

Electricity Markets: Summer Semester 2016,

Lecture 1

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11th April 2016

Frankfurt Institute of Advanced Studies (FIAS), Goethe-Universität Frankfurt
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FIAS Frankfurt Institute
for Advanced Studies

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Administration

Who we are

- Tom Brown, Postdoctoral Researcher, FIAS (Riedberg),
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- Mirko Schäfer, Postdoctoral Researcher, FIAS (Riedberg),
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We are both physicists who have specialised in the optimisation of energy systems and the interactions of complex networks.

Lectures and Exercise Classes

Lectures:

- Weekly, Mondays 16:00 - 18:00 Seminarhaus SH 0.109
- 11.04.2016 until 11.07.2016 (with exception of Pfingsten 16.05.2016)

Exercise Classes:

- Biweekly, Mondays 18:00 - 20:00 Seminarhaus SH 0.109
- 18.04.2016 until 11.07.2016
- Exercise Sheets: Given out the week before, starting today

Exam and Exercise Sheets

Only the final exam is graded.

Every two weeks there will be an **Exercise Sheet**, which we will go through in the Exercise Class in the following week.

The exam will be in the style and at the difficulty level of non-starred question in the Exercise Sheet.

Therefore if you want to do well in the exam, you need to do the Exercise Sheets.

Exercise classes are biweekly, starting 18.04 (next week).

There are also starred questions which are non-compulsory but should be interesting - you'll learn better by doing these exercises.

Literature

There are lots of textbooks covering the material of this course and **lots** of supporting material on the internet. We will roughly follow:

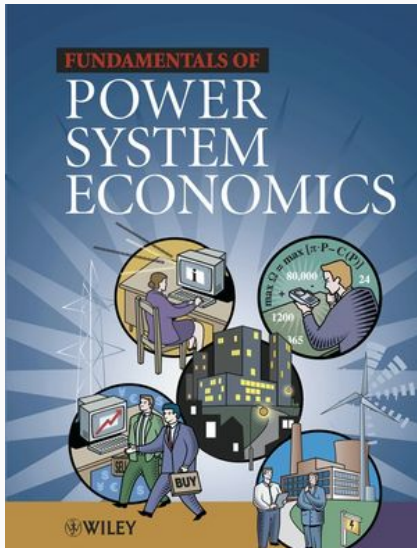
- Daniel Kirschen and Goran Strbac, “Fundamentals of Power System Economics,” Wiley, 2004

The following books are also useful:

- D.R. Biggar, M.R. Hesamzadeh, “The Economics of Electricity Markets,” Wiley, 2014
- Steven Stoft, “Power System Economics: Designing Markets for Electricity,” Wiley, IEEE Press, 2002
- Joshua Adam Taylor, “Convex Optimization of Power Systems,” CUP, 2015
- J.M. Morales et al., “Integrating Renewables in Electricity Markets,” Springer, 2014

Kirschen and Strbac

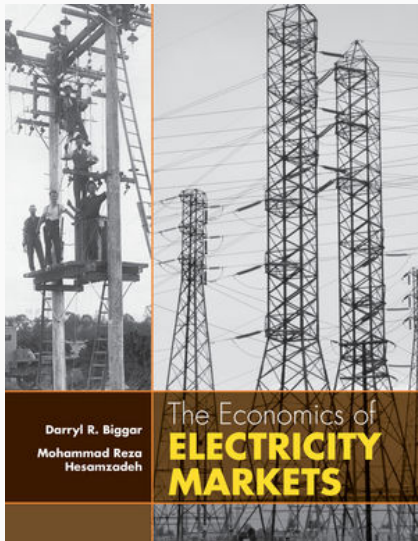
Daniel Kirschen and Goran Strbac, "Fundamentals of Power System Economics," Wiley, 2004



- Focus on examples, concrete calculations
- Less mathematical theory
- We will broadly follow this book

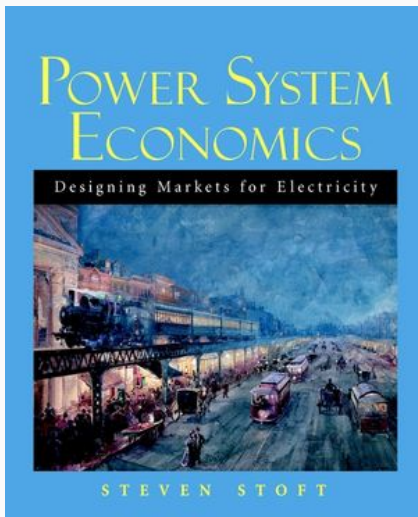
Biggar and Hesamzadeh

D.R. Biggar, M.R. Hesamzadeh, "The Economics of Electricity Markets,"
Wiley, 2014



- More detail on the optimisation theory, engineering background and coupling electricity markets with transmission networks

Steven Stoft, "Power System Economics: Designing Markets for Electricity," Wiley, IEEE Press, 2002



- Economics focus
- Looks at real-world implementations of electricity markets

On the course website:

`http://fias.uni-frankfurt.de/~brown/courses/electricity\_markets/`

we will publish links to material on the internet and to the course notes and exercise sheets.

Introduction: Why Electricity Markets are Important

Why is electricity useful?

Electricity is a versatile form of energy carried by electrical charge which can be consumed in a wide variety of ways (with selected examples):

- Lighting (lightbulbs, halogen lamps, televisions)
- Mechanical work (hoovers, washing machines, electric vehicles)
- Heating (cooking, resistive room heating, heat pumps)
- Cooling (refrigerators, air conditioning)
- Electronics (computation, data storage, control systems)
- Industry (electrochemical processes)

Compare the convenience and versatility of electricity with another energy carrier: the chemical energy stored in natural gas (methane), which can only be accessed by burning it.

How is electricity generated?

Conservation of Energy: Energy cannot be created or destroyed: it can only be converted from one form to another.

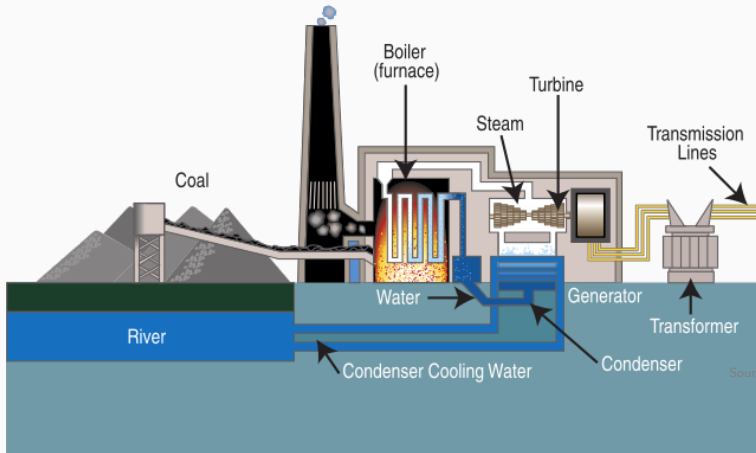
There are several 'primary' sources of energy which are converted into electrical energy in modern power systems:

- Chemical energy, accessed by combustion (coal, gas, oil, biomass)
- Nuclear energy, accessed by fission reactions
- Hydroelectric energy, allowing water to flow downhill
- Wind energy
- Solar energy (accessed with photovoltaic (PV) panels or concentrating solar thermal power (CSP))
- Geothermal energy

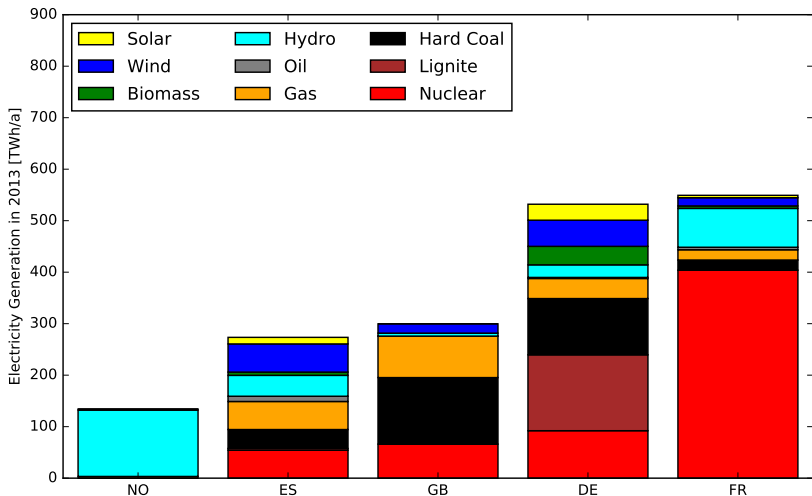
NB: The definition of 'primary' is somewhat arbitrary.

Generators

With the exception of solar photovoltaic panels (and electrochemical energy and a few other minor exceptions), all generators convert to electrical energy via rotational kinetic energy and electromagnetic induction in an *alternating current generator*.

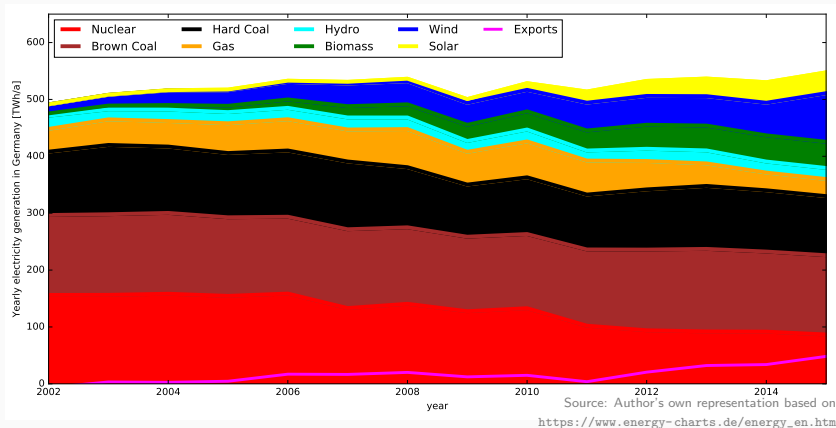


Example of electricity generation across major EU countries in 2013



Electricity generation in Germany per year

In 15 years Germany has gone from a system dominated by nuclear and fossil fuels, to one with 33% renewables in electricity consumption.



The Economic Operation of the Electricity Sector

Given the many different ways of consuming and generating electricity, the questions naturally arise:

- What is the most efficient way to deploy consuming and generating assets in the short-run?
- How should we invest in assets in the long-run to maximise economic welfare?

In the past and still in many countries today, this was done centrally by 'vertically-integrated' monopoly utilities that owned generating assets, the electricity networks and retailing.

Given that these utilities owned all the infrastructure, it was hard for third-party generators to compete, even if they were allowed to.

From the 1980s onwards, countries began to liberalise their electricity sectors, separating generation from transmission, and allowing regulated competition in the generation sector.

Electricity Markets

Electricity markets have several important differences compared to other commodity markets.

At every instant in time, consumption must be balanced with generation.

If you throw a switch to turn on a light, somewhere a generator will be increasing its output to compensate.

If the power is not balanced in the grid, the power supply will collapse and there will be blackouts.

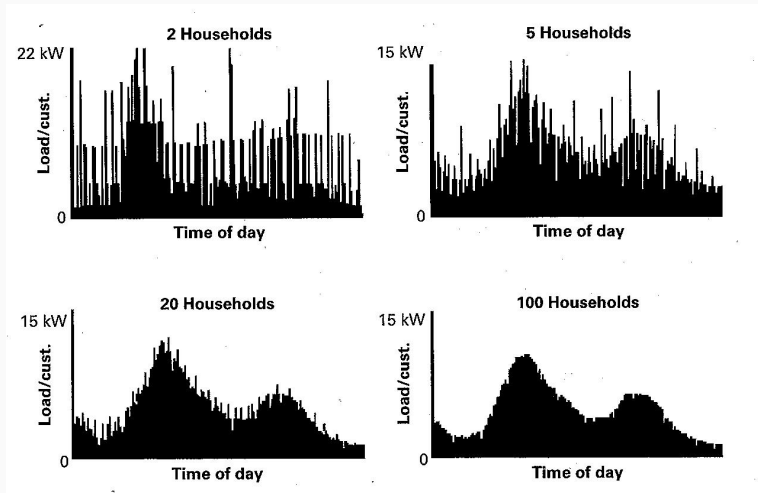
It is not possible to run an electricity market for every single second, for practical reasons (the network must be checked for stability, etc.).

So electricity is traded in blocks of time, e.g. hourly, 14:00-15:00, or quarter-hourly, 14:00-14:15, well in advance of the time when it is actually consumed (based on forecasts).

Further markets trade in backup balancing power, which step in if the forecasts are wrong.

Discrete Consumers Aggregation

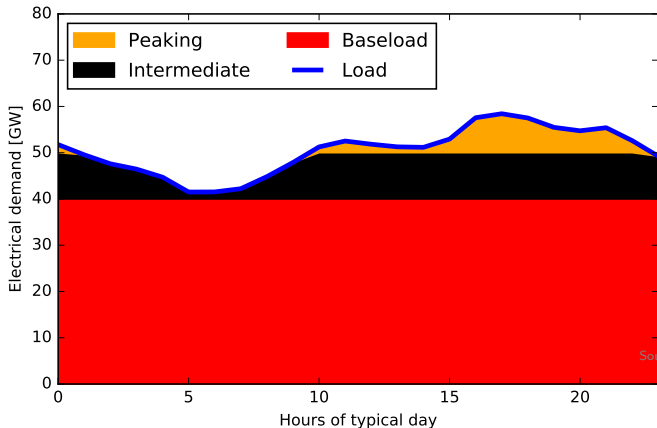
The discrete actions of individual consumers smooth out statistically if we aggregate over many consumers.



Baseload versus Peaking Plant

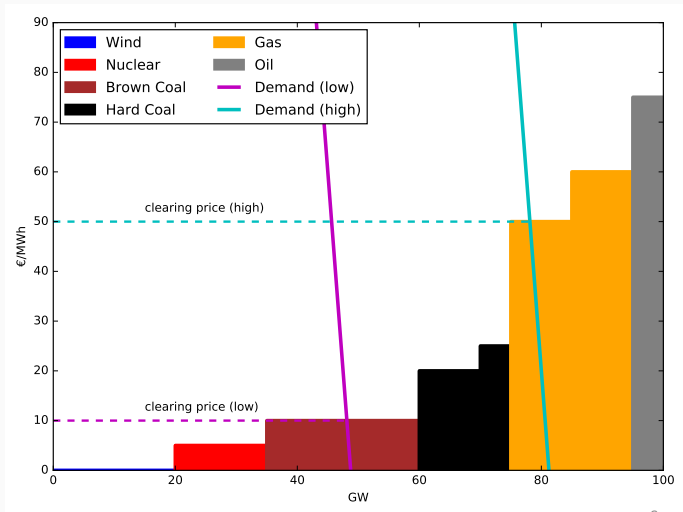
Load (= Electrical Demand) is low during night; in Northern Europe in the winter, the peak is in the evening.

To meet this load profile, cheap **baseload** generation runs the whole time; more expensive **peaking plant** covers the difference.



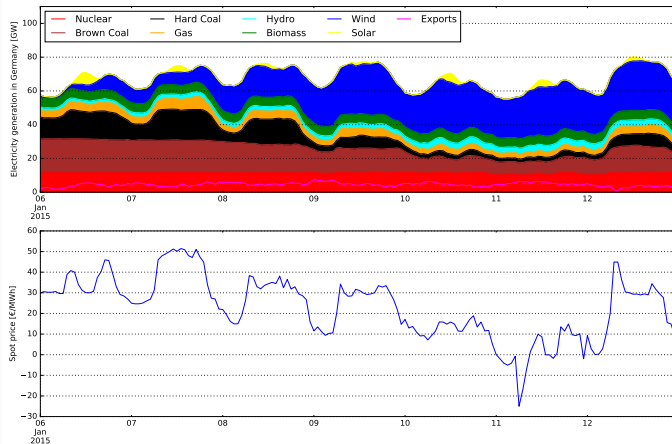
Source: Tom Brown

Effect of varying demand for fixed generation

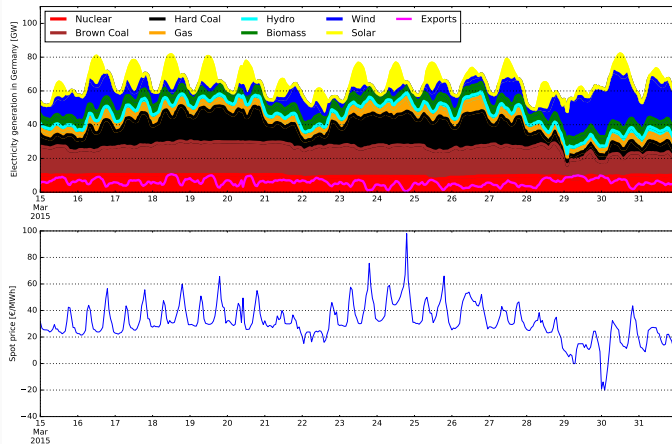


Source: Tom Brown

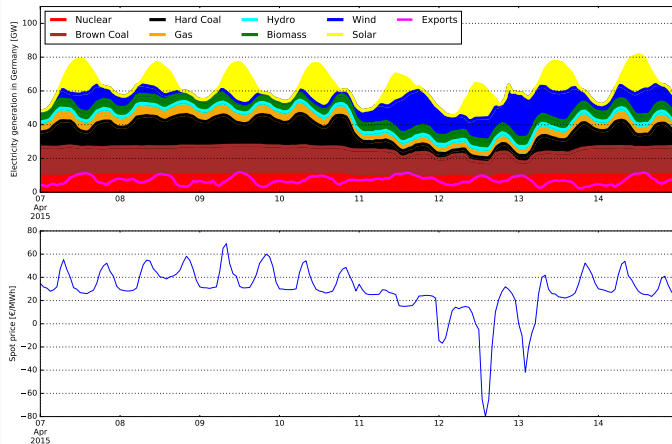
Example market 1/3



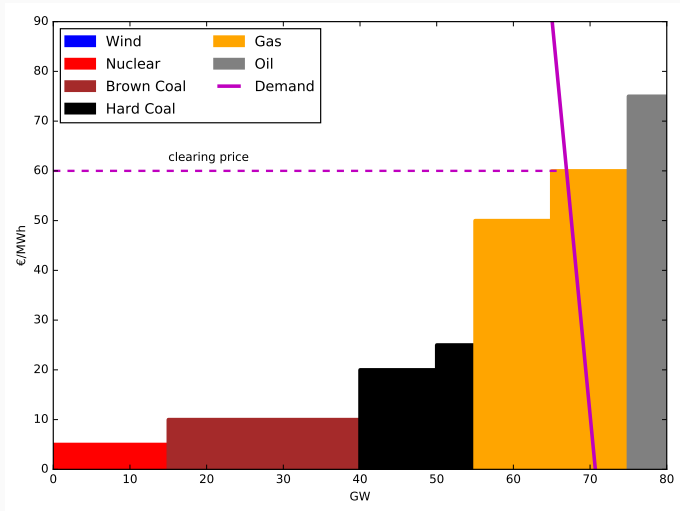
Example market 2/3



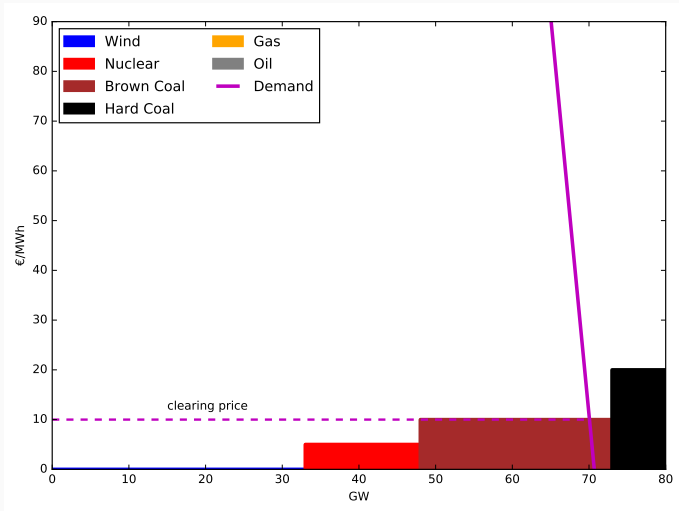
Example market 3/3



Effect of varying renewables: fixed demand, no wind

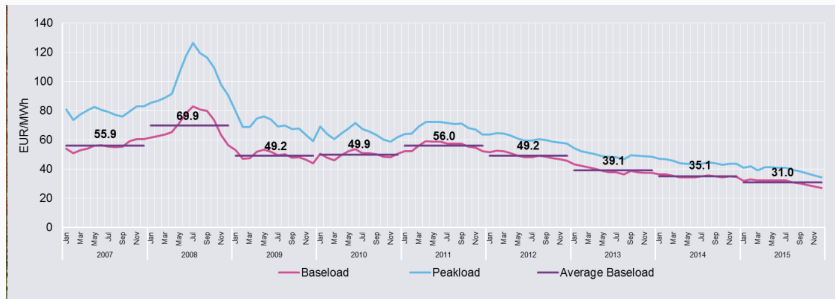


Effect of varying renewables: fixed demand, 35 GW wind



Spot market price development

As a result of so much zero-marginal-cost renewable feed-in, spot market prices have been steadily decreasing:



Source: Agora Energiewende

Merit Order Effect

To summarise:

- Renewables have zero marginal cost
- As a result they enter at the bottom of the merit order, reducing the price at which the market clears
- This pushes non-CHP gas and hard coal out of the market
- This is unfortunate, because among the fossil fuels, gas and hard coal are the most flexible and produce the *lowest* CO₂ per MWh
- It also massively reduces the profits that nuclear and brown coal make
- Will there be enough backup power plants for times with no wind/solar?

This has led to lots of political tension...

Consumption metering



- Look for your electricity meter at home
- Mine showed last Friday 42470.3 kWh
- Check what the value is a week later

Electricity bill

My bill: 1900 kWh for a year, at a cost of €570, which corresponds to 0.3 €/kWh or 300 €/MWh. But the spot market price is 30 €/MWh, so what's going on??

Verbrauchsermittlung

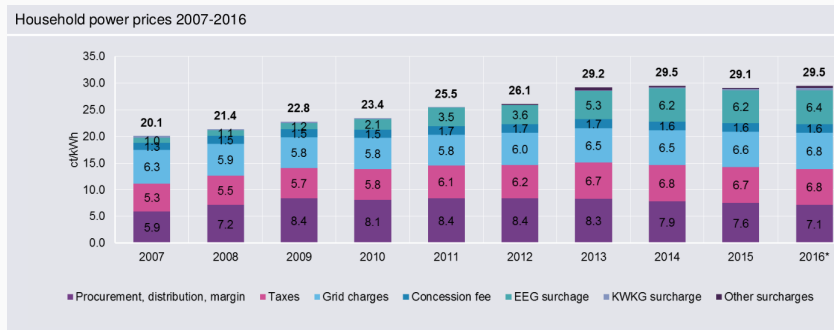
Produktbezeichnung Abrechnungszeitraum	Zähler-Nr.	Zählerstand alt	Zählerstand neu	Verbrauch (kWh)	Umrech- faktor	Verbrauch (kWh)
Strom Direkt	795 388	39.493	41.399	1.906		
31.08.14 - 07.09.15	Tag-/Gesamtverbrauch	Kundenangabe	Kundenangabe			
Verbrauch in kWh - Strom						1.906

Betragsermittlung

Abrechnungszeitraum von bis	Tage	Preisart	Preis in EUR/je	Verbrauch (kWh)	Betrag (EUR)
31.08.14 - 31.12.14 =	123	Arbeitspreis	0,205800/kWh	x	629 = 129,45
01.01.15 - 07.09.15 =	250	Arbeitspreis *)	0,195800/kWh	x	1.277 = 250,04
					1.906
31.08.14 - 31.12.14 =	123	Stromsteuer	0,020500/kWh	x	629 = 12,89
01.01.15 - 07.09.15 =	250	Stromsteuer **)	0,020500/kWh	x	1.277 = 26,18
					1.906
31.08.14 - 07.09.15 =	373	Grundpreis	57,98/Jahr	: 365 x 373 Tage	= 59,25
Nettobetrag					477,81
19% Mehrwertsteuer					90,78
Rechnungsbetrag Strom					568,59

Household price breakdown

Although the wholesale price is going down, other taxes, grid charges and renewables subsidy (EEG surcharge) have kept the price high.

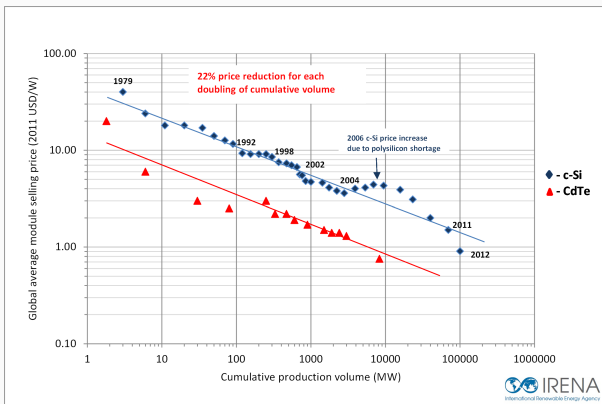


HOWEVER the EEG is only high because it is paying for solar panels bought at a time when they were still comparatively expensive; but through the German subsidy, production volumes were high and the learning curve has brought the costs down exponentially.

Source: Agora-Energiewende

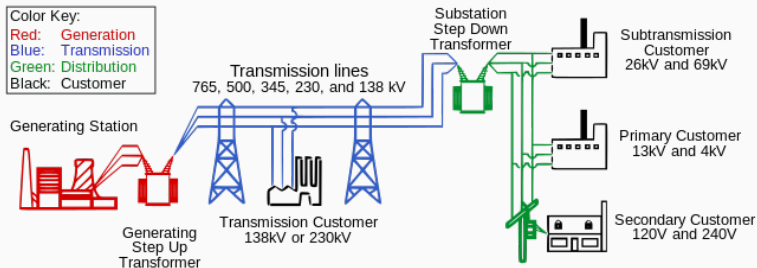
Learning curve for renewables

Onshore wind has already been through the learning process and has been for some years highly competitive with conventional power sources. In some parts of the world wind is being installed without subsidies. Solar has also reaped the benefits of mass production, including the 40 GW of PV installed in Germany:



Electricity Transport from Generators to Consumers

Electricity is also easy to transport over long distances using the high voltage transmission grid:



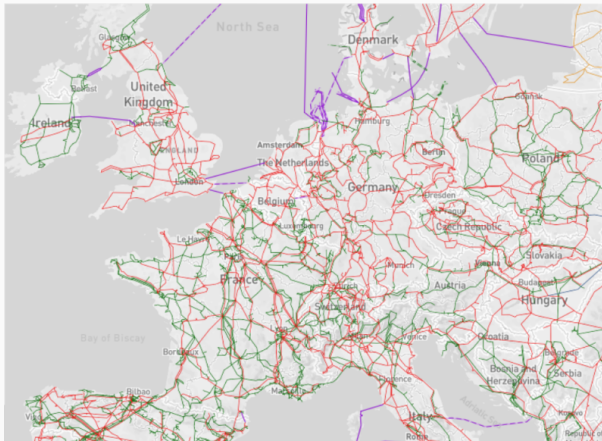
Usually in houses the voltage is 230 V, but in the transmission grid it is hundreds of thousands of Volts.

Source: Wikipedia

European transmission network

Flows in the European transmission network must respect both Kirchoff's laws for physical flow and the thermal limits of the power lines.

Taking account of network flows and constraints in the electricity market is a major and exciting topic at the moment.

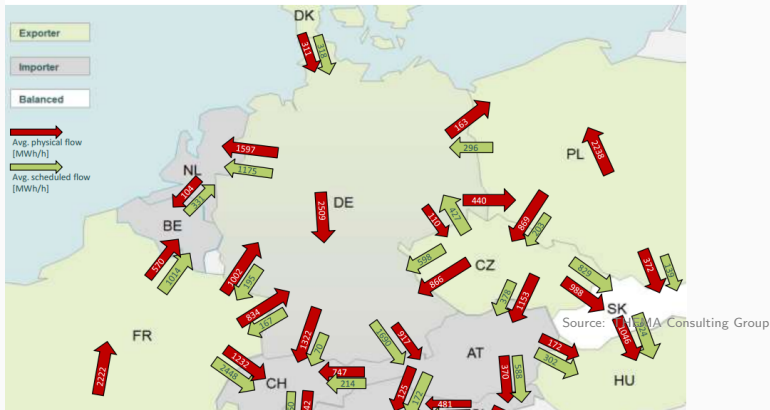


Source: ENTSO-E

Network Bottlenecks and Loop Flows

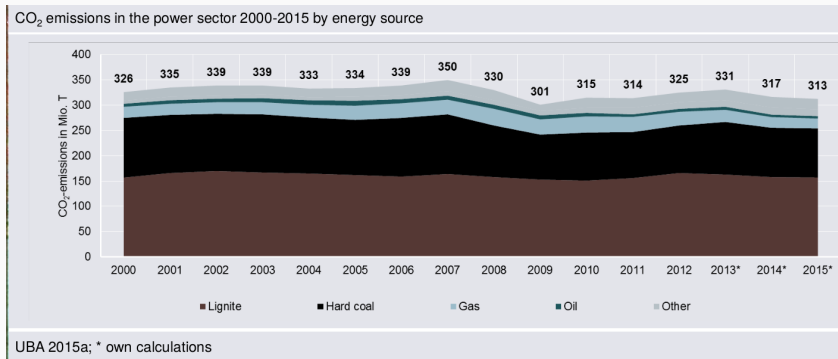
Electricity is traded in large market zones. Power trades between zones (“scheduled flows”) do not always correspond to what flows according to the network physics (“physical flows”). This leads to political tension as wind from Northern Germany flows to Southern Germany via Poland and the Czech Republic.

Figure 7: Average physical and scheduled flows [MWh/h], 01.01.2011 – 31.12.2012



CO₂ emissions from electricity sector

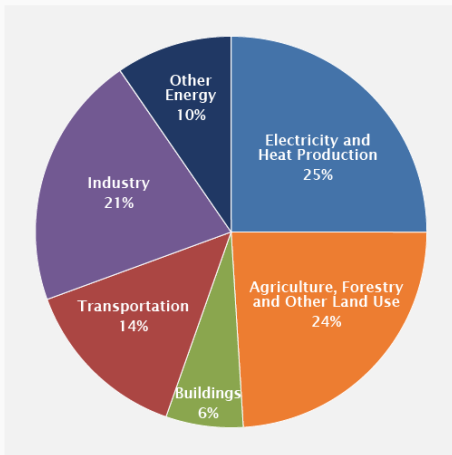
Despite the increase in renewables in the electricity sector, CO₂ emissions have not been reduced substantially in Germany. This is partly because German exports have also increased.



Source: Agora Energiewende

Other sectors

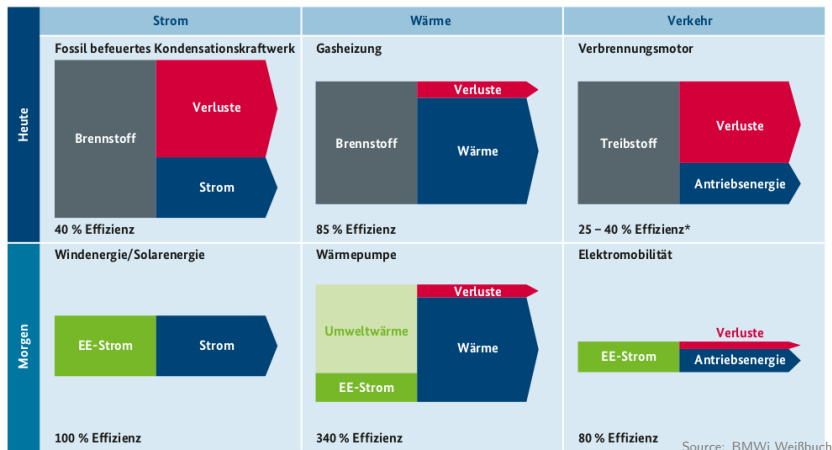
The CO₂ emissions from electricity generation contribute only a fraction to total global greenhouse gas emissions. However electricity generation is one of the easiest places to reduce emissions, aside from directly reducing energy consumption.



Source: EPA, IPCC (2014)

Sector integration

Replacing heating and transport fossil fuels with renewable sources and renewable electricity increases efficiency and reduces emissions of GHG.



* Die Effizienz von Verbrennungsmotoren in anderen Anwendungen (z. B. Seeverkehr, Motorkraftwerke) kann über 50 % liegen.

Quelle: Eigene Darstellung nach Fraunhofer IWES (2015a)

Several changes happening simultaneously

Energiewende: The Energy Transition

- Renewables replace conventional generation
- Increasing integration of international electricity markets
- Better integration of transmission constraints in electricity markets
- Sector coupling: heating and transport electrify
- **Reform of electricity market to allow these transactions and investments to take place efficiently**

In summer 2016 both the German Electricity Market Law and the Renewable Energy Law will be reformed. Fun times!

Newspaper Articles: Lots going on

Climate change

Why fossil fuel power plants will be left stranded

Far from having years to work out how to curb the risks of climate change, we face a moment of truth

Martin Wolf



Newspaper Articles: Lots going on

National Grid

National Grid accused of energy 'panic' over coal deals

UK electricity network provider denies being worried about shortages next winter



Handelsblatt

Dienstag, 5. April 2016, Nr. 65

WIRTSCHAFT & POLITIK **13**

Kampf gegen Kohlendioxid

Franzosen fordern einen
Mindestpreis für
Emissionszertifikate.

Kohlekraftwerk
Jämschwalde:

Die Zertifikatepreise
sind seit Jahren
niedrig.



Klaus Stratmann
Berlin

schlechten Zeiten Zertifikate gehor-
tet. Daher müssen sie heute nur we-
nige Zertifikate nachkaufen. Das
drückt die Preise. Auf Lausitz war,

Ein Mindestpreis entspreche zwar
nicht der reinen Lehre und sei da-
her „nur die zweitbeste Lösung“,
sagte Hans Kerkhoff, Präsident der

sich die Zertifikatepreise am Markt
bilden. Mindestpreise wären daher
ein unzulässiger Eingriff“, sagte
Hans Kerkhoff, Präsident der

le Wettbewerbsfähigkeit der Stahl-
industrie zu sichern, ist vor allem
eine wirtschaftlich und technisch
erreichbare Zuteilung entschei-
dend“, erklärte Kerkhoff.

Die Sorgen der Stahlindustrie ha-
ben ihre Ursache in der laufenden
Debatte über die Reform des Emis-
sionshandels für die Zeit nach
2020. Derzeit arbeitet Brüssel an
entsprechenden Plänen. Nach den
Vorstellungen der EU-Kommission
soll die Zuteilung kostenfreier Emis-
sionszertifikate für Branchen, die
im internationalen Wettbewerb ste-
hen, deutlich restriktiver werden.
So sollen die Benchmarks, an de-
nen sich die Zuteilung der Zertifika-
te bemisst, deutlich verschärft wer-
den. Die knapp 60 Benchmarks –
individuell zugeschnitten auf be-
stimmte Produktionsverfahren und
Produkte – sind aber schon heute
zum Teil so bemessen, dass sie eu-
ropaweit auch von der modernsten
und effizientesten Anlage nicht er-
reicht werden können. Die Bench-
marks sollen jährlich mit einem
„universellen Kürzungsfaktor“ ver-
schärft werden. Hinzu kommt, dass
die Gesamtmenge der Zertifikate
jährlich künftig nicht mehr nur um
1,7 Prozent, sondern um 2,2 Pro-
zent gekürzt werden soll.

Aus Sicht der Industrie sind die

Handelsblatt

HandelsblattNr. 068 vom 08.04.2016 Seite 030

Unternehmen & Märkte

Kleine Motoren groß gefragt

Die Energiewende erfordert flexibel einsetzbare Kleinkraftwerke.

- Das Strommarktgesetz soll die Investitionen ankurbeln.
- Anbieter wie Wärtsilä und GE sowie MAN profitieren.

Axel Höpner, Klaus Stratmann
München, Berlin

Bei allen Klagen über die Schwierigkeiten bei der Energiewende: Es gibt einige Unternehmen, die davon profitieren. Die Hersteller von Windrädern haben seit Jahren volle Auftragsbücher. Auch mit dem Ausbau der Stromnetze lässt sich viel Geld verdienen.

In naher Zukunft werden zudem die Anbieter fossiler Kraftwerke profitieren. Klassische Großkraftwerke sind dabei allerdings out, gefragt sind im Zeitalter der Dezentralisierung vielmehr kleine Motoren-

kraftwerke an verschiedenen Orten. Die Hersteller solcher Anlagen - Firmen wie GE Jenbacher, Wärtsilä und MAN - rechnen mit guten Geschäften. Der Gesetzgeber schafft den passenden Rahmen.

Das Strommarktgesetz befindet sich derzeit im parlamentarischen Verfahren. Es soll dafür sorgen, dass der wachsende Anteil erneuerbarer Energien mit der restlichen Stromerzeugung harmonisiert. Bislang ist das nicht immer der Fall. Schwerfällige Großkraftwerke lassen sich nur mit großer Mühe mit der volatilen Erzeugung aus Windkraft- und Photovoltaikanlagen in Einklang bringen. Kleine, schnell regelbare Kraftwerke würden besser mit den Erneuerbaren harmonisieren. Das neue Gesetz will Investitionen ankurbeln, indem es Preisspitzen zulässt: Wer schnell auf Preissignale reagieren kann, soll damit Geld verdienen können.

„Das deutsche Strommarktgesetz weist in die richtige Richtung“, sagte Karl Hietanen dem Handelsblatt. „Daraus lassen sich neue Geschäftsmodelle entwickeln.“ Hietanen ist Vorstandsmitglied bei Wärtsilä, einem finnischen Hersteller von Motorenkraftwerken. Wärtsilä erzielt mit 18 800 Mitarbeitern einen Umsatz von fünf Milliarden Euro und ist in 70 Ländern aktiv. Die Finnen



Das deutsche Strommarktgesetz weist in die richtige Richtung.

Karl Hietanen
Vorstandsmitglied bei Wärtsilä

nehmen nun den deutschen Markt ins Visier. Zugleich ist Hietanen Präsident der European Engine Power Plants Association, in der sich die Hersteller von Motorenkraftwerken zusammengeschlossen haben. Mitglieder sind neben Wärtsilä auch MAN, Caterpillar, GE und Liebherr.

„Die Rolle der konventionellen Kraftwerke wird sich komplett ändern. Im Zusammenspiel mit erneuerbaren Energien brauchen wir kleine und effiziente Kraftwerke, die sich blitzschnell zuschalten lassen. Diese Anlagen können wir anbieten“, sagt Hietanen. Eine Megawattstunde Strom, die man innerhalb von fünf Minuten liefern könne, sei wesentlich wertvoller als die Megawattstunde, die erst nach einer Stunde abrufbar sei. Tatsächlich braucht eine klassische Gasturbine leicht bis zu einer halben Stunde, ehe sie die volle Leistung bringt. Ein kleiner Gasmotor liefert bereits

nach Strom den die Wärtsilä-bilder zum E
Von s
Seit d
Electr
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facht.
wurde
liefere
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Handelsblatt

HandelsblattNr. 065 vom 05.04.2016 Seite 021

Unternehmen & Märkte

Goldene Zeiten für Stromspeicher

Der Boom von Lithium-Ionen-Akkus erfasst bald auch Unternehmen.

Franz Hubik
Düsseldorf

Detlef Neuhaus kann seit fünf Jahren nichts als Verluste vorweisen. Schlimmer noch: Auch 2016 wird der Chef des Solarmodulherstellers Solarwatt wohl wieder rote Zahlen melden. Das Siechtum kennt kein Ende. Mehr als 200 Mitarbeiter mussten bereits gehen. Der Umsatz der Dresdener Firma zerbröselte von einst mehr als 320 Millionen Euro auf 60 Millionen Euro. Dennoch blickt Neuhaus voll Optimismus in die Zukunft.

Der Manager mit dem grau melierten Haar und dem spitzbühnischen Lächeln arbeitet schließlich an der Energie-Revolution. Und dabei genießt er die volle Rückende-



Anlage von Solarwatt: Hoffen auf intelligente Energiesysteme.

shaving“. Dabei wird Strom aus dem Netz in Lithium-Ionen-Akkus gespeichert, wenn Strom billig ist.

zum Tarifpreis ergibt den Einspareffekt. Und der ist mitunter gewaltig. Ab 2020 können Unternehmen

größte Potenzial ergibt: für Firmen aus den Sektoren, Dienstleistungen und

In den zehn größten Ländern und den zehn größten, die nicht der Organ wirtschaftliche Zusammengehören, beträgt das Einsparvolumen für die 439 Milliarden Dollar pro Jahr in Schwellenländern ergeben sich überproportionale Effekte durch die Vermeidung von Stromausfallkosten.

Bis jetzt haben sich alle sehr wenige Firmen und Batteriespeicher zugeeignet. Deutschland gibt es aktuell Akkus, die zum Strompreis genutzt werden. Der Durchbruch der Technologie wird mittler-

Units: Watts, Kilowatts, Megawatts,
WTF?

Power: Flow of energy

Power is the rate of consumption of energy.

It is measured in Watts:

$$1 \text{ Watt} = 1 \text{ Joule per second}$$

The symbol for Watt is W, $1 \text{ W} = 1 \text{ J/s}$.

$$1 \text{ kilo-Watt} = 1 \text{ kW} = 1,000 \text{ W}$$

$$1 \text{ mega-Watt} = 1 \text{ MW} = 1,000,000 \text{ W}$$

$$1 \text{ giga-Watt} = 1 \text{ GW} = 1,000,000,000 \text{ W}$$

$$1 \text{ tera-Watt} = 1 \text{ TW} = 1,000,000,000,000 \text{ W}$$

Power: Examples of consumption and generation

At full power, the following items consume/generate:

Item	Power
New efficient lightbulb	10 W
Old-fashioned lightbulb	70 W
Single room air-conditioning	1.5 kW
Kettle	2 kW
Wind turbine	3 MW
Coal power station	1 GW
Germany total demand	50-80 GW

Energy

In the electricity sector, energy is usually measured in 'Watt-hours', Wh.

1 kWh = power consumption of 1 kW for one hour

E.g. a 10 W lightbulb left on for two hours will consume

$$10 \text{ W} * 2 \text{ h} = 20 \text{ Wh}$$

It is easy to convert this back to the SI unit for energy, Joules:

$$1 \text{ kWh} = (1000 \text{ W}) * (1 \text{ h}) = (1000 \text{ J/s}) * (3600 \text{ s}) = 3.6 \text{ MJ}$$

Yearly energy to power

Germany consumes around 600 TWh per year, written 600 TWh/a.

What is the *average* power consumption?

$$\begin{aligned} 600 \text{ TWh/a} &= \frac{(600 \text{ TW}) * (1 \text{ h})}{(365 * 24 \text{ h})} \\ &= \frac{600}{8760} \text{ TW} \\ &= 68.5 \text{ GW} \end{aligned}$$

Efficiency

When fuel is consumed, much/most of the energy of the fuel is lost as waste heat rather than being converted to electricity.

The thermal energy, or calorific value, of the fuel is given in terms of MWh_{th} , to distinguish it from the electrical energy MWh_{el} .

The ratio of input thermal energy to output electrical energy is the **efficiency**.

Fuel	Calorific energy $\text{MWh}_{\text{th}}/\text{tonne}$	Per unit efficiency $\text{MWh}_{\text{el}}/\text{MWh}_{\text{th}}$	Electrical energy $\text{MWh}_{\text{el}}/\text{tonne}$
Lignite	2.5	0.4	1.0
Hard Coal	6.7	0.45	2.7
Gas	15.4	0.4	6.16
Uranium	150000	0.33	50000

Fuel costs to marginal costs

The cost of a fuel is often given in €/kg or €/MWh_{th}.

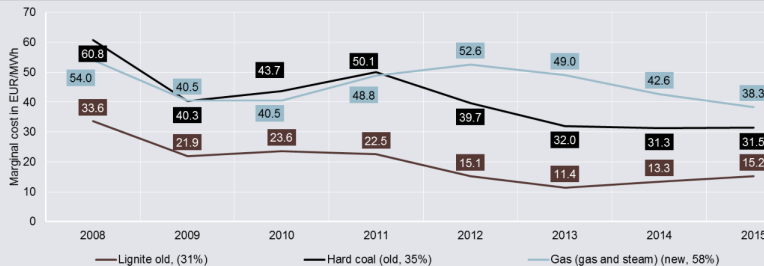
Using the efficiency, we can convert this to €/MWh_{el}.

Fuel	Per unit efficiency MWh _{el} /MWh _{th}	Cost per thermal €/MWh _{th}	Cost per elec. €/MWh _{el}
Lignite	0.4	4.5	11
Hard Coal	0.45	10	22
Gas	0.4	23	58
Uranium	0.33	3.3	10

Real marginal costs

Some actual data on marginal costs for different fuels; these don't agree fully with the previous table due to differing assumptions on efficiencies, etc.

Marginal costs for new gas, lignite and old hard coal power plants 2008-2015



BAFA 2015b, BAFA 2015c, DEHSt2015, EEA 2015, Lazard2015, Statistisches Bundesamt 2015, UBA 2015, own calculations

Source: Agora Energiewende

CO₂ emissions per MWh

The CO₂ emissions of the fuel.

Fuel	t _{CO2} /t	t _{CO2} /MWh _{th}	t _{CO2} /MWh _{el}
Lignite	0.9	0.36	0.9
Hard Coal	2.4	0.36	0.8
Gas	3.1	0.2	0.5
Uranium	0	0	0

Current CO₂ price in EU Emissions Trading Scheme (ETS) €5.27/t_{CO2}

Electricity Markets from the Consumer Perspective

Consumer behaviour: Theory

Suppose for some given period a consumer consumes electricity at a rate of Q MW.

Their **utility or value function** $U(Q)$ in €/h is a measure of their benefit for a given consumption rate Q .

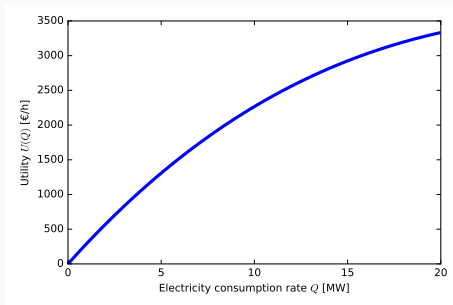
For a firm this could be the profit related to this electricity consumption from manufacturing goods.

Typical the consumer has a higher utility for higher Q , i.e. the first derivative is positive $U'(Q) > 0$. By assumption, the rate of value increase with consumption decreases the higher the rate of consumption, i.e. $U''(Q) < 0$.

Utility: Example

A widget manufacturer has a utility function which depends on the rate of electricity consumption Q [€/h] as

$$U(Q) = 0.0667 Q^3 - 8 Q^2 + 300 Q$$



Note that the slope is always positive, but becomes less positive for increasing Q .

Optimal consumer behaviour

We assume to begin with that the consumer is a price-taker, i.e. they cannot influence the price by changing the amount they consume.

Suppose the market price is $\lambda \text{ €/MWh}$. The consumer should adjust their consumption rate Q to maximise their **net surplus**

$$\max_Q U(Q) - \lambda Q$$

This optimisation problem is optimised for $Q = Q^*$ where

$$U'(Q^*) \equiv \frac{dU}{dQ}(Q^*) = \lambda$$

[Check units: $\frac{dU}{dQ}$ has units $\frac{\text{€/h}}{\text{MW}} = \text{€/MWh.}$]

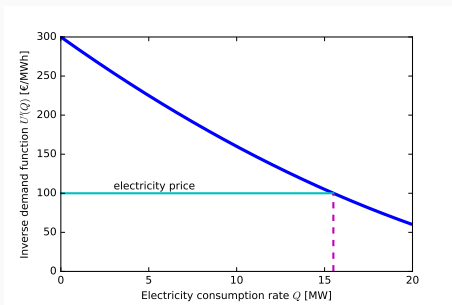
I.e. the consumer increases their consumption until they make a net loss for any increase of consumption.

$U'(Q)$ is known as the **inverse demand curve**, which shows, for each rate of consumption Q what price λ the consumer is willing to pay.

Inverse demand function: Example

For our example the inverse demand function is given by

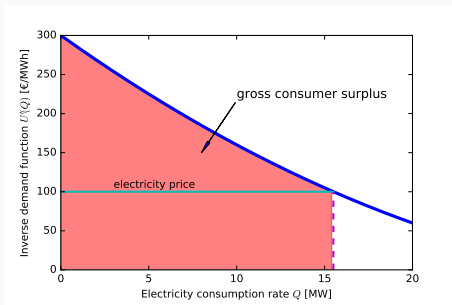
$$U'(Q) = 0.2 Q^2 - 16 Q + 300$$



[It's called the *inverse* demand function, because the demand function is the function you get from reversing the axes. The **demand function** $Q(\lambda)$ gives the demand as a function of the price.]

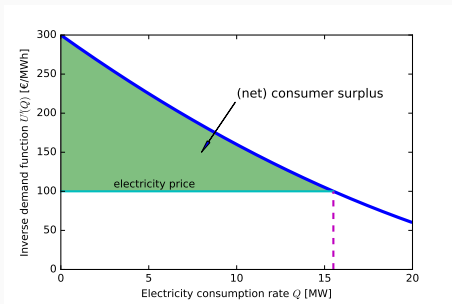
Gross consumer surplus

The area under the curve is the **gross consumer surplus**, which as the integral of a derivative, just gives the utility function $U(Q)$ again, up to a constant.



Net consumer surplus

The more relevant **net consumer surplus**, or just **consumer surplus** is the 'net profit' the consumer makes by having utility above the electricity price.

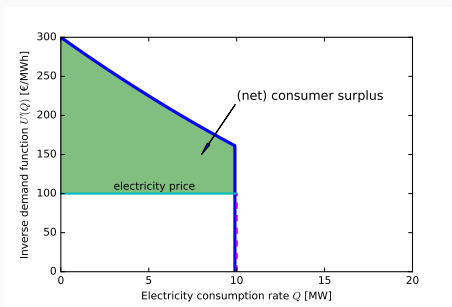


Limits to consumption

Note that it is quite common for consumption to be limited by other factors before the electricity price becomes too expensive, e.g. due to the size of electrical machinery. This gives an upper bound

$$Q \leq Q^{\max}$$

In the following case the optimal consumption is at $Q^{\max} = 10$ MW.

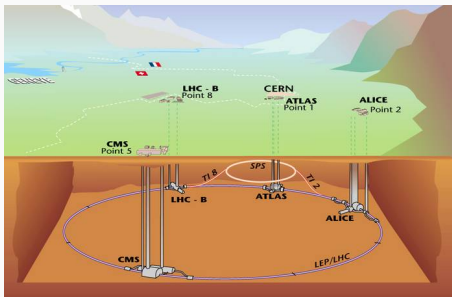


Consumers can delay their consumption

Besides changing the amount of electricity consumption, consumers can also shift their consumption in time.

For example electric storage heaters use cheap electricity at night to generate heat and then store it for daytime.

The LHC particle accelerator does not run in the winter, when prices are higher (see <http://home.cern/about/engineering/powering-cern>). Summer demand: 200 MW, corresponds to a third of Geneva, equal to peak demand of Rwanda (!); winter only 80 MW.



Source: CERN

Consumers can also move location

Aluminium smelting is an electricity-intensive process. Aluminium smelters will often move to locations with cheap and stable electricity supplies, such as countries with lots of hydroelectric power. For example, 73% of Iceland's total power consumption in 2010 came from aluminium smelting.

Aluminium costs around US\$ 1500/ tonne to produce.

Electricity consumption: 15 MWh/tonne.

At Germany consumer price of €300 / MWh, this is €4500 / tonne.

Uh-oh!!!

If electricity is 50% of cost, then need \$750/tonne to go on electricity $\Rightarrow$
 $750/15 \text{ \$/MWh} = 50 \text{ \$/MWh}$.

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`http://fias.uni-frankfurt.de/~brown/courses/electricity\_markets/`

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