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Electricity Markets (ELMA)
6. Exercise Sheet, 04.07.2016

This **mock** exam contains 8 problems, with a total of 140 points. A score of 100 points corresponds to a mark of 100%, so you do not need to solve all exercises. Please indicate all the steps of the solution, and not only the final result. You may answer the questions in any order.

Problem 1: Efficient dispatch in a two-node system with constraints (26 points)

Consider two nodes representing two markets, each with different total demand, and one generator at each node. At node 1 the (inelastic) demand is $Q_1^B = 500$ MW, whereas at node 2 the (inelastic) demand is $Q_2^B = 200$ MW. The supply curve for the generator at node 1 is given by

$$\pi_1 = MC_1 = 15 + 0.05 Q_1 \text{ [€/MWh] },$$

whereas the supply curve for the generator at node 2 reads

$$\pi_2 = MC_2 = 12 + 0.03 Q_2 \text{ [€/MWh] }.$$

Here Q_1 and Q_2 are the rates of production in MW for generators 1 and 2, respectively. The generators have no capacity constraints.

1. Assume that nodes 1 and 2 are separate markets with no transmission line between them. Calculate the optimal dispatch and the market prices at both nodes.
2. Assume that nodes 1 and 2 are connected by a transmission line with unlimited capacity so that the nodal prices equalise. Calculate the optimal dispatch, the market price, and the power flow between the nodes.
3. Assume that nodes 1 and 2 are connected by a transmission line with capacity $K = 100$ MW. Assuming nodal pricing, calculate the optimal dispatch, the market price at each node, and the power flow between the nodes.
4. What is the congestion revenue?

Problem 2: Variable Renewables and electricity markets (12 points)

Describe the effects of increasing shares of wind and solar power on the behaviour of other generators, particularly their usage factors, and the electricity market prices.

Problem 3: Constrained optimisation theory (20 points)

Consider an electricity market with two generator types, one with variable cost $c = 20$ €/MWh, capacity $K = 300$ MW and a dispatch rate of Q_1 [MW] and another with variable cost $c = 50$ €/MWh, capacity $K = 400$ MW and a dispatch rate of Q_2 [MW]. The demand has utility function $U(Q) = 8000Q - 5Q^2$ [€/h] for a consumption rate of Q [MW].

1. What are the objective function and constraints required for an optimisation problem to maximise short-run social welfare in this market?
2. Write down the Karush-Kuhn-Tucker (KKT) conditions for this problem.
3. Determine the optimal rate of production of the generators and the value of all KKT multipliers. What is the interpretation of the respective KKT multipliers?

Problem 4: Current affairs (12 points)

There is currently a debate about splitting Germany and Austria into two price zones. Describe the benefits and disadvantages of this change to the European electricity market.

Problem 5: Managing price risk (15 points)

Consider a load in a market with price duration curve $\pi = 50 - 40z$ for $0 \leq z \leq 1$. The load would like to remove all price risk by purchasing a financial instrument that hedges against the changing price.
What instrument should they buy and what should it cost if the seller were to make no profit?

Problem 6: Power flows Feasible injection patterns (10 points)

Consider a network of three connected nodes with nodal imbalances (Z_1, Z_2, Z_3). Assuming unit line impedances, the PTDF matrix, which relates the nodal imbalances with the power flow on the lines, reads

$$\mathbf{H} = \begin{matrix} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{matrix} \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix}$$

Here node 3 has been chosen as the reference node.

1. Assume that $Z_1 = -10$ MW, $Z_2 = -40$ MW, and $Z_3 = 50$ MW. What are the power flows over the three transmission lines?

2. If all transmission lines are restricted to a capacity of 20 MW, draw the constraints on Z_1 and Z_2 in the $Z_1 - Z_2$ plane.

Problem 7: Efficient operation of a market under network constraints (20 points)

Write down a general optimisation problem for the efficient short-term operation of an electricity market under network constraints, in terms of a nodal benefit function B_k and the nodal power imbalance Z_k. Assume there is a given Power Transfer Distribution Function $H_{\ell k}$ and transmission line capacities K_ℓ. Give an interpretation of the objective function and all constraints.

Problem 8: Duration Curves and Generation Investment (25 points)

Describe the concept of a screening curve and how it helps to determine generation investment, given a demand-duration curve.
Question 4 from Exercise Sheet 4:
Let us suppose that demand is inelastic. The demand-duration curve is given by $Q = 1000 - 1000z$. Suppose that there are three different types of generation with a variable cost of 10, 20 and 50 €/MWh, together with load-shedding at 1000€/MWh. The fixed costs of these generation types are 15, 5 and 1 €/MWh, respectively. Find the optimal mix of generation in this industry.