# Electricity Markets: Summer Semester 2016, Lecture 4

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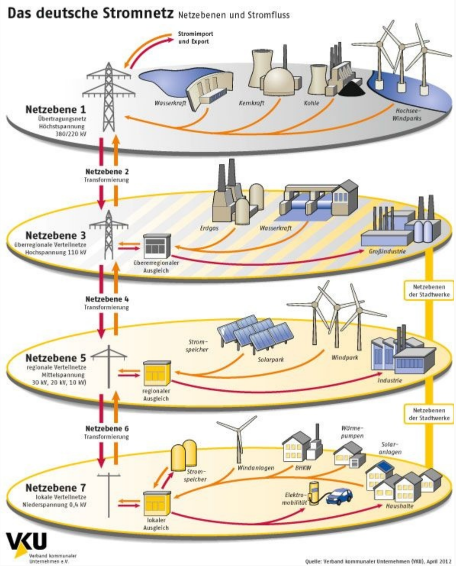
# Transmission and distribution networks

Electricity usually is not consumed where it is produced, so it has to be transported via transmission and distribution networks.

Transmission networks: Transport large volumes of electric power over relatively long distances.

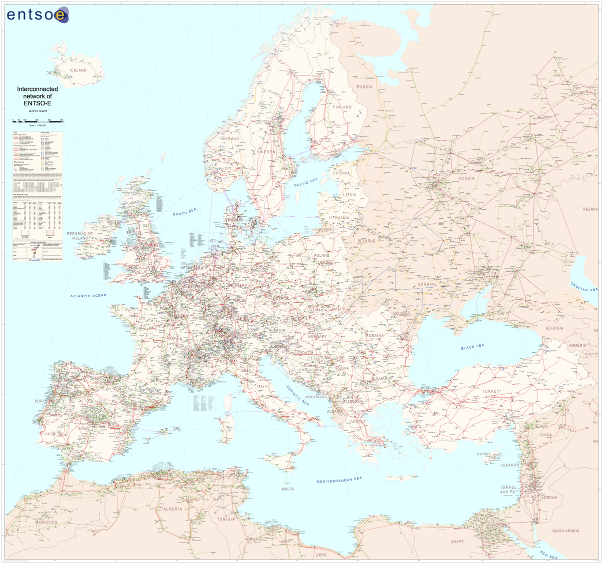
Distribution networks: Take power from the transmission network and deliver it to a large number of end points in a certain geographic area.

# Transmission and distribution networks



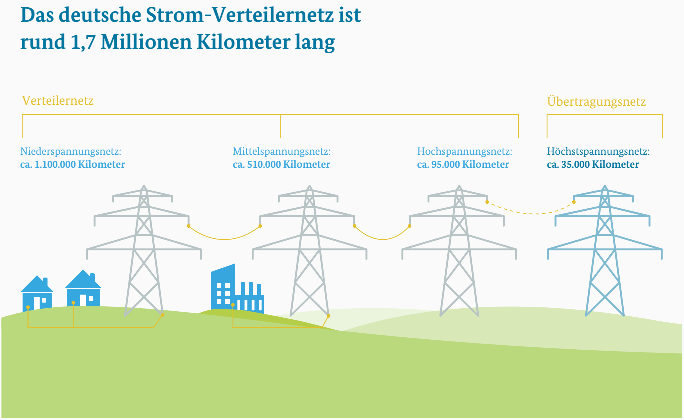

Source: VKU

# European Transmission Grid




Source: ENTSO-E

# Transmission and distribution networks in Germany



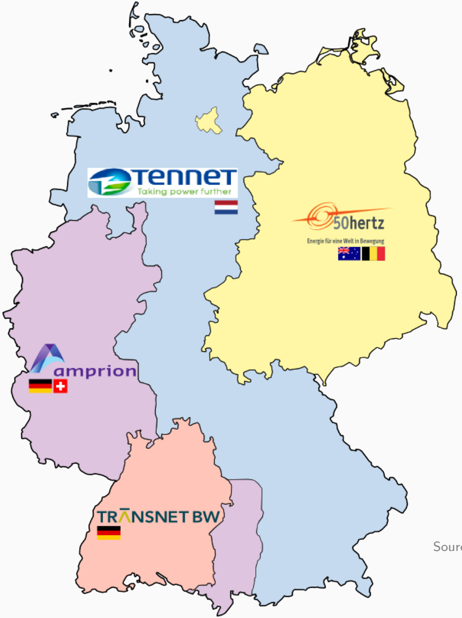

Source: BMWi

# Transmission and distribution networks in Germany

| Sector | Leading Companies | Market Share | Total Number of Providers |
|---|---|---|---|
| **Transmission** | Amprion<br>Transnet BW (ENBW)<br>TenneT<br>50Hertz Transmission | **100%** Combined | 4 |
| **Distribution** | EnBW<br>E.ON<br>RWE<br>Vattenfall | The big 4 distribution companies own and operate a significant portion of the distribution system, though the exact level is not clear. | approximately 890* DSOs, about 700 of which are municipally owned *Stadtwerke* |
| **Total Generation** | EnBW<br>E.ON<br>RWE<br>Vattenfall | **56%** installed capacity** (June 2014)<br>**~59 %** of electricity generated (2012).*** | over 1000 producers (not including individuals) |
| **Retail Suppliers** | EnBW<br>E.ON<br>RWE<br>Vattenfall | **45.5%**<br>of total electricity offtake (TWh).**** | over 900 suppliers |

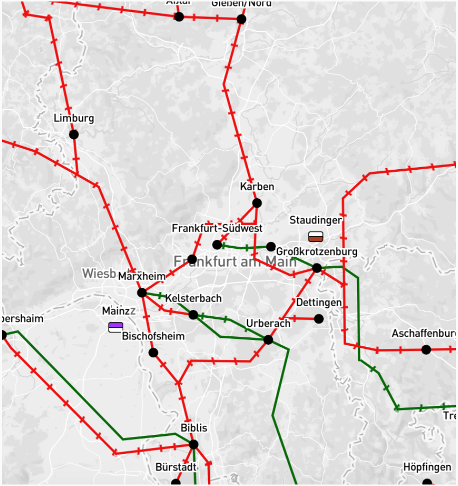
Source: Agora Energiewende / RAP

# TSOs in Germany



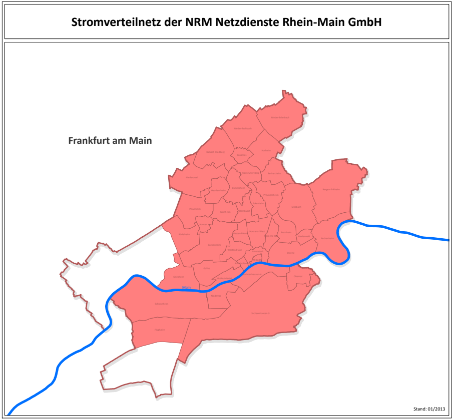

Source: Wikipedia (Francis McLloyd)

# Transmission grid near Frankfurt




Source: ENTSO-E

# Frankfurts DSO




NRM Netzdienste Rhein-Main (subsidiary company of Mainova)

Source: NRM Netzdienste Rhein-Main

# Power grids and electricity markets

The (physical) balancing of supply and demand has to respect the network constraints of the system. These constraints have to be implemented by the system operator, but to some extent can also be included into the market design.

Transmission and distribution networks are (almost?) natural monopolies, which leads to substantial market power. These networks are typically state owned, cooperatives or heavily regulated (many interesting problems with respect to incentives, tariffs, etc.).

Network expansion is part of the long-term efficient operation of the system. Note the interdependency between network and generation investment.

# Representing network constraints

# Physical limits on networks

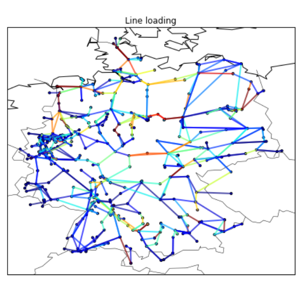
**Thermal limits:** Relate to the maximum amount of power which can be transmitted via a transmission line.

**Voltage stability limits**: Relate to the supply of reactive power to keep the system voltage close to a specific level.

**Dynamic and transient stability limits:** Relate to the stability of the system frequency, and the stability of the synchronized operation of the generators.

# Representing transmission networks

**Terminology:** We represent the transmission grid as an network, consisting of nodes and links. The nodes may represent individual generators, groups of generators and consumers, whole geographic regions or just a point where different transmission lines meet. The links represent transmission lines, or more generally the possibility to transfer electric power between the respective nodes connected to the respective link.




Source: PyPSA

# Basic implementation of thermal limits

A link $l$ connecting two nodes allows to transport electrical power as a power flow $F_l$ from one node to the other.

We implement the thermal limits on a line $l$ as the capacity $K_l$, which gives the upper limit of power flow $F_l$ on $l$:

$$F_l \leq K_l$$
$$-F_l \leq K_l$$

This looks just like another constraint for KKT. Unfortunately, the power flows $F_l$ are usually not free parameters, but are connected via physical laws from the generation and consumption pattern $Q_i^S, Q_i^B$ at the nodes.

# Efficient dispatch in a two-node system with constraints

# Example

[The following example is taken from the book by Strbac and Kirschen.]

Consider two nodes representing regions, each with different total demand, using different types of generators:

**First node:** Fixed demand $Q_1^B = 500$ MW. The (inverse) supply function for the generators is given by

$$\pi_1 = MC_1 = 10 + 0.01Q_1 \text{ [€/MWh]}$$

**Second node:** Fixed demand $Q_2^B = 1500$ MW. The (inverse) supply function for the generators is given by

$$\pi_2 = MC_2 = 13 + 0.02Q_2 \text{ [€/MWh]}$$

For simplicity we assume that at both nodes the total generation limit is 5 GW.

**Transmission line** from node 1 to node 2 with capacity $K$.

# Example: Separate markets

**Transmission capacity $K = 0$:**

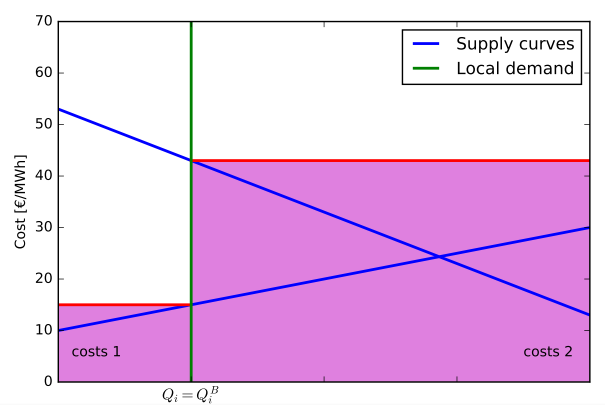
**First node:** Fixed demand $Q_1^B = 500$ MW. The competitive price is

$$\lambda_1 = MC_1(Q_1^B) = 10 + 0.01 \times 500 = 15 \text{ [€/MWh]}$$

**Second node:** Fixed demand $Q_2^B = 1500$ MW. The competitive price is

$$\lambda_2 = MC_2(Q_2^B) = 13 + 0.02 \times 1500 = 43 \text{ [€/MWh]}$$

# Example: Separate markets



# Example: Single market

**Transmission capacity $K = \infty$:**

There is now a total demand $Q^B = Q_1^B + Q_2^B$, leading to a single market clearing price $\lambda$. The generators at node 1 and node 2 adjust their output such that

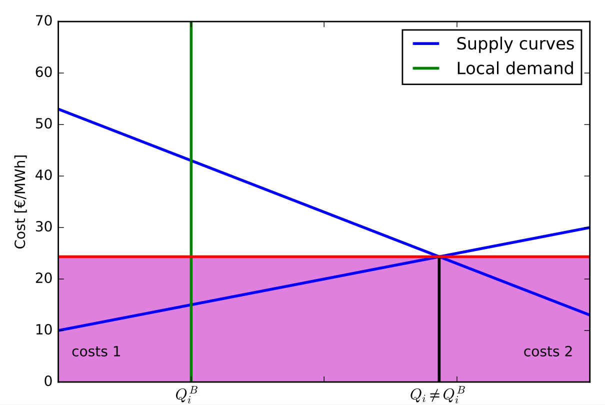
$$MC_1(Q_1) = MC_2(Q_2) = \lambda,$$

under the constraint that

$$Q_1 + Q_2 = Q^B$$

This leads to $Q_1 = 1433$ MW, $Q_2 = 567$ MW, and $\lambda = 24.33$ €/MWh.

# Example: Single market



# Example: Single market

The power flow $F$ from node 1 to node 2 is given by

$$\begin{aligned} F &= \left(Q_1 - Q_1^B\right) = -\left(Q_2 - Q_2^B\right) \\ &= (1433 - 500)\ \text{MW} = -(567 - 1500)\ \text{MW} \\ &= Z_1 = -Z_2 \\ &= 933\ \text{MW} \end{aligned}$$

with $\{Z_1, Z_2\} = \{933\ \text{MW}, -933\ \text{MW}\}$ denoted as the injection pattern.

From the balancing condition it follows $Z_1 + Z_2 = 0$. We call a node with $Z > 0$ a source, and a node with $Z < 0$ a sink.

If $Z_1 > Z_2$, we have a flow from node 1 to node 2, if $Z_2 > Z_1$, we have a flow from node 2 to node 1 (source to sink).

# Example: Single market

| | Separate markets | Single market |
|---|---|---|
| $Q_1^B$ [MW] | 500 | 500 |
| $Q_1$ [MW] | 500 | 1433 |
| $Z_1$ [MW] | 0 | $+933$ |
| $\lambda_1$ [€/MWh] | 15 | 24.33 |
| $Q_2^B$ [MW] | 1500 | 1500 |
| $Q_2$ [MW] | 1500 | 567 |
| $Z_2$ [MW] | 0 | -933 |
| $\lambda_2$ [€/MWh] | 43 | 24.33 |
| $F_{1\to 2}$ [MW] | 0 | 933 |
| $\sum_i \lambda_i \times Q_i$ [€] | 72000 | 48660 |
| $\sum_i \lambda_i \times Q_i^B$ [€] | 72000 | 48660 |

# Another example: Separate markets

Consider two nodes representing two regions with different total demand, using different types of generators:

**First node:** Fixed demand $Q_1^B = 200$ MW, one type of generators with marginal costs $c_1 = 10$ €/MWh and total generation limit $\hat{Q}_1 = 300$ MW.

**Second node:** Fixed demand $Q_2^B = 300$ MW, one type of generators with marginal costs $c_2 = 30$ €/MWh and total generation limit $\hat{Q}_2 = 400$ MW.

**Transmission line** from node 1 to node 2 with capacity $K$.

**Optimal dispatch:**

The generators at node 1 provide 200 MW to the consumers at node 1. Depending on the capacity of the transmission line, they export a power flow $F$ between zero and 100 MW to the consumers at node 2. The generators at node 2 provide the remaining consumption at node 2, that is 300 MW $- F$.

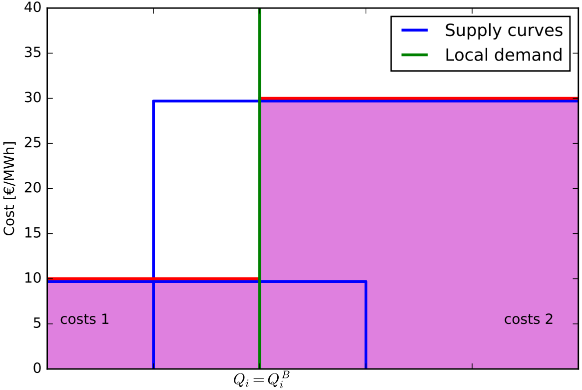
# Another example: Separate markets

**Capacity $K = 0$:**

First node generators produce the entire supply of consumers at node 1, $Q_1 = 200$ MW, but cannot export to node 2. Second node generators provide the entire supply of consumers at node 2, $Q_3 = 200$ MW. The competitive price at node 1 is $\lambda_1 = 10$ €/MWh, at node 2 it is $\lambda_2 = 30$ €/MWh.

Total cost to consumers:
200 MW $\times$ 10 €/MWh $+$ 300 MW $\times$ 30 €/MWh $=$ 11000 €.
Generators at node 1: Revenue 200 MW $\times$ 10 €/MWh $=$ 2000 €.
Generators at node 2: Revenue 300 MW $\times$ 30 €/MWh $=$ 9000 €.

# Another example with two nodes, separate markets



# Another example: Single market

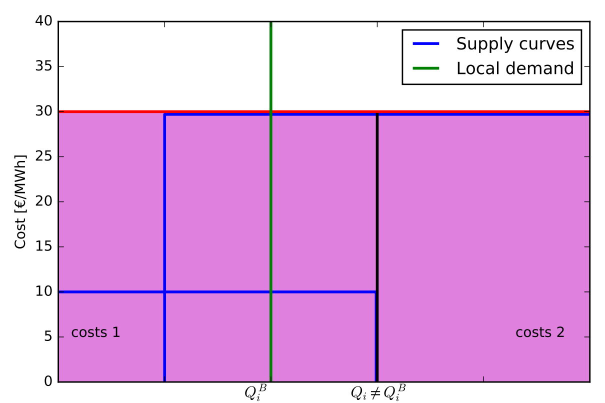
**Capacity $K = \infty$:**

First node generators produce at the limit, $Q_1 = 300$ MW, second node generators provide the remaining $Q_2 = 200$ MW. The power flow is 100 MW. The competitive price at both nodes is $\lambda = 30$ €/MWh.

Total cost to consumers: 500 MW $\times$ 30 €/MWh $=$ 15000 €.
Generators at node 1: Revenue 300 MW $\times$ 30 €/MWh $=$ 9000 €.
Generators at node 2: Revenue 200 MW $\times$ 30 €/MWh $=$ 6000 €.

Due to the particular structure of the supply curves and the inelastic demand, market coupling has led to a higher price.

# Another example: Single market



# Back to the first example

Two nodes representing two regions, each with different total demand, using different types of generators:

**First node:** Fixed demand $Q_1^B = 500$ MW. The (inverse) supply function for the generators is given by

$$\pi_1 = MC_1 = 10 + 0.01 Q_1 \text{ [€/MWh]}$$

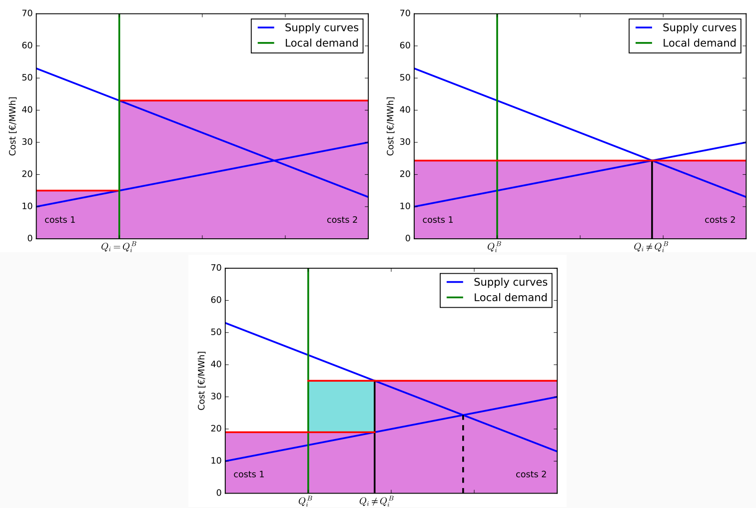
**Second node:** Fixed demand $Q_2^B = 1500$ MW. The (inverse) supply function for the generators is given by

$$\pi_2 = MC_2 = 13 + 0.02 Q_2 \text{ [€/MWh]}$$

For simplicity we assume that at both nodes the total generation limit is 5 GW.

**Transmission line** from node 1 to node 2 with capacity $K = 400$ MW.

# Example: Constrained single market



# Example: Constrained market

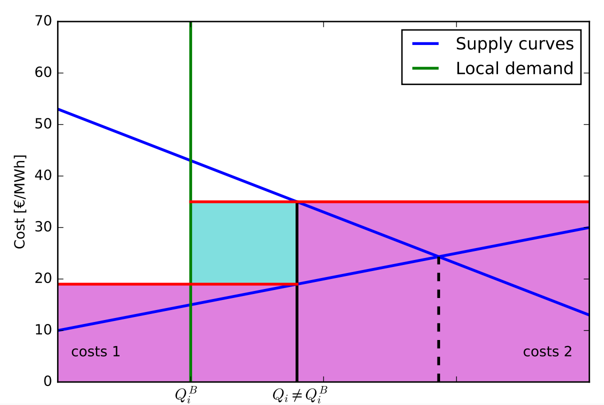
**Transmission capacity $K = 400$ MW:**

The transmission capacity is less than the power flow occuring for a single market with unconstrained transmission. The (cheaper) generators at node 1 export power until the line is congested. The (more expensive) generators then cover the remaining load at node 2.

$$MC_1(Q_1^B + K) = \lambda_1$$
$$MC_2(Q_2^B - K) = \lambda_2$$

This leads to $Q_1 = 900$ MW, $\lambda_1 = 19$ €/MWh, $Q_2 = 1100$ MW, and $\lambda_2 = 35$ €/MWh.

# Example: Constrained market



# Example: Constrained market

| | Separate markets | Single market | Constrained market |
|---|---|---|---|
| $Q_1^B$ [MW] | 500 | 500 | 500 |
| $Q_1$ [MW] | 500 | 1433 | 900 |
| $Z_1$ [MW] | 0 | $+933$ | $+400$ |
| $\lambda_1$ [€/MWh] | 15 | 24.33 | 19 |
| $Q_2^B$ [MW] | 1500 | 1500 | 1500 |
| $Q_2$ [MW] | 1500 | 567 | 1100 |
| $Z_2$ [MW] | 0 | -933 | -400 |
| $\lambda_2$ [€/MWh] | 43 | 24.33 | 35 |
| $F_{1\rightarrow 2}$ [MW] | 0 | 933 | 400 |
| $\sum_i \lambda_i \times Q_i$ [€] | 72000 | 48660 | 55600 |
| $\sum_i \lambda_i \times Q_i^B$ [€] | 72000 | 48660 | 62000 |

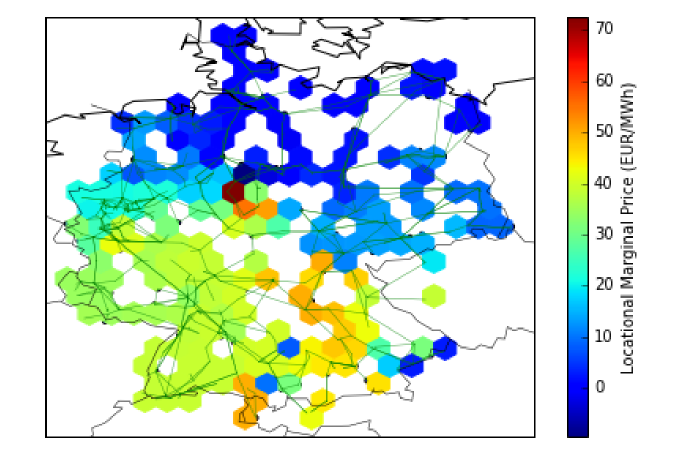
# Locational marginal pricing

Due to the congestion of the transmission line, the marginal cost of producing electricity is different at node 1 and node 2. The competitive price at node 2 is higher than at node 1 – this corresponds to locational marginal pricing, or nodal pricing.

Since consumers pay and generators get paid the price in their local market, in case of congestion there is a difference between the total payment of consumers and the total revenue of producers – this is the merchandising surplus or congestion rent, collected by the market operator. For each line it is given by the price difference in both regions times the amount of power flow between them:

$$\text{Congestion rent} = \Delta\lambda \times F$$

# Spoiler: LMP in a meshed network




Source: PyPSA
(Python for Power System Analysis)

# Redispatch

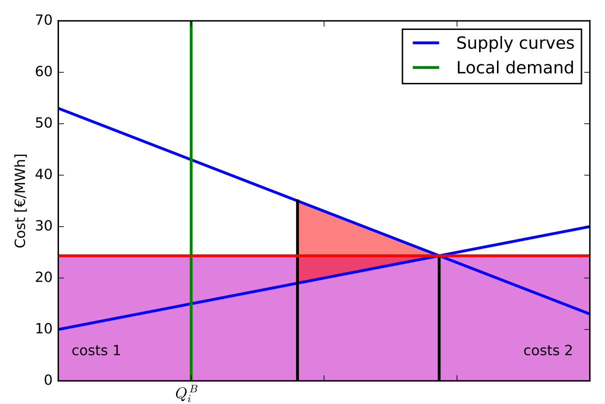
Another way to handle congestion is to correct the single market outcome retrospectively using redispatch. Consider the previous example with line capacity $K = 400$ MW.

Single market result: $Q_1 = 1433$ MW, $Q_2 = 567$ MW, market price $\lambda = 24.33$ €/MWh, power flow 933 MW.

System operator has to adjust the dispatch:

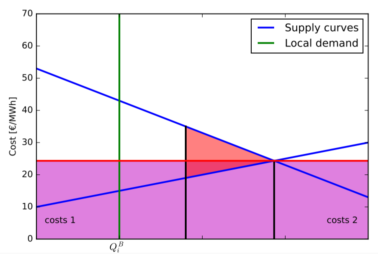
$$\Delta Q_1 = -533 \text{ MW}$$
$$\Delta Q_2 = +533 \text{ MW}$$

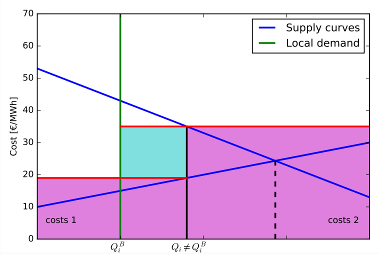
# Redispatch




Cost of redispatch: $0.5 \times (35 - 19) \times 533 = 4264$ [€/MWh]

# Redispatch vs. Nodal pricing




Note that the cost of dispatch for the generators is identical for redispatch (left) and nodal pricing (right).

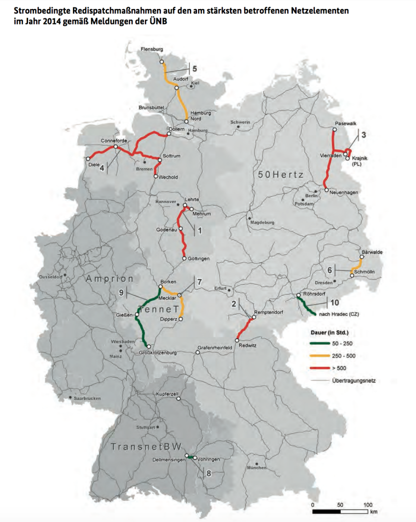
# Redispatch in Germany

**Redispatchmaßnahmen im Jahr 2014**

| Netzgebiet | Dauer in Std. | Menge getätigte Maßnahme in GWh | Gesamtmenge (getätigte Maßnahmen zzgl. Gegenschäft zum bilanziellen Ausgleich) in GWh | Saldierte Kosten für Redispatch in Mio. Euro |
|---|---|---|---|---|
| Regelzone TenneT | 5.000 | 813 | 1.629 | 186,7 |
| Regelzone 50Hertz | 3.230 | 1.751 | 3.502 | |
| Regelzone Transnet BW | 119 | 16 | 25 | |
| Regelzone Amprion | 104 | 20 | 41 | |

Source: Bundesnetzagentur/
Bundeskartellamt

# Redispatch in Germany




Source: Bundesnetzagentur/ Bundeskartellamt

# Efficient market operation in a two-node system with constrained transmission: KKT

# Optimal dispatch

Assume that there are some consumers and generators $i \in N_1$ at node 1, with generation $Q_i^S$ and consumption $Q_i^B$, and some consumers and generators $j \in N_2$ at node 2, with generation $Q_j^S$ and consumption $Q_j^B$. Node 1 and node 2 are connected by a transmission line with capacity $K$. What is the respective market price at node 1 and 2?

What is the optimal level of consumption and generation depending on the transmission capacity $K$?

Recall: "Optimal" corresponds to maximisation of total economic wellfare.

# Optimal dispatch for a single market

For unconstrained transmission we have a single market and obtain the "standard" optimisation problem:

$$\max_{\{Q_i^B, Q_j^B, Q_i^S, Q_j^S\}} \left[ \sum_i U_i(Q_i^B) + \sum_j U_i(Q_i^B) - \sum_i C_i(Q_i^S) - \sum_i C_i(Q_i^S) \right]$$

subject to the balance constraint:

$$\sum_i Q_i^B + \sum_j Q_j^B - \sum_i Q_i^S - \sum_j Q_j^S = 0 \qquad \leftrightarrow \qquad \lambda$$

and any other constraints (e.g. limits on generator capacity, etc.), which we assume to be included in the cost functions $C_i(Q_i^S)$ and utility functions $U_i(Q_i^B)$.

# Optimal dispatch for a single market

We get from stationarity

$$U_i'(Q_i^B) = \lambda^* \quad , \quad U_j'(Q_j^B) = \lambda^*$$
$$C_i'(Q_i^S) = \lambda^* \quad , \quad C_j'(Q_j^S) = \lambda^*$$

under the constraint (primal feasibility)

$$\sum_i Q_i^B + \sum_j Q_j^B - \sum_i Q_i^S - \sum_j Q_j^S = 0$$

We still assume that any other constraints (e.g. limits on generator capacity, etc.) are included in the cost functions $C_i(Q_i^S)$ and utility functions $U_i(Q_i^B)$, so we don't have any further explicit constraints.

# The benefit function

The coupling of node 1 and 2 to a single market leads to transmission between them, determined by the injection pattern $\{Z_1, Z_2\}$:

$$Z_1 = \sum_{i \in N_1} Q_i^S - \sum_{i \in N_1} Q_i^B = -Z_2 = -\left( \sum_{j \in N_2} Q_j^S - \sum_{j \in N_2} Q_j^B \right)$$

We define the benefit function $B_k(Z_k)$ of node $k$ as follows:

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[ \sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k = \sum_{i \in N_k} Q_i^S - \sum_{i \in N_k} Q_i^B \qquad \leftrightarrow \qquad \alpha_k$$

# The benefit function

We define the benefit function $B_k(Z_k)$ of node $k$ as follows:

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[ \sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k = \sum_{i \in N_k} Q_i^S - \sum_{i \in N_k} Q_i^B \qquad \leftrightarrow \qquad \alpha_k$$

The optimisation of the benefit function $B_k(Z_k)$ yields the optimal dispatch for the consumers and generators at node $k$ under the constraint, that this dispatch leads to a net injection $Z_k$ at this node.

The parameter $\alpha_k$ gives the change in the objective function when we relax the respective constraint - that is, the competitive price at this node.

# Optimal dispatch for a single market

We can rewrite the optimal dispatch task for the single market now using the benefit function:

$$\max_{\{Z_k\}} \left[ \sum_k B_k(Z_k) \right]$$

$$\text{subject to } \sum_k Z_k = 0 \qquad \leftrightarrow \qquad \lambda$$

with

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[ \sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k = \sum_{i \in N_k} Q_i^S - \sum_{i \in N_k} Q_i^B \qquad \leftrightarrow \qquad \alpha_k$$

# Optimal dispatch for a constrained market

The optimal dispatch task for the two connected nodes, with transmission constraint $K$ is then simply given by

$$\begin{aligned}
&\max_{\{Z_1, Z_2\}} \left[B_1(Z_1) + B_2(Z_2)\right] \\
&\text{subject to } Z_1 + Z_2 = 0 \qquad \leftrightarrow \qquad \lambda \\
&\text{subject to } Z_1 = -Z_2 \leq K \qquad \leftrightarrow \qquad \bar{\mu} \\
&\text{subject to } -Z_1 = Z_2 \leq K \qquad \leftrightarrow \qquad \underline{\mu}
\end{aligned}$$

with

$$\begin{aligned}
B_k(Z_k) = &\max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S)\right] \\
\text{subject to } Z_k = &\sum_{i \in N_k} Q_i^S - \sum_{i \in N_k} Q_i^B \qquad \leftrightarrow \qquad \alpha_k
\end{aligned}$$

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