T. Brown
M. Schäfer

6. Exercise Sheet Solutions
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Problem 1: Efficient dispatch in a two-node system with constraints

1. With no transmission at node 1 we have $Q_1 = Q_1^B = 500$ MW and therefore the price is $\pi_1 = 15 + 0.05 * 500 = 40$ €/MWh. At node 2 we have $Q_2 = Q_2^B = 200$ MW and therefore the price is $\pi_2 = 12 + 0.03 * 200 = 18$ €/MWh.

2. With infinite transmission the flow F from node 2 to node 1 is determined by the point at which the prices equalise, i.e.

$$15 + 0.05 * (500 - F) = 12 + 0.03 * (200 + F)$$

   which solves to $F = 275$ MW, i.e. $Q_1 = 225$ MW and $Q_2 = 475$ MW. At this point the market price at both nodes is 26.25 €/MWh.

3. If the flow is limited to $F = 100$ MW, then $Q_1 = 500 - 100 = 400$ MW, $\pi_1 = 15 + 0.05 * 400 = 35$ €/MWh, $Q_2 = 200 + 100 = 300$ MW, $\pi_1 = 12 + 0.03 * 300 = 21$ €/MWh.

4. In this case at node 1 the consumer payment is $500 * 35 = 17500$ and at node 2 is $200 * 21 = 4200$, 21700 in total. The generator revenue at node 1 is $400 * 35 = 14000$ and at node 2 is $300 * 21 = 6300$, 20300 in total. The difference is the congestion revenue $1400 = 100 * (35 - 21)$.

Problem 2: Variable Renewables and electricity markets

Since renewable generators have variables costs that are close to zero, in the merit order they dispatch before conventional generators that have higher variable costs (due to fuel costs, etc.). Because the demand remains the same, this reduces the effective residual demand that the conventional generators see. This means that cheaper generators are dispatched first, shifting the supply curve to the right, lowering the market price and forcing expensive conventional generators out of the market. The result is that the market price is reduced, since the intersection of the supply-demand curves is now with cheaper conventional generators.
This reduction in the market price is called the "merit order effect".
More expensive generators, such as gas plants, are run less and thus get lower usage factors, which may lower their profits. Similarly baseload plants such

as coal and nuclear may have less opportunity to run and make lower profits, since the market price is on average lower.
A diagram of the supply-demand curves with and without wind and solar generation would be good here. A diagram of the change to the residual load duration curve would also help.

Problem 3: Constrained optimisation theory

1. The optimisation problem has objective function:

$$\max_{Q,Q_1,Q_2} [U(Q) - C_1(Q_1) - C_2(Q_2)] = \max_{Q,Q_1,Q_2} \left[8000Q - 5Q^2 - c_2 Q_1 - c_2 Q_2\right]$$

with constraints:

$$\begin{aligned} Q - Q_1 - Q_2 = 0 &\leftrightarrow \lambda \\ Q_1 \leq K_1 &\leftrightarrow \bar{\mu}_1 \\ Q_2 \leq K_2 &\leftrightarrow \bar{\mu}_2 \\ -Q_1 \leq 0 &\leftrightarrow \underline{\mu}_1 \\ -Q_2 \leq 0 &\leftrightarrow \underline{\mu}_2 \end{aligned}$$

2. Stationarity gives for Q:

$$\frac{\partial U}{\partial Q} - \lambda = 8000 - 10Q - \lambda = 0$$

Stationarity for Q_1 gives:

$$-\frac{\partial C_1}{\partial Q_1} + \lambda - \mu_1 = -c_1 + \lambda - \bar{\mu}_1 + \underline{\mu}_1 = 0$$

Stationarity for Q_2 gives:

$$-\frac{\partial C_2}{\partial Q_2} + \lambda - \mu_2 = -c_2 + \lambda - \bar{\mu}_2 + \underline{\mu}_2 = 0$$

Primal feasibility is just the constraints above. Dual feasibility is $\bar{\mu}_i, \underline{\mu}_i \geq 0$ and complementary slackness is $\bar{\mu}_i(Q_i - K) = 0$ and $\underline{\mu}_i Q_i = 0$ for $i = 1, 2$.

3. The marginal utility at the full output of the generators, $K_1 + K_2 =$ 700 MW is $U'(700) = 8000 - 10 * 700 = 1000$ €/MWh, which is higher than the costs c_i, so we'll find optimal rates $Q_1^* = K_1$ and $Q_2^* = K_2$ and $Q^* = K_1 + K_2$. This means $\lambda = U'(K_1 + K_2) = 1000$ €/MWh, which is

the market price. Because the lower constraints on the generator output are not binding, from complementary slackness we have $\underline{\mu}_i = 0$. The upper constraints are binding, so $\bar{\mu}_i \geq 0$. From stationarity $\bar{\mu}_i = \lambda - c_i$, which is the increase in social welfare if Generator i could increase its capacity by a marginal amount.

Problem 4: Current affairs

A price zone is an area of a network with a single market clearing process and a single market price. It is assumed that there is no congestion inside the price zone; if there is congestion it much be dealt with after market clearing by redispatch.
Currently Germany and Austria are part of the same price zone. This means that for example consumers in Austria can purchase cheap wind energy from northern Germany even though there may not be enough transmission capacity to transport it across Germany. Instead, the wind may have to be curtailed and expensive conventional power plants in southern Germany or Austria must be dispatched to compensate. In addition, the power between northern Germany and Austria may not flow inside Germany at all, but in loop flows through neighbouring countries such as Poland, the Czech Republic and Slovakia. This blocks their networks, meaning they cannot import the cheap north-German wind power themselves.
A market split makes the congestion transparent at the boundaries of the price zones, since limits are introduced on the trades between the zones.
Benefits: This makes it more difficult to dispatch unphysical trades, reducing loop flows through neighbouring countries and the need for dispatch within the German-Austria zone.
Disadvantages: Administrative costs of reconfiguration of market, smaller zones are less liquid and may make it easier to abuse market power, splitting Germany and Austria doesn't fix the possibly more significant bottlenecks within Germany. (In addition, Germany may rely on Austria hydro plants for black start capabilities.).

Problem 5: Managing price risk

The market price varies between 10 and 50, averaging 30.
The load can buy a swap for their load of X MW with strike price S for the price of $P_{Swap}X$. The cash flow to the load from the swap is then $(P-S)X - P_{Swap}X$, which when added to the price of the electricity gives $-PX + (P - S)X - P_{Swap}X = -SX - P_{Swap}X$.

If the seller makes no profit on average then $0 = \mathbb{E}\left[(P - S) - P_{Swap}\right]$ so that $P_{Swap} = \mathbb{E}[P] - S = 30 - S$.

Problem 6: Power flows Feasible injection patterns

We using the equation $F_\ell = \sum_k H_{\ell k} Z_k$ to calculate the flows:

$$F = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix} \begin{pmatrix} -10 \\ -40 \\ 50 \end{pmatrix} = \begin{pmatrix} 10 \\ -20 \\ -30 \end{pmatrix}$$

It's useful at this stage to check the consistency. We require $\sum_k Z_k = 0$ to balance the power in the network. This holds since $-10 - 40 + 50 = 0$.
Similarly the nodal power balances much equal the exports minus imports at each node, giving:

$$Z_1 = F_{1\to 2} + F_{1\to 3} \tag{6.1}$$
$$Z_2 = F_{2\to 3} - F_{1\to 2} \tag{6.2}$$
$$Z_3 = -F_{1\to 3} - F_{2\to 3} \tag{6.3}$$

(The minus signs take account of the direction of the flow to or from the node.)
The constraints follow from $|F_\ell| = |\sum_k H_{\ell k} Z_k| \leq K_\ell$:

$$|Z_1 - Z_2| \leq 60$$
$$|2Z_1 + Z_2| \leq 60$$
$$|Z_1 + 2Z_2| \leq 60$$

Draw these on a diagram.

Problem 7: Efficient operation of a market under network constraints

The efficient short-term operation of an electricity market under network constraints can be formulated as the following constrained optimisation problem. One fixes Z_k (nodal imbalances) by maximising total economic welfare given constraints for the nodal injections (determined by the transmission constraints):

$$\max_{\{Z_k\}} \left[\sum_k B_k(Z_k) \right] \tag{7.1}$$

subject to

$$\sum_k Z_k = 0 \qquad \leftrightarrow \qquad \lambda \tag{7.2}$$

$$\left| \sum_k H_{\ell k} Z_k \right| \leq d_\ell \qquad \leftrightarrow \qquad \mu_\ell^\pm \tag{7.3}$$

The first constraint is that total power balances in the network. The second constraint says that the flows derived from the nodal power imbalances using the PTDF H cannot exceed the capacities d_ℓ.
The benefit function is the welfare at each node given the power imbalance Z_k:

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right] \tag{7.4}$$

$$\text{subject to } Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k \tag{7.5}$$

This constraint is the power balance at each node: generation minus load must equal nodal imbalance.

Problem 8: Duration Curves and Generation Investment

A screening curve plots the costs of different generators as a function of their utilization/capacity/usage factor so that they can be compared based on their fixed and variable costs. The utilization factor is plotted along the x axis from 0 to 1, 0 corresponding to no running time, 1 corresponding to the power plant running 100% of the time. The intercept of the curve of each generator with the y axis is given by the fixed cost f [€/MWh] (i.e. the cost with no variable costs) and the slope is given by the variable cost c [€/MWh].
The interception points of the linear curves of the different generators determine the ranges of utilization factors in which one generator is cheaper than another.
By comparing the screening curves with the demand duration curve, the correct generator capacities for different utilisation factors can be determined (e.g. how much baseload power is required, how much peaking power is required, how much load shedding).
We now answer the specific generation investment optimisation question using screening curves. First we work out the intersection points of the generators as a function of their capacity factors (percentage of time that they

operate at full power per year), then we work out the capacities K_* of the generators.
The screening curves tell us above which capacity factor it costs less to run one type of generator rather than another.

Generator	c_i [€/MWh]	f_i [€/MW/h]
A	10	15
B	20	5
C	50	1
LS	1000	0

Generators A and B intersect at x_{AB} given by

$$15 + 10x_{AB} = 5 + 20x_{AB}$$

i.e. $x_{AB} = 1$. This means that above x_{AB} it is cheaper to run generator A, i.e. the capacity is set by

$$1000 - 1000x_{AB} = K_A = 0 \tag{8.1}$$

Since $x_{AB} = 1$, this means that in ALL circumstances B is cheaper than A, so A will never get built, $K_A = 0$.
Generators B and C intersect at x_{BC} given by

$$5 + 20x_{BC} = 1 + 50x_{BC}$$

i.e. $x_{BC} = 2/15$. This means that if a generator can run more than 2/15 of the time, then it should be generator B. The amount of load that is present at least x_{BC} of the time gives $K_A + K_B$, which we find by solving based on the load duration curve

$$1000 - 1000(x_{BC}) = K_A + K_B \tag{8.2}$$

to find $K_B = 866.6667$ (since $K_A = 0$).
Generator C and load-shedding LS intersect at x_{CLS} given by

$$1 + 50x_{CLS} = 1000x_{CLS}$$

i.e. $x_{CLS} = 1/950$. This means that for 1/950 of the time we have load-shedding because it's not economical to cover the rare times of very high load. To get the capacity of generator C we solve based on the load duration curve

$$1000 - 1000(x_{CLS}) = K_A + K_B + K_C \tag{8.3}$$

to find $K_C = 132.3$.
Load above $K_A + K_B + K_C = 999.067$ is shed.