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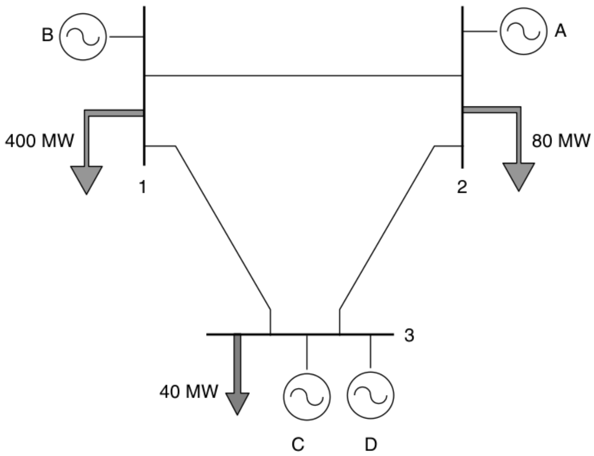
Electricity Markets
4. Exercise Sheet
06.06.2016

Exercise 1: Another three-bus system (copied from 3. exercise sheet

Consider the three-bus system:



The generators have the following data:

Generator	Capacity [MW]	Marginal Cost [$/MWh]
A	150	12
B	200	15
C	150	10
D	400	8

1. Calculate the unconstrained economic dispatch and the market clearing price of the three-node system.

2. *Bonus non-exam question: Given the branch data:

Branch	Reactance [p.u.]	Capacity [MW]
1-2	0.2	250
1-3	0.3	250
2-3	0.3	250

confirm that, using bus 1 as the reference bus, the PTDF is:

$$H = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{8} \\ 0 & \frac{1}{4} & \frac{5}{8} \\ 0 & -\frac{1}{4} & \frac{3}{8} \end{pmatrix}$$

3. Calculate the flow that would result if the generating units were dispatch for the unconstrained case in Problem 1. Identify all the violations of security constraints.

4. Determine two ways of removing the constraint violations that you identified by redispatching generating units. Which redispatch is preferable?

5. Calculate the nodal prices for the three-bus system when the generating units have been optimally redispatched to relieve the constraint violations. Calculate the merchandising surplus and show that it is equal to the sum of the surpluses of each line.

6. Suppose now that the capacity of branch 1-2 is reduced to 140 MW while the capacity of the other lines remains unchanged. Calculate the optimal dispatch and the nodal prices for these conditions. [Hint: the optimal solution involves a redispatch of generating units at all three buses.]

Exercise 2: Negative nodal prices

Consider the network of three connected nodes from lecture 6 with the following PTDF matrix (reference node 3):

$$H = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{4}{5} & 0 \end{pmatrix}$$

Construct an example with a negative nodal price at node 2. (Hint: place a constant demand at node 3, generators with different marginal costs at node 1 and 3, and a transmission constraint on line $2 \to 3$.)

Exercise 3: Levelised cost of electricity for wind power

The levelised cost of electricity (LCOE) of a generator is defined as:

$$LCOE = \frac{I_0 + \sum_{t=1}^{n} \frac{A_t}{(1+i)^t}}{\sum_{t=1}^{n} \frac{Q_t}{(1+i)^t}} \, ,$$

where I_0 [€] are the investment costs, A_t [€] are the annual costs in year t, Q_t [kWh] is the electricity generated in year t, i is the interest rate, and n is the lifetime of the generator.

The following table is adapted from a study from Fraunhofer ISE (2013):

Type	Onshore (low wind)	Onshore (strong wind)	Offshore (low wind)	Offshore (strong wind)
Investment cost (€/kW)	1400	1400	3900	3900
Lifetime	20	20	20	20
annual var. cost (€/kWh)	0.018	0.018	0.035	0.035
annual full-load hours	1300	2700	2800	4000
interest rate	3.8%	3.8 %	6.7%	6.7%

1. Give an interpretation of the LCOE.
2. Calculate the LCOE for all four cases given in the table.
3. Give estimates on the LCOE for PV, lignite, hard coal and combined cycle gas turbines.
4. Assume that the feed-in tariff for onshore wind turbines in Germany going online in the first quartal of 2017 is 8.72 ct/kWh for the first five years, and then 4.95 ct/kWh. Convert these tariffs to an effective feed-in tariff over the 20 years. For onshore wind power (strong/low wind), for what investment costs this tariff corresponds to the LCOE? How does the result change if the inital FIT is reduced by 5% or 7.5%?

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Exercise 4: Duration Curves and Generation Investment

Let us suppose that demand is inelastic. The demand-duration curve is given by $Q = 1000 - 1000z$. Suppose that there are three different types of generation with a variable cost of 10, 20 and 50 €/MWh, together with load-shedding at 1000€/MWh. The fixed costs of these generation types are 15, 5 and 1 €/MWh, respectively. Find the optimal mix of generation in this industry.

Exercise 5: Screening curves (*Bonus non-exam question)

Consider the expression

$$f_i = (V - c_i)P(Q > S_N) + \sum_{j=i+1}^{N} (c_j - c_i)P(S_{j-1} < Q \leq S_j)$$

from lecture 7 (screening curve analysis for the optimal mix of generation with inelastic demand). Show that these equations can be rewritten recursively using the substitution $\theta_i = P(Q > S_i)$

$$\begin{aligned} f_N + \theta_N c_N &= V\theta_N \\ f_i + \theta_i c_i &= f_{i+1} + \theta_i c_{i+1} \qquad \forall i = 1, \dots N-1 \; . \end{aligned}$$