Electricity Markets: Summer Semester 2016, Lecture 9

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Managing price risks – basic instruments

Why risk managing?

- Sources of risk: Volatile spot prices, also geographic variation of prices due to network constraints
- Risk exposure can dampen incentives to make valuable investments
- Market particants can reduce their exposure to price volatility by participating in the forward market
- Forward market provides signals about future expected market prices
- Instruments: swaps, caps, floors, collars,. . .

Swaps (contracts for differences)

Under a swap the seller of the swap agrees to pay the buyer the difference between the spot price P at a pretermined time and a pretermined fixed price S multiplied by a pretermined quantity X.
Pay flow to the seller:

$$\mathrm{Swap}(P, X, S) = (S - P)\, X \ .$$

Pay flow to the buyer:

$$-\mathrm{Swap}(P, X, S) = (P - S)\, X \ .$$

A swap (contract for difference) thus basically allows the buyer to secure electricity from the seller for the fixed swap price S, while both parties participate in a centralised market.

Caps (call options)

Under a cap the seller of the cap agrees to pay the buyer the difference between the spot price P at a pretermined time and a pretermined strike price S, but only if the spot price is higher than the strike price. As a compensation, the buyer pays the seller a fixed fee.
Pay flow to the seller:

$$\mathrm{Cap}(P, X, S, P_{\mathrm{Cap}}) = -(P - S)X\mathbb{I}(P \geq S) + P_{\mathrm{Cap}}X \ .$$

Pay flow to the buyer:

$$-\mathrm{Cap}(P, X, S, P_{\mathrm{Cap}}) = (P - S)X\mathbb{I}(P \geq S) - P_{\mathrm{Cap}}X \ .$$

Here $\mathbb{I}(\cdot)$ is the indicator function. It takes the value 1 when the expression in the brackets is true and 0 otherwise. A cap (call option) would allow the buyer to buy electricity at or below the strike price via the centralised market.

Floors (put options)

A floor is the opposite of a cap, that is now the difference is only paid if the spot price is below the strike price.

Pay flow to the seller:

$$\text{Floor}(P, X, S, P_{\text{Floor}}) = -(S - P)X\mathbb{I}(P < S) + P_{\text{Floor}}X \ .$$

Pay flow to the buyer:

$$-\text{Floor}(P, X, S, P_{F\text{Floor}}) = (S - P)X\mathbb{I}(P < S) - P_{\text{Floor}}X \ .$$

A floor (put option) would allow the buyer to sell electricity at or above the strike price via the centralised market.

Swaps, caps and floors

Selling a swap is identical to selling a cap and buying a floor at the same strike price and quantity:

$$\mathrm{Cap}(P, X, S, P_{\mathrm{Cap}}) - \mathrm{Floor}(P, X, S, P_{\mathrm{Floor}}) = \mathrm{Swap}(P, X, S + P_{\mathrm{Cap}} - P_{\mathrm{Floor}}) \; .$$

Hedging by generators

Profit function for a generator

Consider a generator participating in the wholesale market. Assume that the generator is a price-taker and can be represented by a cost function $C(Q, \epsilon)$. Here Q is the rate of production, and ϵ are some uncertain cost-shifting factors affecting the total cost of production.

Given an uncertain wholesale spot price equal to P, the generator makes the profit

$$\pi(P, Q, \epsilon) = PQ - C(Q, \epsilon) \ .$$

If the generator additionally sells a portfolio of hedge contracts with payout $H(P, \epsilon)$, the hedged profit is given by

$$\pi(P, Q, \epsilon) = PQ - C(Q, \epsilon) + H(P, \epsilon) \ .$$

Here the terms in $H(P, \epsilon)$ are negative if the generator has to make a payment, and positive if it receives a pay flow under the respective contracts.

Rate of production

For a given market price P the generator adjusts its rate of production to the optimal level Q^*:

$$\frac{\partial C}{\partial Q}\left(Q^*(P,\epsilon),\epsilon\right) = P \ .$$

Profit function:

$$\pi(P,\epsilon) = PQ^*(P,\epsilon) - C\left(Q^*(P,\epsilon),\epsilon\right) + H(P,\epsilon) \ .$$

Sources of uncertainty

Profit function:

$$\pi(P, \epsilon) = PQ^*(P, \epsilon) - C\left(Q^*(P, \epsilon), \epsilon\right) + H(P, \epsilon)$$

1. Uncertainty in the generator cost function $C(Q, \epsilon)$ (reflected in the variable ϵ)
2. Uncertainty in output $Q^*(P, \epsilon)$
3. Uncertainty in the spot price P

Perfect price hedge

A perfect price hedge eliminates the variability of the (hedged) profit $\pi(P, \epsilon)$ with respect to the spot price P:

$$\frac{\partial \pi}{\partial P}(P, \epsilon) = 0 \ .$$

Using the formula for the hedged profit, it can be shown that this leads to the following condition:

$$-\frac{\partial H}{\partial P}(P, \epsilon) = Q^*(P, \epsilon) \ .$$

The hedge portfolio of a generator thus provides a perfect price hedge, if the rate of change of the hedge pay flow with respect to the spot price is equal to the optimal rate of production of the generator.

Perfect price hedge – example

Generator with a constant variable cost c, capacity K, and no cost-shifting factors. Output Q is zero if spot price P is below variable cost, and K if the price is above the variable cost:

$$Q(P) = \begin{cases} 0 & P < c \\ 0 \leq Q \leq K & P = c \\ K & P > c \end{cases}$$

Profit:

$$\pi(P) = (P - c)K\mathbb{I}(P \geq c) + H(P) \ .$$

Perfect price hedge – example

The generator obtains a perfect price hedge by selling a cap with a strike price equal to its variable cost and a volume equal to its capacity:

$$\begin{aligned} H(P) &= \mathrm{Cap}(P, c, K, P_{\mathrm{Cap}}) \\ &= -(P - c)K\mathbb{I}(P \geq c) + P_{\mathrm{Cap}}K \ . \end{aligned}$$

The generator trades the uncertain profit $K(P - c)$ against the certain revenue $P_{\mathrm{Cap}}K$:

$$\begin{aligned} \pi(P) &= (P - c)K\mathbb{I}(P \geq c) + H(P) \\ &= (P - c)K\mathbb{I}(P \geq c) + \mathrm{Cap}(P, c, K, P_{\mathrm{Cap}}) \\ &= P_{\mathrm{Cap}}(c)K \ . \end{aligned}$$

Price of the cap contract

Cap contract – payout to the seller:

$$\text{Cap}(P, X, S, P_{\text{Cap}}) = -(P - S)X\mathbb{I}(P \geq S) + P_{\text{Cap}}X \ .$$

Given a competitive market for cap contracts with a sufficient large number of large, risk-neutral traders, the price for a cap contract with strike price S should be equal to the expected payoff:

$$\begin{aligned} P_{\text{Cap}} &= \mathbb{E}\left[\text{Cap}(P, S, 1)\right] \\ &= \mathbb{E}\left[(P - S)\mathbb{I}(P \geq S)\right] \ . \end{aligned}$$

Hedging cost-shifting risks: capacity

Assume that the capacity of a generator occasionally falls to $K - \epsilon$. Treating ϵ as a random variable, this leads to the following profit function:

$$\pi(P, \epsilon) = (P - c)(K - \epsilon)\mathbb{I}(P \geq c) \ .$$

To hedge both the price risk and the risk of the drop in capacity, the generators can construct a portfolio with the pay flow:

$$H(P, \epsilon) = \mathrm{Cap}(P, c, K, P_{\mathrm{Cap}}) - \mathrm{Cap}(P, c, \epsilon, P_{\mathrm{Cap}}) \ .$$

Its (certain) profit is

$$\pi(P, \epsilon) = P_{\mathrm{Cap}}(c)K - P_{\mathrm{Cap}}(c)\epsilon = P_{\mathrm{Cap}}(c)\,(K - \epsilon) \ .$$

The second cap works as a kind of outage insurance. But is this really a perfect hedge?

Hedging cost-shifting risks: fuel costs

Assume that due to a change in the fuel costs the variable costs occasionally rise to $c + \epsilon$.
Profit function (ϵ random variable):

$$\pi(P, \epsilon) = (P - (c + \epsilon))K\mathbb{I}(P \geq (c + \epsilon)) \ .$$

To hedge both the price risk and the risk related to the variable cost, the generator can sell a cap contract:

$$H(P, \epsilon) = \text{Cap}(P - \epsilon, c, K, P_{\text{Cap}}) \ .$$

Notice that this not a standard cap contract – it depends on the difference between P and $(c + \epsilon)$, where $(c + \epsilon)$ are the now uncertain variable costs of the generator.

Hedging by customers

Utility function for a customer

Consider a consumer participating in the wholesale market. Assume that the consumer is a price-taker and can be represented by a utility function $U(Q, \epsilon)$. Here Q is the rate of consumption, and ϵ are some uncertain utility-shifting factors.

Given an uncertain wholesale spot price equal to P, the consumer receives the utility

$$\varphi(P, Q, \epsilon) = U(Q, \epsilon) - PQ \ .$$

If the generator additionally sells a portfolio of hedge contracts with pay flow $H(P, \epsilon)$, the hedged profit is given by

$$\varphi(P, Q, \epsilon) = U(Q, \epsilon) - PQ + H(P, \epsilon) \ .$$

Perfect price hedge

For a given market price P the consumer adjusts its rate of production to maximise utility:

$$\frac{\partial U}{\partial Q}\left(Q^*(P,\epsilon),\epsilon\right) = P \ .$$

Utility function:

$$\varphi(P,\epsilon) = U\left(Q^*(P,\epsilon),\epsilon\right) - PQ^*(P,\epsilon) + H(P,\epsilon) \ .$$

Perfect price hedge cancels the variability of the payoff with respect to the price:

$$\frac{\partial H}{\partial P}(P,\epsilon) = Q^*(P,\epsilon) \ .$$

Perfect price hedge – example

Consider a consumer with the following (step) demand function:

$$Q(P) = \begin{cases} 0 & P > A_1 \\ Q_1 & P \leq A_1 \end{cases}$$

Here Q_1 is a fixed consumption level.
Utility function:

$$\varphi(P) = (A_1 - P)Q_1\mathbb{I}(P \leq A_1) \ .$$

The customer obtains a perfect price hedge by constructing a portfolio which exchanges this (uncertain) profit by a certain price-independent profit.

Perfect price hedge – example

The consumer makes profit for low prices, but no profit for high prices. This can be matched in a first step by buying a swap with a strike price $S < A_1$:

$$-\mathrm{Swap}(P, Q_1, S) = (P - S)Q_1 \ ,$$

which leads to the utility

$$\begin{aligned} \varphi(P) &= (A_1 - P)Q_1\mathbb{I}(P \leq A_1) + (P - S)Q_1 \\ &= \begin{cases} (P - S)Q_1 & P > A_1 \\ (A_1 - S)Q_1 & P \leq A_1 \end{cases} \end{aligned}$$

Perfect price hedge – example

First step hedged utility:

$$\begin{aligned}\varphi(P) &= (A_1 - P)Q_1\mathbb{I}(P < A_1) + (P - S)Q_1 \\ &= \begin{cases}(P - S)Q_1 & P > A_1 \\ (A_1 - S)Q_1 & P \leq A_1\end{cases}\end{aligned}$$

For low prices there is no price variability. For high prices the variability can be matched by selling a cap:

$$\mathrm{Cap}(P, Q_1, A_1, P_{\mathrm{Cap}}) = -(P - A_1)Q_1\mathbb{I}(P > A_1) + P_{\mathrm{Cap}}(A_1)Q_1$$

This yields the final hedged utility

$$\varphi(P) = \begin{cases}(A_1 - S)Q_1 + P_{\mathrm{Cap}}(A_1)Q_1 & P > A_1 \\ (A_1 - S)Q_1 + P_{\mathrm{Cap}}(A_1)Q_1 & P \leq A_1\end{cases}$$

Hedging utility-shifting risks

Assume now that the level of consumption of the consumer is uncertain, i.e. the fixed level Q_1 is replaced by the variable L (corresponding to the utility-shifting factor):

$$U(Q, L) = A_1 Q \mathbb{I}(Q \leq L) \; .$$

Maximised utility:

$$\begin{aligned} \varphi(P, Q^*, L) &= U(Q^*, L) - PQ^* \\ &= (A_1 - P) L \mathbb{I}(P \leq A_1) \; . \end{aligned}$$

The variability of the utility with respect to the level L is hedged by a so-called *load-following hedge* with the same payout

$$\mathrm{LHF}(P, A_1, L) = -(A_1 - P) L \mathbb{I}(P \leq A_1) \; .$$

Note that here L is a variable.

The role of the trader

Trading of contracts – example

Consider a situation with two generators $G1$,$G2$ and two consumers $C1$, $C2$. There is some uncertainty in the consumption of $C1$: in state A, it is $L = 100$ MW, whereas in state B it is $L = 200$ MW. In state A this leads to a market price P_A, whereas in state the price is $P_B > P_A$.

Trading of contracts – example

- $C1$ values each unit of consumption at A_1, with $A_1 > P_B > P_A$. Consumption level is $L = 100$ MW in state A, and $L = 200$ MW in state B
- $C2$ values each unit of consumption at A_2, with $P_B > A_1 > P_A$. Consumption level is 80 MW in state A, and 0 MW in state B
- $G1$ has a variable cost c_1 with $P_B > P_A > c_1$ and a capacity $K_1 = 180$ MW. It produces 180 MW in both states A and B
- $G2$ has a variable cost c_2 with $P_B > c_2 > P_A$ and a capacity $K_2 = 20$ MW. It produces 0 MW in state A and 20 MW in state B

Trading of contracts – example

Profit of generators / utility of consumers:

Participant	Rate of Production/ Consumption (MW)	Profit/ Utility (€)
$G1$	180	$\pi^{G1}(P) = (P - c_1)180$
$G2$	$20\mathbb{I}(P \geq c_2)$	$\pi^{G2}(P) = 20(P - c_2)\mathbb{I}(P \geq c_2)$
$C1$	L	$\varphi^{C1}(P, L) = (A_1 - P)L$
$C2$	$80\mathbb{I}(P < c_2)$	$\varphi^{C2} = 80(A_2 - P)\mathbb{I}(P < c_2)$

Trading of contracts – example

A hedge market trader purchases hedge contracts from generators and customers (in the following we do not consider the fixed payments). Here: Trader α purchases a load-following hedge from $C1$:

$$H_{\alpha}^{P1}(P, L) = LFH(P, A_1, L) = (A_1 - P)L \ .$$

This is a perfect hedge for $C1$ – but now the trader is exposed to price risk. The trader now purchases hedge contracts from $G1$ and $G2$ to cancel this price risk:

$$H_{\alpha}^{G1}(P) = (P - c_1)(100 + 80\mathbb{I}(P \geq c_2))$$
$$H_{\alpha}^{G2}(P) = (P - c_2)20\mathbb{I}(P \geq c_2)$$

Trading of contracts – example

A hedge market trader purchases hedge contracts from generators and customers.
Here: Trader β purchases the following contract from $C2$:

$$H_{\beta}^{C2}(P, L) = 80(A_2 - P)\mathbb{I}(P < c_2),$$

which is a perfect hedge for $C2$. The trader β cancels the price risk by purchasing the following hedge from $G1$:

$$H_{\beta}^{G1}(P) = (P - c_1)80\mathbb{I}(P < c_2)\ .$$

Trading of contracts – example

By purchasing the hedge contracts from generators and consumers, the traders collectively offer them perfect hedges. The traders do not take on any price risk by this portfolio on contracts. Nevertheless, they take on a residual risk based on the uncertain consumption level L of $C1$ (their pay flow is different for state A than for state B).

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