Electricity Markets: Summer Semester 2016, Lecture 5

Tom Brown, Mirko Schäfer

9th May 2016

Frankfurt Institute of Advanced Studies (FIAS), Goethe-Universität Frankfurt
FIAS Renewable Energy System and Network Analysis (FRESNA)

{brown,schaefer}@fias.uni-frankfurt.de

FIAS Frankfurt Institute for Advanced Studies

Table of Contents

Supporting slides to Press Review: Splitting Germany

Splitting Germany-Austria market zone

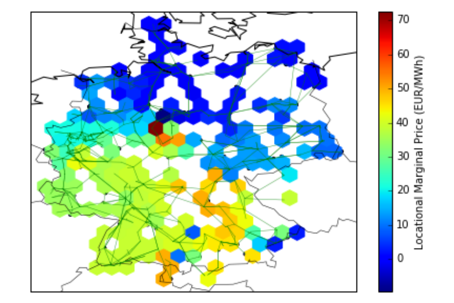
See press and research paper links at http://fias.uni-frankfurt.de/~brown/courses/electricity_markets/

- Scandinavia and Italy are split up into multiple bidding zones
- On the other hand, Germany and Austria form a joint bidding zone
- To our knowledge, Sweden was split following a complaint to the European Commission from Denmark that Danish producers could not export electricity into Sweden because of internal Swedish network bottlenecks.

Source: Ofgem

Splitting Germany-Austria market zone

In windy hours Germany also shows price divergence between North and South when looking at Locational Marginal Prices. Time to split the Germany-Austria bidding zone so that the market sees the transmission bottlenecks? Alternative is increasingly expensive redispatch measures...



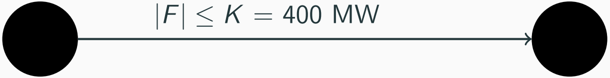
Source: PyPSA

Recap of two-node example from last time

Two-node transmission example

Revisit example from Kirschen and Strbac 6.3.1.2, page 152.

We have two nodes with fixed consumption and differently-priced producers, connected via a transmission line of limited capacity:



Node 1	Node 2
$Q_1^B = 500$ MW	$Q_2^B = 1500$ MW
$C_1'(Q_1) = 10 + 0.01 Q_1$	$C_2'(Q_2) = 13 + 0.02 Q_2$
$Z_1 = Q_1 - Q_1^B$	$Z_2 = Q_2 - Q_2^B$

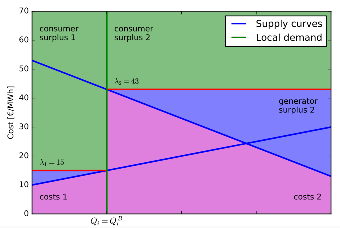
In addition we can determine the flow between the nodes from the nodal imbalances Z_i:

$$F = Z_1 = -Z_2$$

Outcomes for different values of transmission capacity K

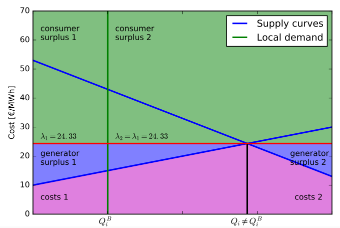
$K = 0, F = 0,$

$Q_1^* = 500, Q_2^* = 1500$



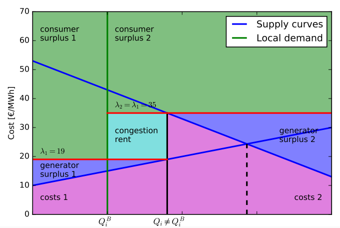
$K = \infty, F = 933$

$Q_1^* = 1433, Q_2^* = 567$



$K = 400, F = 400$

$Q_1^* = 900, Q_2^* = 1100$



Example: Numbers for different values of K

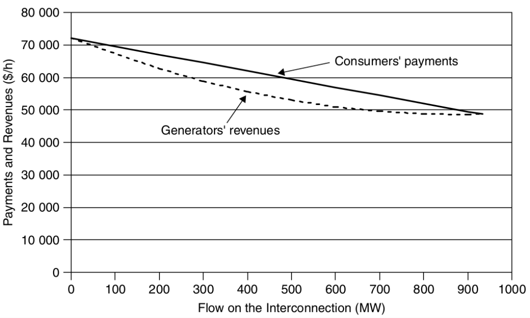
	Separate markets	Single market	Constrained market
Q_1^B [MW]	500	500	500
Q_1 [MW]	500	1433	900
Z_1 [MW]	0	$+933$	$+400$
λ_1 [€/MWh]	15	24.33	19
Q_2^B [MW]	1500	1500	1500
Q_2 [MW]	1500	567	1100
Z_2 [MW]	0	-933	-400
λ_2 [€/MWh]	43	24.33	35
$F_{1 \to 2}$ [MW]	0	933	400
$\sum_i \lambda_i \times Q_i$ [€]	72000	48660	55600
$\sum_i \lambda_i \times Q_i^B$ [€]	72000	48660	62000
Congestion rent	0	0	6400

Example: Congestion rent for different values of K

The congestion rent for the two-node example is given by

$$\text{Congestion rent} = |\lambda_1 - \lambda_2| \times |F|$$

As a function of K:



Efficient market operation in a multi-node system with constrained transmission: KKT

Optimising a multi-node system

We want answers to the following questions:

1. What is the most efficient configuration of production and consumption when there are transmission constraints between nodes?
2. How should the market price be set at each node to guarantee that decentralised actors reach a system-optimal solution?
3. How does this fit in the Karush-Kuhn-Tucker framework?

Recap of optimisation for a single node

Without transmission we maximised the total economic welfare, the sum of the consumer and the producer surplus for consumers with consumption Q_i^B and generators generating with rate Q_i^S:

$$\max_{\{Q_i^B\},\{Q_i^S\}} \left[\sum_i U_i(Q_i^B) - \sum_i C_i(Q_i^S) \right]$$

subject to the supply equalling the demand in the balance constraint:

$$\sum_i Q_i^B - \sum_i Q_i^S = 0 \qquad \leftrightarrow \qquad \lambda$$

where λ gave us the market price.

How do we then extend this scheme to multiple nodes with transmission constraints inbetween?

Answer: Maximise the combined sum of welfare at each node while implementing transmission constraints.

Nodal benefit function

Suppose at node k there are some consumers and generators $i \in N_k$, with generation Q_i^S and consumption Q_i^B.

We define the benefit function $B_k(Z_k)$ of node k as follows:

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

where we have introduced a new variable Z_k for the total nodal power imbalance (supply - demand) at the node

$$Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

The optimisation of the benefit function $B_k(Z_k)$ yields the optimal dispatch for the consumers and generators at node k under the constraint that this dispatch leads to a net injection Z_k at this node.

The parameter λ_k gives the change in the objective function when we relax the respective constraint - i.e. the marginal price at this node.

Full optimisation problem

Note: the values of the Z_k are not yet fixed by the scheme. Now we fix the values by maximising total economic welfare given constraints for the nodal injections (determined by the transmission constraints):

$$\max_{\{Z_k\}} \left[\sum_k B_k(Z_k) \right]$$

subject to

$$\sum_k Z_k = 0 \qquad \leftrightarrow \qquad \lambda$$

$$h_\ell(\{Z_k\}) \leq d_\ell \qquad \leftrightarrow \qquad \mu_\ell$$

with

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

Optimal dispatch for two-nodes

We now return to our two-node example. We have a flow on the single transmission line $F = Z_1 = -Z_2$ restricted by $|F| \leq K$.

The optimal dispatch is given by

$$\max_{\{Z_1, Z_2\}} [B_1(Z_1) + B_2(Z_2)]$$

$$\text{subject to } Z_1 + Z_2 = 0 \qquad \leftrightarrow \qquad \lambda$$

$$\text{subject to } Z_1 \leq K \qquad \leftrightarrow \qquad \bar{\mu}$$

$$\text{subject to } -Z_1 \leq K \qquad \leftrightarrow \qquad \underline{\mu}$$

with

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

KKT analysis

Considering the single total optimisation over all variables Q_i^B, Q_i^S, Z_k, we get from stationarity

$$\frac{\partial \mathcal{L}}{dQ_i^B} \Rightarrow U_i'(Q_i^B) - \lambda_k = 0$$
$$\frac{\partial \mathcal{L}}{dQ_i^S} \Rightarrow -C_i'(Q_i^S) + \lambda_k = 0$$
$$\frac{\partial \mathcal{L}}{dZ_1} \Rightarrow +\lambda - \lambda_1 - \bar{\mu} + \underline{\mu} = 0$$
$$\frac{\partial \mathcal{L}}{dZ_2} \Rightarrow +\lambda - \lambda_2 = 0$$

and from complementary slackness:

$$\bar{\mu}(K - Z_1) = 0$$
$$\underline{\mu}(K + Z_1) = 0$$

Our three cases

For a solution where typically $\lambda_2^* \geq \lambda_1^*$ we have:

For the separate markets ($K = 0$):

$$F = Z_1^* = -Z_2^* = 0, \lambda_1^* \neq \lambda_2^*, \bar{\mu}^* = \lambda_2^* - \lambda_1^*, \underline{\mu}^* = 0$$

For the constrained markets ($K = 400$):

$$F = Z_1^* = -Z_2^* = 400, \lambda_1^* \neq \lambda_2^*, \bar{\mu}^* = \lambda_2^* - \lambda_1^*, \underline{\mu}^* = 0$$

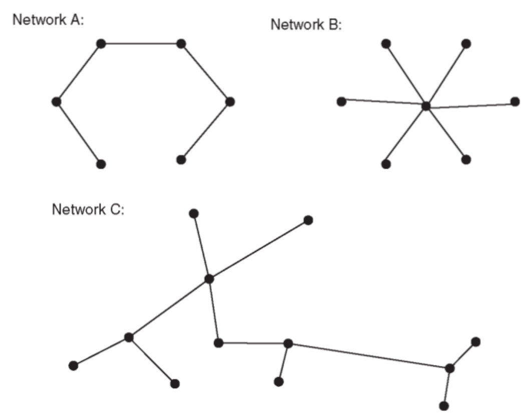
For the unconstrained markets ($K = \infty$):

$$F = Z_1^* = -Z_2^* = 933, \lambda_1^* = \lambda_2^*, \bar{\mu}^* = 0, \underline{\mu}^* = 0$$

Beyond two nodes: radial networks

In a radial network there is only one path between any two nodes on the network.

The power flow is a simple function of the nodal power imbalances.

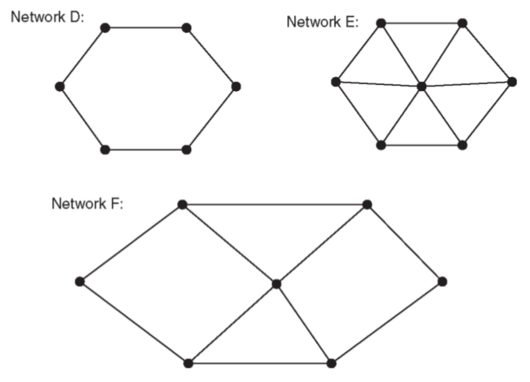


Source: Biggar & Hesamzadeh

Beyond two nodes: meshed networks

In a meshed network there are at least two nodes with multiple paths between them.

The power flow is now a function of the impedances in the network.



Source: Biggar & Hesamzadeh

Long-run efficiency: Investment in Generation

Definition of long-run efficiency

Up until now we have considered short-run equilibria that ensure short-run efficiency (static), i.e. they make the best use of presently available productive resources.

Long-run efficiency (dynamic) requires in addition the optimal investment in productive capacity.

Concretely: given a set of options and constraints for different generators (nuclear/gas/wind/solar) what is the optimal generation portfolio for maximising long-run welfare?

From an indivdual generators' perspective: how best should I invest in extra capacity?

We will show again that with perfect competition and no barriers to entry, the system-optimal situation can be reached by individuals following their own profit.

Simple example: Single generator type with downward sloping demand

Consider the long-run efficiency of a market with a single generator type with linear cost function and downward-sloping demand (taken from Biggar-Hesamzadeh pages 21 and 183).

We have to consider marginal costs arising from each unit of production Q and capital costs that arise from fixed costs regardless of the rate of production (such as the investment in building capacity K).

For a given production rate Q and capacity K we have in this simple example a cost

$$C(Q, K) = cQ + fK$$

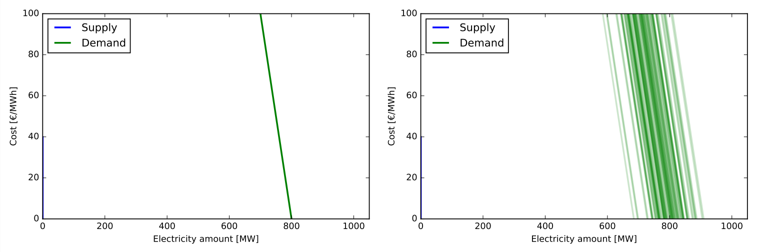
with $0 \leq Q \leq K$, where $C(Q, K)$ has units €/h, c has units €/MWh, Q and K have units MW and f has units €/MW/h ('hourised' capital cost).

Note again: the term fK is constant regardless of production rate Q.

Can't just consider just one load situation

Up until now, in our considerations of short-run efficiency, we've considered just a single demand situation.

Now that we're considering long-term investment, we have to consider many or even all demand situations.

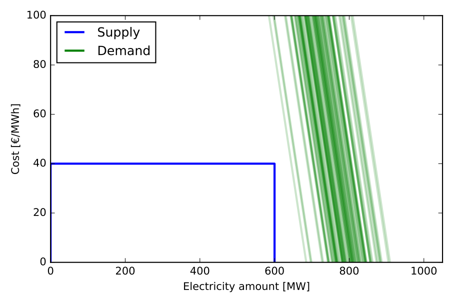


We consider many different utility curves $U_t(Q)$ for different times t, each of which occurs with probability $p_t > 0$, $\sum p_t = 1$.

Simple example: Consumer with downward sloping demand

Suppose the generators have a marginal cost of $c = 40$ €/MWh and the downward-sloping demand fluctuates over time.

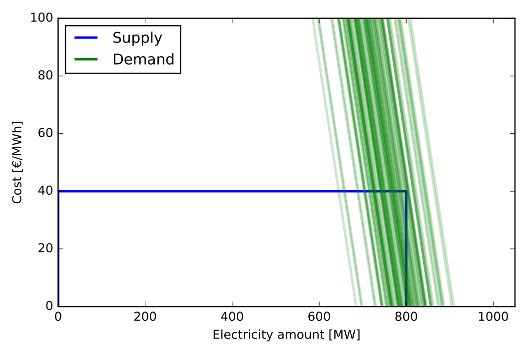
If total generation capacity is always below demand, the demand will set the price at MCB (Marginal Consumer Benefit) and the generators will always earn above their Marginal Generation Cost (MGC):



But then why don't they build more capacity to make even more profit?

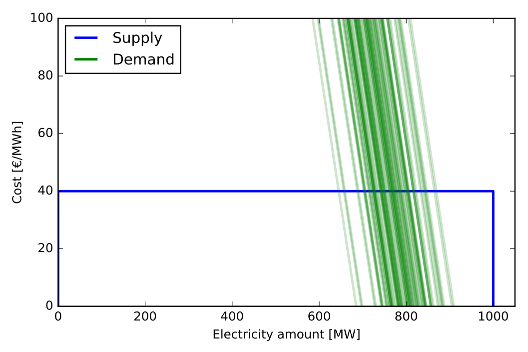
Simple example: Consumer with downward sloping demand

If sometimes the price is set by MCB and sometimes by the MGC then the generators might still earn enough to cover their capital costs:



Simple example: Consumer with downward sloping demand

If generation capacity is so large that it can always cover the demand, regardless of the MCB, then generators will never earn enough money to regain their capital costs, because the price will always be set by the marginal generation cost:



Simple example: optimisation problem

Now consider the maximisation of long-run welfare, including the capital costs:

$$\max_{\{Q_t^B\},\{Q_t^S\},K} \sum_t p_t \left[U_t(Q_t^B) - C(Q_t^S, K) \right]$$

i.e. with cost $C(Q,K) = cQ + fK$ we optimise

$$\max_{\{Q_t^B\},\{Q_t^S\},K} \sum_t p_t \left[U_t(Q_t^B) - (cQ_t^S + fK) \right]$$

given

$$\begin{aligned} Q_t^B - Q_t^S &= 0 & \leftrightarrow & \quad p_t \lambda_t & \forall t \\ -Q_t^S &\leq 0 & \leftrightarrow & \quad p_t \underline{\mu}_t & \forall t \\ Q_t^S &\leq K & \leftrightarrow & \quad p_t \bar{\mu}_t & \forall t \end{aligned}$$

(We have taken the liberty to multiply the KKT multipliers by a constant $p_t > 0$, to make the resulting equations easier to read.)

Simple example: KKT

From stationarity we get:

$$\frac{\partial \mathcal{L}}{\partial Q_t^B} \Rightarrow p_t U_t'(Q_t^B) - p_t \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial Q_t^S} \Rightarrow -p_t c + p_t \lambda_t + p_t \underline{\mu}_t - p_t \bar{\mu}_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial K} \Rightarrow -f + \sum_t p_t \bar{\mu}_t = 0$$

From primal feasibility we get $Q_t^B = Q_t^S = Q_t^*$ and from complementary slackness we have $\underline{\mu}_t^* = 0$, assuming the demand is always positive, and $\bar{\mu}_t^* \geq 0$. Thus we get

$$\lambda_t^* = U_t'(Q_t^*)$$

$$\lambda_t^* = c + \bar{\mu}_t^*$$

$$f = \sum_t p_t \bar{\mu}_t^*$$

Simple example: KKT interpretation

We have

$$\lambda_t^* = U_t'(Q_t^*)$$
$$\lambda_t^* = c + \bar{\mu}_t^*$$
$$f = \sum_t p_t \bar{\mu}_t^*$$

So $\bar{\mu}_t^*$ is the difference between the Marginal Generation Cost (MGC) c and the Marginal Consumer Benefit (MCB) $U_t'(Q_t^*)$.

If the constraint $Q_t \leq K$ is binding, then $\bar{\mu}_t^* \geq 0$.

The optimal investment level happens when the average value of $\bar{\mu}_t^*$, $\sum_t p_t \bar{\mu}_t^*$, is equal to the capital cost f.

Different types of generators

Different types of generators

Fuel/Prime mover	Marginal cost	Capital cost	Controllable	Predictable days ahead	CO2
Oil	V. High	Low	Yes	Yes	Medium
Gas OCGT	High	Low	Yes	Yes	Medium
Gas CCGT	Medium	Medium	Yes	Yes	Medium
Hard Coal	Medium	Lowish	Yes	Yes	High
Brown Coal	Low	Medium	Yes	Yes	High
Nuclear	V. Low	High	Partly	Yes	Zero
Hydro dam	Zero	High	Yes	Yes	Zero
Wind/Solar	Zero	High	Down	No	Zero

Copyright