Electricity Markets: Summer Semester 2016, Lecture 6

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'Homework'

We have some flexibility for the later lectures and will try to adjust the topics according to your interests. As homework, please send us by next week three questions you would like us to cover in these lectures.

Examples could be 'How was the electricity system in Japan structured before the Fukushima incident? What has changed?', 'When and how was the EEG in Germany implemented? How did it change over time?', 'What is the role of hydro power in todays electricity system?',etc.

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Table of Contents

Recap: Efficient market operation in a multi-node system with constrained transmission: KKT

Optimising a multi-node system

We want answers to the following questions:

1. What is the most efficient configuration of production and consumption when there are transmission constraints between nodes?
2. How should the market price be set at each node to guarantee that decentralised actors reach a system-optimal solution?
3. How does this fit in the Karush-Kuhn-Tucker framework?

Recap of optimisation for a single node

Without transmission we maximised the total economic welfare, the sum of the consumer and the producer surplus for consumers with consumption Q_i^B and generators generating with rate Q_i^S:

$$\max_{\{Q_i^B\},\{Q_i^S\}} \left[\sum_i U_i(Q_i^B) - \sum_i C_i(Q_i^S)\right]$$

subject to the supply equalling the demand in the balance constraint:

$$\sum_i Q_i^B - \sum_i Q_i^S = 0 \qquad \leftrightarrow \qquad \lambda$$

where λ gave us the market price.

How do we then extend this scheme to multiple nodes with transmission constraints inbetween?

Answer: Maximise the combined sum of welfare at each node while implementing transmission constraints.

Nodal benefit function

Suppose at node k there are some consumers and generators $i \in N_k$, with generation Q_i^S and consumption Q_i^B.

We define the benefit function $B_k(Z_k)$ of node k as follows:

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

where we have introduced a new variable Z_k for the total nodal power imbalance (supply - demand) at the node

$$Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

The optimisation of the benefit function $B_k(Z_k)$ yields the optimal dispatch for the consumers and generators at node k under the constraint that this dispatch leads to a net injection Z_k at this node.

The parameter λ_k gives the change in the objective function when we relax the respective constraint - i.e. the marginal price at this node.

Full optimisation problem

Note: the values of the Z_k are not yet fixed by the scheme. Now we fix the values by maximising total economic welfare given constraints for the nodal injections (determined by the transmission constraints):

$$\max_{\{Z_k\}} \left[\sum_k B_k(Z_k) \right]$$

subject to

$$\sum_k Z_k = 0 \qquad \leftrightarrow \qquad \lambda$$

$$h_\ell(\{Z_k\}) \leq d_\ell \qquad \leftrightarrow \qquad \mu_\ell$$

with

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

Optimal dispatch for two-nodes

We now return to our two-node example. We have a flow on the single transmission line $F = Z_1 = -Z_2$ restricted by $|F| \leq K$.

The optimal dispatch is given by

$$\begin{aligned} &\max_{\{Z_1, Z_2\}} [B_1(Z_1) + B_2(Z_2)] \\ &\text{subject to } Z_1 + Z_2 = 0 \qquad \leftrightarrow \qquad \lambda \\ &\text{subject to } Z_1 \leq K \qquad \leftrightarrow \qquad \bar{\mu} \\ &\text{subject to } -Z_1 \leq K \qquad \leftrightarrow \qquad \underline{\mu} \end{aligned}$$

with

$$\begin{aligned} B_k(Z_k) = &\max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right] \\ \text{subject to } &Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k \end{aligned}$$

KKT analysis

Considering the single total optimisation over all variables Q_i^B, Q_i^S, Z_k, we get from stationarity

$$\frac{\partial \mathcal{L}}{dQ_i^B} \Rightarrow U_i'(Q_i^B) - \lambda_k = 0$$
$$\frac{\partial \mathcal{L}}{dQ_i^S} \Rightarrow -C_i'(Q_i^S) + \lambda_k = 0$$
$$\frac{\partial \mathcal{L}}{dZ_1} \Rightarrow \lambda - \lambda_1 - \bar{\mu} + \underline{\mu} = 0$$
$$\frac{\partial \mathcal{L}}{dZ_2} \Rightarrow \lambda - \lambda_2 = 0$$

and from complementary slackness:

$$\bar{\mu}(K - Z_1) = 0$$
$$\underline{\mu}(K + Z_1) = 0$$

Our three cases

For a solution where typically $\lambda_2^* \geq \lambda_1^*$ we have:

For the separate markets ($K = 0$):

$$F = Z_1^* = -Z_2^* = 0, \lambda_1^* \neq \lambda_2^*, \bar{\mu}^* = \lambda_2^* - \lambda_1^*, \underline{\mu}^* = 0$$

For the constrained markets ($K = 400$):

$$F = Z_1^* = -Z_2^* = 400, \lambda_1^* \neq \lambda_2^*, \bar{\mu}^* = \lambda_2^* - \lambda_1^*, \underline{\mu}^* = 0$$

For the unconstrained markets ($K = \infty$):

$$F = Z_1^* = -Z_2^* = 933, \lambda_1^* = \lambda_2^*, \bar{\mu}^* = 0, \underline{\mu}^* = 0$$

Representing network constraints in meshed networks

Full optimisation problem

We fix the values Z_k (nodal imbalances) by maximising total economic welfare given constraints for the nodal injections (determined by the transmission constraints):

$$\max_{\{Z_k\}} \left[\sum_k B_k(Z_k) \right]$$

subject to

$$\sum_k Z_k = 0 \qquad \leftrightarrow \qquad \lambda$$

$$h_\ell(\{Z_k\}) \leq d_\ell \qquad \leftrightarrow \qquad \mu_\ell$$

with

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

Network constraints

In the full optimisation problem the network constraints are included via the terms

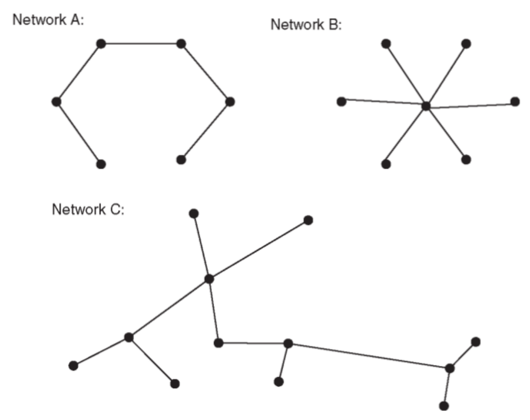
$$h_l(\{Z_k\} \leq d_l) \ .$$

Here Z_k are the nodal imbalances (the injection pattern) and d_l are thermal capacity limits (often written as K_l). The functions h_l relate the injection pattern (node properties) to the transmission constraints by giving the respective power flow (line properties).
What is the structure of the functions $h_l(\{Z_k\})$ for general network topologies?

Beyond two nodes: radial networks

In a radial network there is only one path between any two nodes on the network.

The power flow is a simple function of the nodal power imbalances.

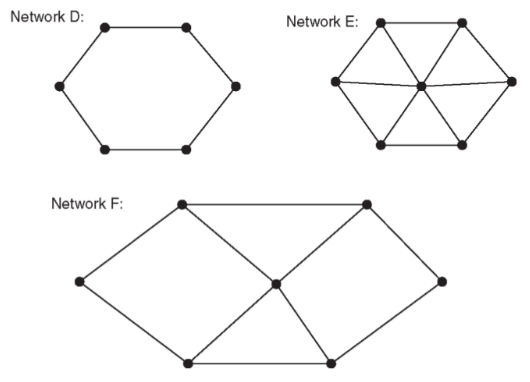


Source: Biggar & Hesamzadeh

Beyond two nodes: meshed networks

In a meshed network there are at least two nodes with multiple paths between them.

The power flow is now a function of the impedances in the network.



Source: Biggar & Hesamzadeh

The DC Load Flow model

Modelling the power flows in the transmission grid accurately requires to solve the physical AC equations, which is complicated.

For stable network operation, the DC Load Flow model is a reasonable approximation. In this model, the power flow F_l over a link l is a linear function of the nodal imbalances Z_k:

$$F_l = \sum_k H_{lk} Z_k \ ,$$

or in matrix notation (with $\mathbf{F}$ the vector of power flows, and $\mathbf{Z}$ the injection pattern)

$$\mathbf{F} = \mathbf{H}\mathbf{Z} \ .$$

The matrix $\mathbf{H}$ is denoted as the power transfer distribution factors (PTDF) matrix. It is calculated from the topology of the network and the physical properties of the connections (impedance) via Kirchhoff's Current and Voltage law (KCL, KVL).

Some degrees of freedom for the PTDF matrix

For a balanced injection pattern the nodal imbalances sum up to zero:

$$\sum_k Z_k = 0 \ .$$

For balanced injection patterns we have some degrees of freedom for the PTDF matrix entries:

$$\begin{aligned} F_l &= \sum_k \left(H_{lk} + c_l\right) Z_k \\ &= \sum_k H_{lk} Z_k + c_l \left(\sum_k Z_k \right) \\ &= \sum_k H_{lk} Z_k \ . \end{aligned}$$

Adding the constant c_l to all entries of row l does not change the result, that is instead of the PTDF matrix H_{lk} we can also use the PTDF matrix $H_{lk} + c_l$ with arbitrary values of c_l.

The reference node

From any PTDF matrix H_{lk} we can construct a PTDF matrix $H_{lk}^{(n)}$ which has zero entries $H_{ln}^{(n)} = 0$ in row n:

$$H_{lk}^{(n)} = H_{lk} - H_{ln} \ .$$

In this case node n is denoted as the reference node.

Example: Two nodes

One line with label $l = 1$ from node 1 to node 2, which have nodal imbalances Z_1 and Z_2. Choose node 2 as the reference node. The PTDF matrix is a 1×2 matrix with entries

$$\mathbf{H} = 1 \rightarrow 2 \begin{pmatrix} 1 & 0 \end{pmatrix}$$

The power flow on line 1 is calculated as

$$\begin{aligned} F_1 &= H_{11} Z_1 + H_{12} Z_2 \\ &= Z_1 \ . \end{aligned}$$

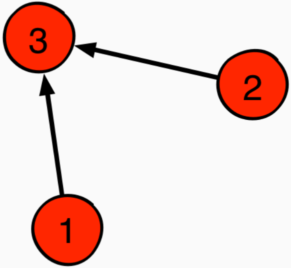
Example: Three nodes I

Reference node 3.

$$\mathbf{H} = \begin{array}{c} 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Power flows:

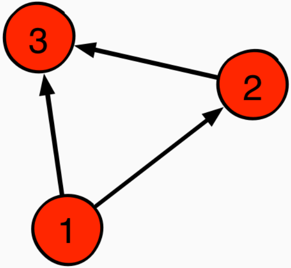
$$F_{1\to 3} = Z_1$$
$$F_{2\to 3} = Z_2$$



Example: Three nodes II

Reference node 3.

$$
\mathbf{H} = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix}
$$



Power flows:

$$
F_{1\to 2} = \frac{1}{3}\left(Z_1 - Z_2\right)
$$

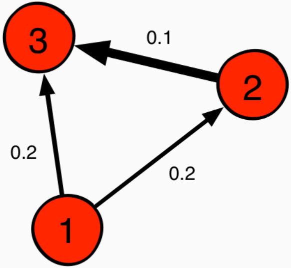
$$
F_{1\to 3} = \frac{1}{3}\left(2Z_1 + Z_2\right)
$$

$$
F_{2\to 3} = \frac{1}{3}\left(Z_1 + 2Z_2\right)
$$

Example: Three nodes III

Remark: Different line impedances (reactances) change the PTDF matrix and thus the power flow!

$$\mathbf{H} = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 2/5 & -1/5 & 0 \\ 3/5 & 1/5 & 0 \\ 2/5 & 4/5 & 0 \end{pmatrix}$$



Power flows:

$$F_{1\to 2} = \frac{1}{5}\left(2Z_1 - Z_2\right)$$

$$F_{1\to 3} = \frac{1}{5}\left(3Z_1 + Z_2\right)$$

$$F_{2\to 3} = \frac{2}{5}\left(Z_1 + 2Z_2\right)$$

Transmission constraints

Thermal limits as transmission constraints for the optimisation problem can be represented as general inequality constraints:

$$h_l(\{Z_k\}) \leq d_l \ .$$

In our treatment, we model these constraints as capacity limits, that is as upper boundaries for the power flow on a link:

$$\begin{aligned} F_l(Z_k) &\leq K_l \\ -F_l(Z_k) &\leq K_l \end{aligned}$$

Using the PTDF matrix, this reads

$$\begin{aligned} \left[\sum_k H_{lk} Z_k\right] &\leq K_l \\ -\left[\sum_k H_{lk} Z_k\right] &\leq K_l \end{aligned}$$

Feasible injections

The set of feasible injections represents the collection of all balanced injection patterns $\{Z_k\}$ which satisfy the transmission constraint equations

$$\left[\sum_k H_{lk} Z_k\right] \leq K_l$$
$$-\left[\sum_k H_{lk} Z_k\right] \leq K_l$$

Example: Three nodes I

Reference node 3.

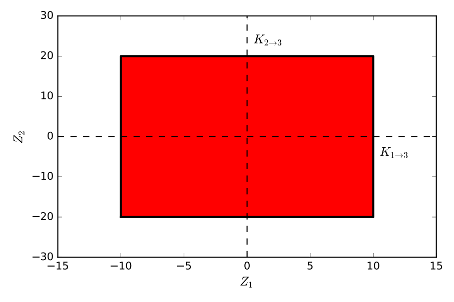
$$\mathbf{H} = \begin{array}{c} 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

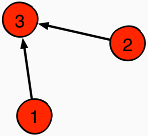
Power flows:

$$F_{1\to 3} = Z_1$$
$$F_{2\to 3} = Z_2$$

Capacity limits:

$$K_{1\to 3} = 10 \text{ MW}$$
$$K_{2\to 3} = 20 \text{ MW}$$





Example: Three nodes II

Reference node 3.

$$\mathbf{H} = \begin{array}{c} 1\to 2 \\ 1\to 3 \\ 2\to 3 \end{array} \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix}$$

Power flows:

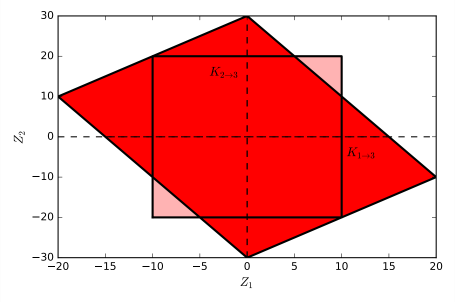
$$F_{1\to 2} = \frac{1}{3}(Z_1 - Z_2)$$

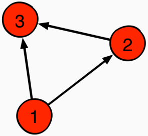
$$F_{1\to 3} = \frac{1}{3}(2Z_1 + Z_2)$$

$$F_{2\to 3} = \frac{1}{3}(Z_1 + 2Z_2)$$

Capacity limits:

$$(K_{1\to 2}, K_{2\to 3}, K_{1\to 3}) = (10, 20, 10) \text{ MW}$$





Full optimisation problem

We fix the values Z_k (nodal imbalances) by maximising total economic welfare given constraints for the nodal injections (determined by the transmission constraints):

$$\max_{\{Z_k\}} \left[\sum_k B_k(Z_k) \right]$$

subject to

$$\sum_k Z_k = 0 \qquad \leftrightarrow \qquad \lambda$$

$$\pm \left[\sum_k H_{\ell k} Z_k \right] \leq K_\ell \qquad \leftrightarrow \qquad \bar{\mu}_\ell, \underline{\mu}_\ell$$

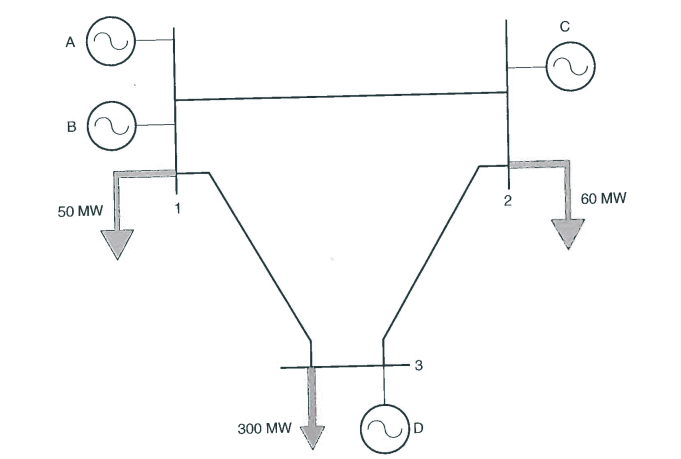
with

$$B_k(Z_k) = \max_{\{Q_i^B, Q_i^S\}} \left[\sum_{i \in N_k} U_i(Q_i^B) - \sum_{i \in N_k} C_i(Q_i^S) \right]$$

$$\text{subject to } Z_k - \sum_{i \in N_k} Q_i^S + \sum_{i \in N_k} Q_i^B = 0 \qquad \leftrightarrow \qquad \lambda_k$$

Example: Efficient market operation in a 3-node system with constrained transmission

An exemplary 3-node system



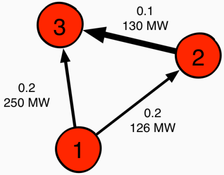
Source: Kirschen and Strbac

An exemplary 3-node system

Example taken from Kirschen and Strbac, Chapter 6.

Generator	Capacity (MW)	Marginal cost (€/MWh)
A	140	7.5
B	285	6
C	90	14
D	85	10
Line	**Reactance (p.u.)**	**Capacity (MW)**
$1 \rightarrow 2$	0.2	126
$1 \rightarrow 3$	0.2	250
$2 \rightarrow 3$	0.1	130

Power flows and feasible injections



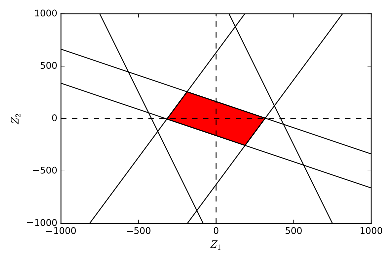
$$\mathbf{H} = \begin{array}{c} 1 \to 2 \\ 1 \to 3 \\ 2 \to 3 \end{array} \begin{pmatrix} 2/5 & -1/5 & 0 \\ 3/5 & 1/5 & 0 \\ 2/5 & 4/5 & 0 \end{pmatrix}$$

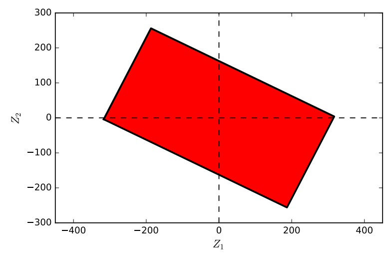
Power flows:

$$F_{1\to2} = \frac{1}{5}(2Z_1 - Z_2)$$

$$F_{1\to3} = \frac{1}{5}(3Z_1 + Z_2)$$

$$F_{2\to3} = \frac{2}{5}(Z_1 + 2Z_2)$$



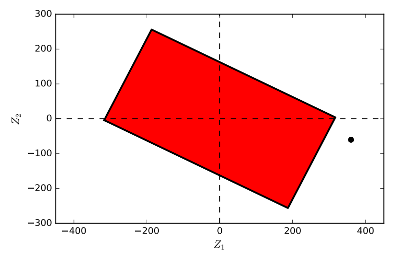


Economic dispatch

Market price: $\lambda = 7.5$ €/MWh

$$Q_A^S = 125 \text{ MW}$$
$$Q_B^S = 285 \text{ MW}$$
$$Q_C^S = 0 \text{ MW}$$
$$Q_D^S = 0 \text{ MW}$$

$$K_{1\to 2} = 126 \text{ MW}$$
$$K_{1\to 3} = 250 \text{ MW}$$
$$K_{2\to 3} = 130 \text{ MW}$$



$Z_1 = 360$ MW, $Z_2 = -60$ MW.

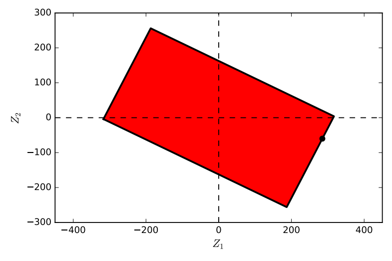
$$F_{1\to 2} = \frac{1}{5}(2Z_1 - Z_2) = 156 \text{ MW}$$
$$F_{1\to 3} = \frac{1}{5}(3Z_1 + Z_2) = 204 \text{ MW}$$
$$F_{2\to 3} = \frac{2}{5}(Z_1 + 2Z_2) = 96 \text{ MW}$$

Economic redispatch

$$Q_A^S = 125 \text{ MW} \rightarrow Q_A^S = 50 \text{ MW}$$
$$Q_B^S = 285 \text{ MW} \rightarrow Q_B^S = 285 \text{ MW}$$
$$Q_C^S = 0 \text{ MW} \rightarrow Q_C^S = 0 \text{ MW}$$
$$Q_D^S = 0 \text{ MW} \rightarrow Q_D^S = 75 \text{ MW}$$

$$K_{1\rightarrow 2} = 126 \text{ MW}$$
$$K_{1\rightarrow 3} = 250 \text{ MW}$$
$$K_{2\rightarrow 3} = 130 \text{ MW}$$

Redispatch cost: 187.5 €/h



$$F_{1\rightarrow 2} = \frac{1}{5}(2Z_1 - Z_2) = 126 \text{ MW}$$
$$F_{1\rightarrow 3} = \frac{1}{5}(3Z_1 + Z_2) = 159 \text{ MW}$$
$$F_{2\rightarrow 3} = \frac{2}{5}(Z_1 + 2Z_2) = 66 \text{ MW}$$

$Z_1 = 275$ MW, $Z_2 = -60$ MW.

Nodal prices

The nodal marginal price is equal to the minimal system cost of supplying an additional megawatt of load at this node.

$$Q_A^S = 50 \text{ MW}$$
$$Q_B^S = 285 \text{ MW}$$
$$Q_C^S = 0 \text{ MW}$$
$$Q_D^S = 75 \text{ MW}$$

$$K_{1\to2} = 126 \text{ MW}$$
$$K_{1\to3} = 250 \text{ MW}$$
$$K_{2\to3} = 130 \text{ MW}$$

Nodal prices:

$$\lambda_1 = c_A = 7.5 \text{ €/MWh}$$
$$\lambda_3 = c_D = 10 \text{ €/MWh}$$
$$\lambda_2 = 1.5 \times c_D - 0.5 \times c_A = 11.25 \text{ €/MWh}$$

$$F_{1\to2} = \frac{1}{5}(2Z_1 - Z_2) = 126 \text{ MW}$$
$$F_{1\to3} = \frac{1}{5}(3Z_1 + Z_2) = 159 \text{ MW}$$
$$F_{2\to3} = \frac{2}{5}(Z_1 + 2Z_2) = 66 \text{ MW}$$

Nodal prices

Economic operation of the three-node system using nodal pricing.

	Node 1	Node 2	Node 3	System
Consumption (MW)	50	60	300	410
Production (MW)	335	0	75	410
Nodal marginal price (€/MWh)	7.5	11.25	10	-
Consumer payments (€/h)	375	675	3000	4050
Generator revenue (€/h)	2512.5	0	750	3262.5
Congestion rent (€/h)				787.5

Nodal prices

Congestion rent:

Connection	Flow (MW)	'From' price (€/MWh)	'To' price (€/MWh)	Surplus (€/h)
$1 \rightarrow 2$	126	7.5	11.25	427.5
$1 \rightarrow 3$	159	7.5	10	397.5
$2 \rightarrow 3$	66	11.25	10	-82.5
Total				787.5

Note the counter-intuitive flow from node 2 (higher price) to node 3 (lower price)!

Example slightly changed

The nodal marginal price is equal to the minimal system cost of supplying an additional megawatt of load at this node.

$$Q_A^S = 47.5 \text{ MW}$$
$$Q_B^S = 285 \text{ MW}$$
$$Q_C^S = 0 \text{ MW}$$
$$Q_D^S = 77.5 \text{ MW}$$

$$K_{1\to2} = 126 \text{ MW}$$
$$K_{1\to3} = 250 \text{ MW}$$
$$K_{2\to3} = \mathbf{65} \text{ MW}$$

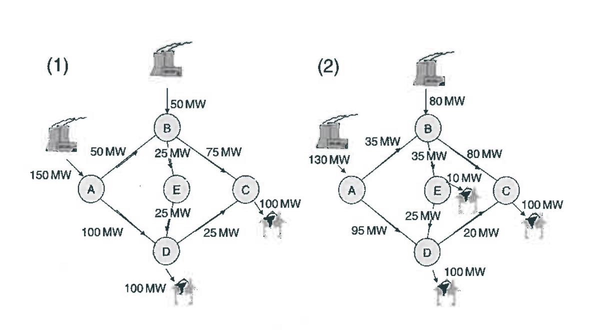
Nodal prices:

$$\lambda_1 = c_A = 7.5 \text{ €/MWh}$$
$$\lambda_3 = c_D = 10 \text{ €/MWh}$$
$$\lambda_2 = 2 \times c_A - 1 \times c_D = 5 \text{ €/MWh}$$

$$F_{1\to2} = \frac{1}{5}(2Z_1 - Z_2) = 125 \text{ MW}$$
$$F_{1\to3} = \frac{1}{5}(3Z_1 + Z_2) = 157.5 \text{ MW}$$
$$F_{2\to3} = \frac{2}{5}(Z_1 + 2Z_2) = 65 \text{ MW}$$

The nodal marginal price at node 2 is lower than the marginal cost of any generator!

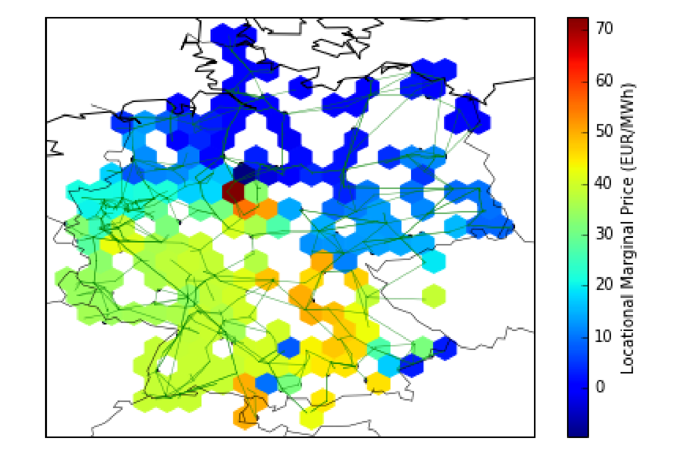
Negative nodal prices



Generator A has marginal costs 60€/MWh, generator B has marginal costs 30€/MWh. The line between E and D is constrained to 25 MW. The additional load of 10 MW at node E reduces the system cost by 300 €/h, so $\lambda_E = -30$ €/MWh!

Source: Harris

Nodal prices for Germany



Source: PyPSA
(Python for Power System Analysis)

Application: Flow Based Market Coupling

ENTSO-E

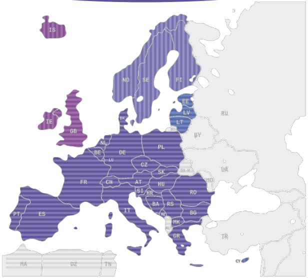
About ENTSO-E

41 TSOs
from 34 countries

532 million
citizens served

828 GW
generation

305 Thousand Km
of transmission lines



Ten-Year Network Development Plans

Adequacy forecasts

R&D plans

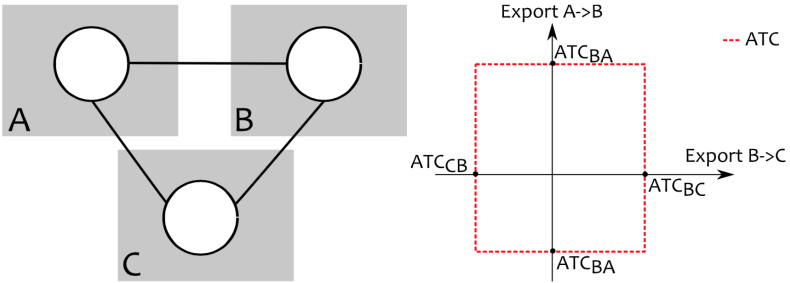
Tools for Market Integration

Network Codes

entsoe networkcodes.eu

Source: ENTSO-E

Available Transfer Capacity

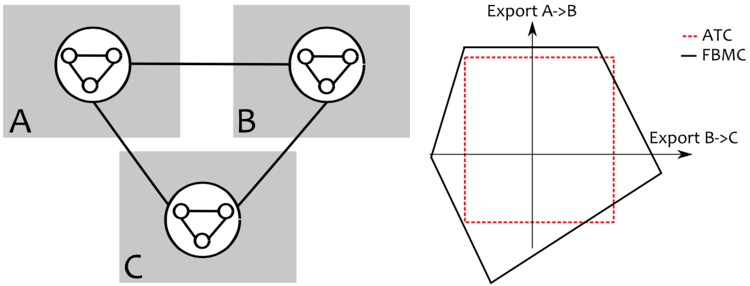


(a) Zonal market model. (b) Flow domain.

Figure 3. In the ATC method, only one equivalent node per zone is considered, with one cross-border link connecting the market zones. In this simple grid, the zonal network consists of 3 nodes and 3 cross-border links. The ATC flow domain is a rectangle, characterized by the ATC-values.

Source: Van den Bergh, Boury, Delarue

Flow Based Market Coupling



(a) Zonal market model. (b) Flow domain.

Figure 4. In the FMBC method, only one equivalent node per zone is considered, but all (critical) lines are taken into account. In this simple grid, the zonal network consists of 3 nodes and 12 lines. The FBMC flow domain is larger than the ATC flow domain as the physical characteristics of the grid are better represented in the FBMC method.

Source: Van den Bergh, Boury, Delarue

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