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Electricity Markets
5. Exercise Sheet Solutions
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Exercise 1: Subsidy

1. A FIT provides a generator with a guaranteed remuneration for each kWh for a fixed number of years regardless of market conditions. This provides investor certainty for new technologies and a guarantee that variable generators aren't subject to the vicissitudes of the market price.

2. It prevents the generator responding to market price movements (e.g. when the price goes below its marginal cost or negative), it may push other otherwise profitable generators out of the market (due to the merit order effect), the pricing of the tariff is not determined by a market process and may be higher than the natural value if the generator was unsupported on the market.

3. The administratively-set tariff is set based on an estimate of the capital cost plus a guaranteed rate of return; the auction provides a means of competition to reach a more efficient tariff rate without any administrative assumptions.

4. See `https://de.wikipedia.org/wiki/Direktvermarktung_erneuerbarer_Energien`. Direct Marketing is different from a FIT, because it forces the generator onto the market; any difference between the market price (based on an average over each month) and the comparable FIT is then paid to the generator (the market bonus). If the generator is producing at times different to the technology-specific average reference value, it can earn more than the FIT, encouraging generators to be responsive to the market. There is also a Management Bonus paid to generators to cover the administration costs. There is no payment if the market price is negative for more than a few hours.

5. Quotas, no support, contract for difference (like direct marketing), CO2 tax - see lecture.

Exercise 2: Hedging with a cap

The profit is

$$\pi(P) = (P - c)K\mathbb{I}(P \geq c)$$

The uniform distribution has a probability density function $f_P(p)$ which takes value $f_P(p) = \frac{1}{50}$ for $0 \leq p \leq 50$ and is zero elsewhere.

The expected value of the profit is therefore

$$\mathbb{E}(\pi(P)) = K \int_c^{50} f_P(p)(p-c)dp = 16K$$

So a cap with a strike price of c should have a price of $P_{cap} = \$16/\text{MWh}$ to replace this expected profit, i.e. the pay flow to the seller for volume K is

$$\text{Cap}(P, K, c, P_{cap}) = -(P-c)K\mathbb{I}(P \geq c) + P_{cap}K$$

which averages to zero.

Exercise 3: Cournot duopoly

This example is take from Taylor (2015) Example 6.2.
Each generator i maximises

$$\max_{x_i} \left[x_i(a - b(x_i + x_{3-i})) - c_i x_i\right]$$

This yields from stationarity for each generator

$$a - b(2x_i + x_{3-i}) - c_i = 0$$

Solve this simultaneous equation to get

$$x_i = \frac{a - 2c_i + c_{3-i}}{3b}$$

and the equilibrium price

$$\pi = \frac{a + c_1 + c_2}{3}$$

Exercise 4: Market power with nodal pricing

1. The generator at node 1 dispatches at its maximum given the line constraints since it is the cheapest generator in the system. This results in an optimal dispatch of $P_1 = 25$ MW and $P_3 = 75$ MW to cover the load of 100 MW at node 3. The price at node 1 is just $\lambda_1 = c_1$ because the cheapest generator is at that node. The price at node 3 is $\lambda_3 = c_3$ because it cannot import any more, because of the congestion on line $2 \to 3$, and must supply the load locally. To determine the price at node 2, consider increasing the demand there by ϵ. Suppose this load is covered by x at Generator 1 and $\epsilon - x$ to Generator 3; to determine x we

see what value respects the thermal limit on line $2 \to 3$ by calculating the flow there:

$$F = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{4}{5} & 0 \end{pmatrix} \begin{pmatrix} 25 + x \\ -\epsilon \\ -25 - x + \epsilon \end{pmatrix} = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} 10 + \frac{2}{5}x + \frac{1}{5}\epsilon \\ 15 + \frac{3}{5}x - \frac{1}{5}\epsilon \\ 10 + \frac{2}{5}x - \frac{4}{5}\epsilon \end{pmatrix}$$

To maintain the security of line $2 \to 3$ we need $x = 2\epsilon$ so that Generator 1 increases by 2ϵ and Generator 3 increases by $\epsilon - x = -\epsilon$. This results in costs of $(2c_1 - c_3)\epsilon$, so $\lambda_2 = 2c_1 - c_3$.

2. 10 MW flows from 1 to 2. The price at node 1 is $\lambda_1 = c_1$ and at node 2 is $\lambda_2 = 2c_1 - c_3$. Since $c_1 < c_3$, $-c_3 < -c_1$, and therefore $\lambda_2 = 2c_1 - c_3 < 2c_1 - c_1 = c_1 = \lambda_1$, which means $\lambda_2 < \lambda_1$. This means that power flows from an expensive bus to a cheaper bus which is counterintuitive (but beneficial to the overall system, because it allows cheap power at node 1 to flow to the load at node 3).

3. The generator at node 3 can exploit the fact that no further power can be imported and increase his price to the VOLL. The generator at node 1 can increase their price until it is just below c_3 and still be guaranteed 25 MW of dispatch. If there were no line constraints, the whole dispatch would come from generator 1 as long as it were cheaper than generator 3.

Exercise 5: Optimal behaviour of generators with storage

See Biggar & Hesamzedah page 108.

Generator	Dispatch 1	Dispatch 2	Dispatch 3	Total [MW]	Cost [$]
A	500	500	500	1500	0
B	300	500	200	1000	10000
C	0	200	0	200	10000
Total	800	1200	700	2700	20000

Generator	Dispatch 1	Dispatch 2	Dispatch 3	Total [MW]	Cost [$]
A	0	200	0	200	0
B	500	500	500	1500	15000
C	300	500	200	1000	50000
Total	800	1200	700	2700	65000

Generator	Dispatch 1	Dispatch 2	Dispatch 3	Total [MW]	Cost [$]
A	300	700	200	1200	0
B	500	500	500	1500	15000
C	0	0	0	0	0
Total	800	1200	700	2700	15000

Generator	Dispatch 1	Dispatch 2	Dispatch 3	Total [MW]	Cost [$]
A	-200	200	0	0	0
B	500	500	500	1500	15000
C	500	500	200	1200	60000
Total	800	1200	700	2700	75000