilon$. In this case the cost is $\lambda_1 = \left(-\frac{2}{3}10 + \frac{5}{3}12\right)$ \$/MWh = 13.33 \$/MWh.

There are two ways of calculating the merchandising surplus. In the first way, we calculate the difference between what consumers pay and the generator revenue:

$$(400*13.33+80*12+40*10)-(0*13.33+80*12+440*10) = 1333.333$$

The other way is to multiply the line flows by the price difference between the connected nodes

$$(150 * 1.33) + (250 * 3.333) + (150 * 2) = 1333.3333 \tag{1.4}$$

6. This is too hard to do by hand...better to let the computer do it :-).

   $P_A = 63.33$ MW; $P_B = 10$ MW; $P_C = 6.67$ MW; $P_D = 400$ MW

   $\lambda_1 = 15$ \$/MWh; $\lambda_2 = 12$ \$/MWh; $\lambda_3 = 10$ \$/MWh

Exercise 2: Negative nodal prices

Suppose there is a load of 100 MW at node 3, a generator A at node 1 with marginal cost 1 €/MWh, a generator B at node 3 with marginal cost 10 €/MWh and a transmission constraint of 10 MW on line 2 → 3. This example is constructed so that increasing the load at node 2 relieves the constrained line and allows more cheap generation to be imported from generator A at node 1 to the load at node 3.

First, let's work out the dispatch of generators A and B. Suppose A dispatches x MW so that B must dispatch $100 - x$ MW. Let's find x. We have flows

$$\begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{4}{5} & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \\ 100-(100-x) \end{pmatrix} = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} \frac{2}{5}x \\ \frac{3}{5}x \\ \frac{2}{5}x \end{pmatrix}$$

If the line 2 → 3 is restricted to 10 MW we want to maximise x to maximise the dispatch from the cheaper generator A, so then we can solve for x

$$10 = \frac{2}{5}x \tag{2.1}$$

so that $x = 25$, $P_A = 25$, $P_B = 75$.

Now suppose we increase the demand at node 2 by ϵ. What is the cost of distributing the generation x to Generator A and $\epsilon - x$ to Generator B? We have flows

$$\begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{4}{5} & 0 \end{pmatrix} \begin{pmatrix} 25 + x \\ -\epsilon \\ -25 - x + \epsilon \end{pmatrix} = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} 10 + \frac{2}{5}x + \frac{1}{5}\epsilon \\ 15 + \frac{3}{5}x - \frac{1}{5}\epsilon \\ 10 + \frac{2}{5}x - \frac{4}{5}\epsilon \end{pmatrix}$$

To maintain the security of line $2 \to 3$ we need $x = 2\epsilon$ so that A increases by 2ϵ and B increases by $\epsilon - x = -\epsilon$. This results in a *cheaper* dispatch because dispatch from A has replaced expensive dispatch from B, with total cost 2ϵ MWh∗1 €/MWh − ϵ MWh∗10 €/MWh = -8ϵ €, i.e. a negative price of -8€/MWh.

Exercise 3: Levelised cost of electricity for wind power

1. The LCOE gives an average cost for producing a kWh of energy over the lifetime of the generator, discounted to present net value. Note that offshore wind has a higher interest rate because it is perceived as a more risky, immature technology than onshore wind.

2. Using the formula we have

Type	Onshore (low wind)	Onshore (strong wind)	Offshore (low wind)	Offshore (strong wind)
LCOE (€/kWh)	0.0958	0.0555	0.1634	0.1249

3. We take approximate values from Lecture 1 for the technologies.

4. The effective FIT is found by spreading the FIT over 20 years evenly and thus solving (with discounting):

$$\sum_{t=1}^{20} \frac{X}{(1+i)^t} = \sum_{t=1}^{5} \frac{8.72ct/kWh}{(1+i)^t} + \sum_{t=6}^{20} \frac{4.95ct/kWh}{(1+i)^t}$$

Pull the X out of the sum and using $i = 0.038$ we get

$$X = \frac{1}{\sum_{t=1}^{20} \frac{1}{(1+i)^t}} \left[\sum_{t=1}^{5} \frac{8.72ct/kWh}{(1+i)^t} + \sum_{t=6}^{20} \frac{4.95ct/kWh}{(1+i)^t} \right] = 6.17ct/kWh$$

Type	Solar PV	Lignite	Hard Coal	CCGT
Investment cost (€/kW)	1000	1500	1200	700
Lifetime	20	40	40	30
annual var. cost (€/kWh)	0.02	0.006	0.025	0.04
annual full-load hours	1000	7000	5000	4000
interest rate	3.8%	3.8 %	3.8%	3.8%
LCOE (€/kWh)	0.0923	0.0165	0.0368	0.0500

Exercise 4: Duration Curves and Generation Investment

This question is from Hesamzedah & Biggar, Q9.4.
We answer this question using screening curves. First we work out the intersection points of the generators as a function of their capacity factors (percentage of time that they operate at full power per year), then we work out the capacities K_* of the generators.
The screening curves tell us above which capacity factor it costs less to run one type of generator rather than another.

Generator	c_i [€/MWh]	f_i [€/MW/h]
A	10	15
B	20	5
C	50	1
LS	1000	0

Generators A and B intersect at x_{AB} given by

$$15 + 10x_{AB} = 5 + 20x_{AB}$$

i.e. $x_{AB} = 1$. This means that in ALL circumstances B is cheaper than A, so A will never get built, $K_A = 0$.
Generators B and C intersect at x_{BC} given by

$$5 + 20x_{BC} = 1 + 50x_{BC}$$

i.e. $x_{BC} = 2/15$. This means that if a generator can run more than 2/15 of the time, then it should be generator B. The amount of load that is present at least x_{BC} of the time gives K_B, which we find by solving based on the load duration curve

$$1000 - 1000(x_{BC}) = K_B \tag{4.1}$$

to find $K_B = 866.6667$.
Generator C and load-shedding LS intersect at x_{CLS} given by

$$1 + 50x_{CLS} = 1000x_{CLS}$$

i.e. $x_{CLS} = 1/950$. This means that for 1/950 of the time we have load-shedding because it's not economical to cover the rare times of very high load. To get the capacity of generator C we solve based on the load duration curve

$$1000 - 1000(x_{CLS}) = K_B + K_C \tag{4.2}$$

to find $K_C = 132.3$.
Load above $K_B + K_C = 999.067$ is shed.

Exercise 5: Screening curves

This question is a proof of the formula from Lecture 7, Slide 34.
We have generators $i = 1, \ldots N$ with costs

$$C_i(Q_i, K_i) = c_iQ_i + f_iK_i \tag{5.1}$$

The sums of the generator capacities (ordered by fixed price) are $S_i = \sum_{k=1}^{i} K_k$.
The equation

$$f_i = (V - c_i)P(Q > S_N) + \sum_{j=i+1}^{N} (c_j - c_i)P(S_{j-1} < Q \leq S_j)$$

expresses the condition for optimal generation investment with inelastic demand with value V: the fixed cost f_i of each generation technology should be equal to the area of the price duration curve above the marginal cost c_i of the generator.
We now use this to prove the screening curve formula, i.e. that the optimal capacities are determined where the costs as a function of capacity factor intersect.
We define $\theta_i = P(Q > S_i)$ and note that $P(S_{j-1} < Q \leq S_j) = P(Q > S_{j-1}) - P(Q > S_j) = \theta_{j-1} - \theta_j$.

For $i = N$ we get

$$f_N = (V - c_N)\theta_N$$

For $i < N$ we have

$$\begin{aligned}
f_i &= (V - c_i)\theta_N + \sum_{j=i+1}^{N} (c_j - c_i)(\theta_{j-1} - \theta_j) \\
&= V\theta_N - c_i\theta_N + \sum_{j=i+1}^{N} c_j(\theta_{j-1} - \theta_j) - c_i \left(\sum_{j=i+1}^{N} \theta_{j-1} - \sum_{j=i+1}^{N} \theta_j \right) \\
&= V\theta_N - c_i\theta_N + c_{i+1}(\theta_i - \theta_{i+1}) + \sum_{j=i+2}^{N} c_j(\theta_{j-1} - \theta_j) - c_i\theta_i + c_i\theta_N \\
&= V\theta_N - c_i\theta_i + c_{i+1}\theta_i - c_{i+1}\theta_{i+1} + \sum_{j=i+2}^{N} c_j(\theta_{j-1} - \theta_j) \\
&= V\theta_N - c_i\theta_i + c_{i+1}\theta_i - c_{i+1}\theta_N + \sum_{j=i+2}^{N} (c_j - c_{i+1})(\theta_{j-1} - \theta_j) \\
&= f_{i+1} - c_i\theta_i + c_{i+1}\theta_i
\end{aligned}$$