Energy Systems, Summer Semester 2021

Lecture 13: Long-Term Dynamics

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The value of money depends on time

Question 1: What would you prefer: €1000 today, or €1000 in 3 years?

The value of money depends on time

Question 1: What would you prefer: €1000 today, or €1000 in 3 years?

€1000 today can be invested in the bank with an interest rate of 5%.

After 3 years you would have

$$1000 \cdot (1 + 0.05)^3 = 1158$$

Answer 1: Best to take the money today and use the opportunity to invest!

"Money in the future is worth less than money today."

The value of money depends on time

Question 2: What would you prefer: €1000 today, or €1300 in 5 years?

The value of money depends on time

Question 2: What would you prefer: €1000 today, or €1300 in 5 years?

If you invested €1000 today, after 5 years you would have only

$$1000 \cdot (1 + 0.05)^5 = 1276$$

Answer 2: Best to wait for the €1300 in 5 years!

Present value

To allow comparison between income and outgoings in different years, we need to agree on a particular point in time to evaluate the cash flows.

The simplest and most frequently used time point: today's value, known as the **present value**.

For an **interest rate** r we multiply the income or outgoings in year t by the **discount factor**

$$\frac{1}{(1+r)^t}$$

to calculate the present value. We have **discounted** the future cash flow.

Future income or outgoings are **worth less** from today's point of view (as long as r is positive).

"Money in the future is worth less than money today."

Example: present value

For our example with interest rate 5% we can now order the options:

Income (€)	Year	Present value (€)
1000	3	$\frac{1000}{(1+0.05)^3} = 863$
1000	0	$\frac{1000}{(1+0.05)^0} = 1000$
1300	5	$\frac{1300}{(1+0.05)^5} = 1019$

Investment calculations

Motivation: Power plant investment

A company is considering investing in a photovoltaic plant on its roof. The key figures:

Size	100 kW
Investment cost	800 €kW^{-1}
Operating cost	20 €$\mathrm{kW}^{-1}\ \mathrm{a}^{-1}$
Feed-In Tariff	0.1 €kWh^{-1}
Full load hours	1000
Period of subsidy	20 years

The company can invest its money elsewhere for a return of 5%.

Is it worthwhile to invest in the photovoltaic plant?

Source: Fraunhofer ISE Stromgestehungskosten 2018, Wikipedia

Investment calculations

An **investment calculation** quantifies the financial costs and benefits of an investment, assuming that future income and outgoings can be predicted.

It considers

- **Capital costs** - Costs for investments and installation
- **Consumption costs** - Fuel, other materials (e.g. lubricants for wind turbine), etc.
- **Operating costs** - Maintenance, wages, insurance, management, etc.
- **Income** - depends on market price, subsidies, and production

Dynamic investment calculation

For a **dynamic investment calculation** we sum the present values of all income and outgoings over the T years of operation taking account of the interest rate r to get the **Net Present Value (NPV)**:

$$NPV = \sum_{t=0}^{T} \frac{-I_t - V_t - B_t + U_t}{(1+r)^t}$$

where I_t is the capital expenditure in year t, V_t the consumption costs (e.g. for fuel cost o_t and annual production Q_t, $V_t = o_t \cdot Q_t$), B_t the operating costs und U_t the income (e.g. average market value λ_t times annual production Q_t, $U_t = \lambda_t \cdot Q_t$).

Conclusion: If $NPV > 0$, the investment is worthwhile.

If $NPV < 0$, better to invest with a rate of return of r elsewhere.

For comparisons between different investments, a higher NPV should be preferred.

Example: Rooftop photovoltaic unit

All cash flows (costs and income) in €:

year t	0	1	2	$\cdots$	20
Capital costs I_t	80,000	0	0		0
Operating costs B_t	0	2,000	2,000		2,000
Income U_t	0	10,000	10,000		10,000
Net cash flow $U_t - I_t - B_t$	-80,000	8,000	8,000		8,000
Discount factor $\frac{1}{(1+r)^t}$	1	$\frac{1}{(1+r)}$	$\frac{1}{(1+r)^2}$		$\frac{1}{(1+r)^{20}}$

NPV simplification

If investments only occur in the first year, and the costs and income for the following years are constant, we can simplify the NPV formula:

$$NPV = -I_0 + (U - V - B)\sum_{t=1}^{T} \frac{1}{(1+r)^t}$$

The sum $\sum$ is called the **Present Value Factor** $PVF(r, T)$.

For a geometric series with $|q| < 1$ we have $\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$. For $q = (1+r)^{-1}$ we can simplify the formula

$$\begin{aligned} PVF(r,T) &= \sum_{t=1}^{T} \frac{1}{(1+r)^t} \\ &= \left[\frac{1}{(1+r)} - \frac{1}{(1+r)^{T+1}}\right] \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \left[\frac{1}{(1+r)} - \frac{1}{(1+r)^{T+1}}\right] \frac{1}{1-(1+r)^{-1}} \\ &= \left[\frac{1}{(1+r)} - \frac{1}{(1+r)^{T+1}}\right] \frac{1+r}{1+r-1} = \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] \end{aligned}$$

Example: Rooftop photovoltaic unit

For our example with $r = 0.05$

$$
\begin{aligned}
NPV &= -80,000 + (10,000 - 2,000) \cdot \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] \\
&= -80,000 + 8,000 * 12.5 \\
&= 19,698
\end{aligned}
$$

Conclusion: It's worthwile to invest in the photovoltaic unit!

Example: Rooftop photovoltaic unit

For our example with $r = 0.05$

$$
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&= -80,000 + 8,000 * 12.5 \\
&= 19,698
\end{aligned}
$$

Conclusion: It's worthwile to invest in the photovoltaic unit!

NB: The calculation is very sensitive to the interest rate, e.g. with $r = 0.08$

$$
\begin{aligned}
NPV &= -80,000 + 8,000 * 9.8 \\
&= -1,454
\end{aligned}
$$

Conclusion: The investment is not worthwhile.

Return On Investment (ROI)

The expected return or **Return On Investment (ROI)** is the required interest rate to reach the point $NPV = 0$.

In our example you can either experiment or use the Newton-Raphson algorithm to determine the ROI r

$$0 = NPV = -I_0 + (U - V - B) \sum_{t=1}^{T} \frac{1}{(1+r)^t}$$

In our example we find an ROI of $r = 7.75\%$.

German example figures for electricity production technologies in 2018

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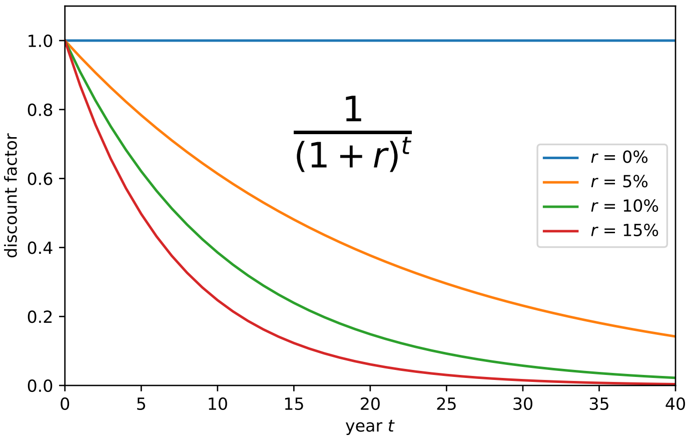
WACC is the **Weighted Average Cost of Capital** over the bank interest rate for borrowed capital (Fremdkapital) and the investor's ROI on their own investment (Eigenkapital).

	PV Dach Klein-anlagen (5-15 kWp)	PV Dach Großanlgen (100-1000 kWp)	PV Frei-fläche (ab 2000 kWp)	Wind Onshore	Wind Offshore	Biogas	Braun-kohle	Stein-kohle	GuD	GT
Lebensdauer in Jahren	25	25	25	25	25	30	40	40	30	30
Anteil Fremdkapital	80%	80%	80%	80%	70%	80%	60%	60%	60%	60%
Anteil Eigenkapital	20%	20%	20%	20%	30%	20%	40%	40%	40%	40%
Zinssatz Fremdkapital	3,5%	3,5%	3,5%	4,0%	5,5%	4,0%	5,5%	5,5%	5,5%	5,5%
Rendite Eigenkapital	5,0%	6,5%	6,5%	7,0%	10,0%	8,0%	11,0%	11,0%	10,0%	10,0%
WACC nominal	3,8%	4,1%	4,1%	4,6%	6,9%	4,8%	7,7%	7,7%	7,3%	7,3%
WACC real	1,8%	2,1%	2,1%	2,5%	4,8%	2,7%	5,6%	5,6%	5,2%	5,2%
OPEX fix [EUR/kW]	2,5% von CAPEX	2,5% von CAPEX	2,5% von CAPEX	30	100	4,0% von CAPEX	36	32	22	20
OPEX var [EUR/kWh]	0	0	0	0,005	0,005	0	0,005	0,005	0,004	0,003

Source: Fraunhofer ISE Stromgestehungskosten 2018

Warning: Discounting over long time periods

Over long time periods the discounting can have a very large effect....



- Long-term benefits aren't seen, e.g. long production life of nuclear power plants or benefits of long-lived efficiency measures
- Long-term costs are also suppressed, e.g. decommissioning, waste disposal, climate damages
- This is a **controversial topic!**

Programming example: photovoltaic plant

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jupyter NPV_examples Last Checkpoint: an hour ago (unsaved changes)

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3

Run Code

PV Example

In [37]:
```
lifetime = 20 #years
discount_rate = 0.08 #per unit
size = 100 #kW
specific_cost = 800 #EUR/kW
fom = 20 #EUR/kW/a
fit = 0.1 #EUR/kWh
flh = 1000 #h/a
flows = pd.DataFrame(index=range(lifetime+1))
flows["investment"] = [-size*specific_cost] + [0]*lifetime
flows["FOM"] = [0] + [-size*fom]*lifetime
flows["income"] = [0] + [size*flh*fit]*lifetime
flows["total_flow"] = flows.sum(axis=1)
flows["discount_factor"] = [(1+discount_rate)**(-t) for t in range(lifetime+1)]
flows["discounted_total_flow"] = flows["total_flow"]*flows["discount_factor"]
```

In [38]:
```
flows.head()
```

Out[38]:

	investment	FOM	income	total_flow	discount_factor	discounted_total_flow
0	-80000	0	0.0	-80000.0	1.000000	-80000.000000
1	0	-2000	10000.0	8000.0	0.925926	7407.407407
2	0	-2000	10000.0	8000.0	0.857339	6858.710562
3	0	-2000	10000.0	8000.0	0.793832	6350.657928
4	0	-2000	10000.0	8000.0	0.735030	5880.238822

In [39]:
```
flows.sum()
```

Out[39]:
```
investment              -80000.000000
FOM                     -40000.000000
income                  200000.000000
total_flow               80000.000000
discount_factor             10.818147
discounted_total_flow    -1454.820740
dtype: float64
```

Programming example: nuclear plant

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Trusted | Python 3

Run | Code

Nuclear Example

In [56]:

```
lifetime = 40 #years
discount_rate = 0.05 #per unit
size = 3e6 #kW
specific_cost = 5000 #EUR/kW
decommissioning_cost = 1000 #EUR/kW
fom = 20 #EUR/kW/a
fuel = 10 #EUR/MWh
market_value = 50 #EUR/MWh
flh = 8000 #h/a
flows = pd.DataFrame(index=range(lifetime+1))
flows["investment"] = [-size*specific_cost] + [0]*(lifetime-1) + [-size*decommissioning_cost]
flows["FOM"] = [0] + [-size*fom]*lifetime
flows["income"] = [0] + [size*flh*market_value/1000]*lifetime
flows["total_flow"] = flows.sum(axis=1)
flows["discount_factor"] = [(1+discount_rate)**(-t) for t in range(lifetime+1)]
flows["discounted_total_flow"] = flows["total_flow"]*flows["discount_factor"]
```

In [57]:

```
flows.head()
```

Out[57]:

	investment	FOM	income	total_flow	discount_factor	discounted_total_flow
0	-1.500000e+10	0.0	0.000000e+00	-1.500000e+10	1.000000	-1.500000e+10
1	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.952381	1.085714e+09
2	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.907029	1.034014e+09
3	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.863838	9.847749e+08
4	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.822702	9.378808e+08

In [59]:

```
flows.sum()
```

Out[59]:

```
investment              -1.800000e+10
FOM                     -2.400000e+09
income                   4.800000e+10
total_flow               2.760000e+10
discount_factor          1.815909e+01
discounted_total_flow    4.135221e+09
dtype: float64
```

Summary

- Future income or costs are worth less from today's point of view
- To calculate the **present value** give the **interest rate** r, multiply the cash flow in year t by the **discount factor** $\frac{1}{(1+r)^t}$
- To calculate the **net present value (NPV)** for an investment, sum the present values of all income and costs
- If $NPV > 0$, the investment is worthwhile compared to investing with interest rate r
- For two different investments, a higher NPV should be preferred
- **Long-term** costs or benefits are **suppressed** by discounting

Levelised Cost Of Electricity (LCOE)

You can also solve for the market value or feed-in tariff that's necessary to cover all the costs of the investment, i.e. the point where the present value of all income balances the present value of all costs. You solve for the price λ such that

$$0 = NPV = -I_0 + (\lambda Q - oQ - B) PVF(r, T)$$

(using $V = oQ$). We find:

$$\lambda = \frac{1}{Q}\left(\frac{I_0}{PVF(r,T)} + B + oQ\right) = \frac{1}{Q}\left(\frac{I_0}{PVF(r,T)} + B\right) + o$$

In our example we find a price of $\lambda = 89$ €/MWh for $i = 0.05$.

This value corresponds to the average long-term costs of the unit, since we've divided the total yearly costs by the total production Q. It is called the the **Levelised Cost Of Energy (LCOE)**.

It is also called the **Long-Run Marginal Cost (LMRC)**, since we've added to the short-run marginal cost o an annualised contribution to the capital cost and the operating costs.

Check: The higher I_0 or B are, the higher the LCOE. The higher Q is, the lower the LCOE.

The **annuity** is the annualised investment cost $a = \frac{I_0}{PVF(r,T)}$ and $a(r,T) = \frac{1}{PVF(r,T)}$ is the **annuity factor**, which spreads the capital costs I_0 evenly over the operational years of the investment (like a mortgage for a house).

For a loan I_0 from the bank, the bank is compensated for the **opportunity cost** of investing elsewhere at a rate of r by an annual fixed sum a so that the NPV for the bank is zero

$$0 = NPV = -I_0 + \sum_{t=0}^{T} \frac{a}{(1+r)^t} = -I_0 + PVF(r,T)\frac{I_0}{PVF(r,T)}$$

The formula for the annuity factor is derived from that for the PVF:

$$a(r,T) = \frac{1}{PVF(r,T)} = \frac{r}{1-(1+r)^{-T}}$$

Examples of annuity factor

AF = Annuity Factor, $a(r, T)$

Lifetime T years	Discount Rate r %	AF $a(r, T)$ per unit
20	0	0.05
20	5	0.08
20	10	0.12
20	20	0.21
40	0	0.025
40	5	0.06
40	10	0.10
40	20	0.20

Things to notice:

- AF reduce to $1/T$ in limit $r \to 0$
- AF climbs steeply with r
- For long lifetimes, AF is similar to short lifetimes for high r - in reality investors try to pay off investments faster than lifetime
- In reality, an investor would provide some capital themselves, e.g. 10-20% of the capital cost, and borrow the rest from the bank. The weighted average of the investor's desired internal rate of return and that of the bank loan is the **weighted average cost of capital** (WACC).

Parameters for different generation technologies

Here are some typical investment and operational parameters projected for 2020:

Source	Lifetime years	Capital Cost €kW^{-1}	Fix O&M €$\mathrm{kW}^{-1}\mathrm{a}^{-1}$	Var O&M €$\mathrm{MWh}_{\mathrm{el}}^{-1}$	η [%]	Fuel Cost €/$\mathrm{MWh}_{\mathrm{th}}$	Marg. Cost €/$\mathrm{MWh}_{\mathrm{el}}$
Hard Coal	40	1200	30	6	39	10	32
Gas OCGT	30	400	15	3	39	20	54
Gas CCGT	30	800	20	4	60	20	37
Nuclear	40-60	6000	0	6	33	3.3	16
Wind Onshore	25	1240	35	0		0	0
Solar PV	25	750	25	0		0	0

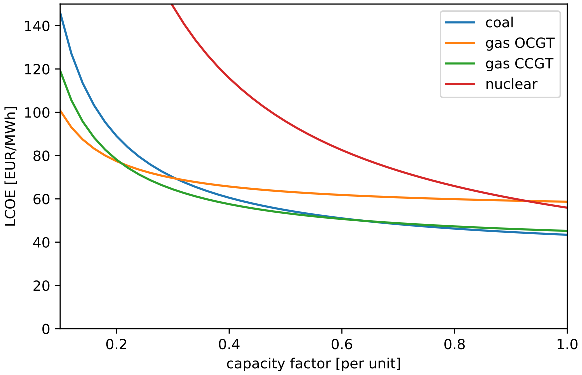
O&M = Operation and Maintenance, Var. = Variable, Fix. = Fixed, η = efficiency

For a plant with capacity G_s in MW and yearly production Q in $\mathrm{MWh}_{\mathrm{el}}$, we have $I_0 = 1000 \cdot G_s \cdot (\text{Capital Cost})$, $B = 1000 \cdot G_s \cdot (\text{Fix O\&M})$, $V = Q \cdot o$ where o is the marginal cost $o = (\text{Marg. Cost}) = (\text{Var O\&M}) + (\text{Fuel Cost})/\eta$.

Source: DIW Data Documentation, 2013

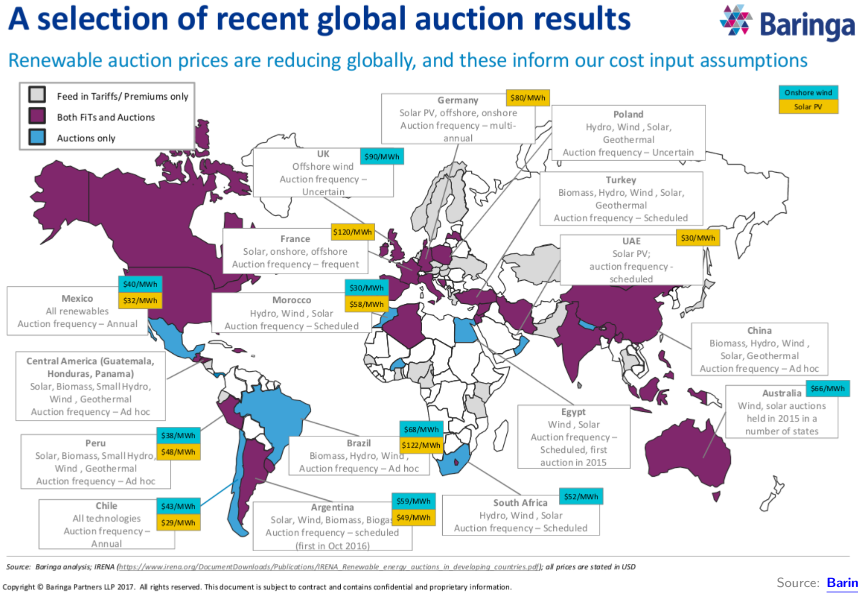
LCOE for dispatchable generators depends on capacity factor

The LCOE had the form (Marg. Cost) + (Yearly Fixed Costs)/(Yearly Production). Therefore it decreases with increasing capacity factor:



- LCOE > marginal cost
- LCOE starts high then reduces as fixed costs are spread over more hours
- There are **crossing points** where some types of generators become cheaper for a given capacity factor
- NB: All generators need downtime for regular maintenance, so cf < 0.9
- NB: Carbon pricing would alter this graphic by adding to the marginal cost

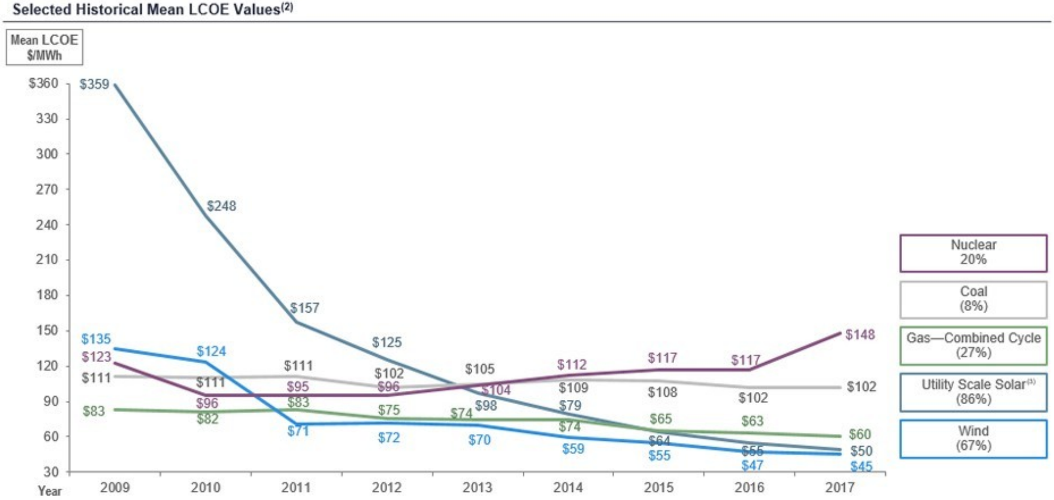
LCOE for wind and solar depends on location: worldwide auction results 2017



Source: Baringa Partners LLP 2017

Levelised Cost of Electricity Since 2009 in US

NB: Treat with care since LCOE doesn't take account of time or place of generation!



Source: **Lazard's LCOE Analysis V11**

Multi-horizon investment: Motivation

Short-run efficiency

Short-run efficiency is concerned with the **efficient operation** of the existing energy system, assuming that the capacities of all investments are fixed.

Example: Power plant dispatch for inelastic demand d. All capacities G_s [MW] are fixed. We optimise the **dispatch** g_s [MW], assuming that the **marginal costs** o_s [€/MWh] scale linearly with the dispatch. We minimise **total operational costs**:

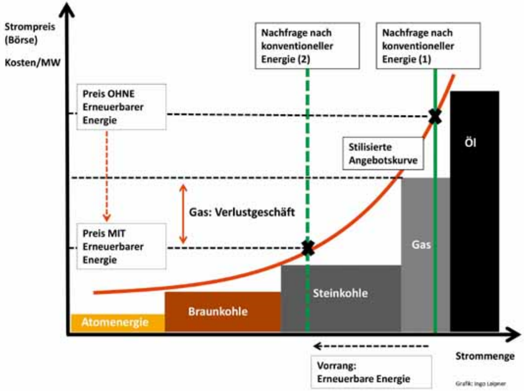
$$\min_{\{g_s\}} \sum_s o_s g_s$$

with constraints

$$\sum_s g_s = d \qquad \leftrightarrow \qquad \lambda$$

$$g_s \leq G_s \qquad \leftrightarrow \qquad \bar{\mu}_s$$

$$-g_s \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_s$$



Long-run efficiency is concerned with the **efficient operation** and **the efficient dimensioning of investments** in the energy system.

Example: Power plant **dispatch** $g_{s,t}$ (costs o_s) and **capacities** G_s (annualised costs c_s) are optimised over a year of hourly time periods t with demand d_t:

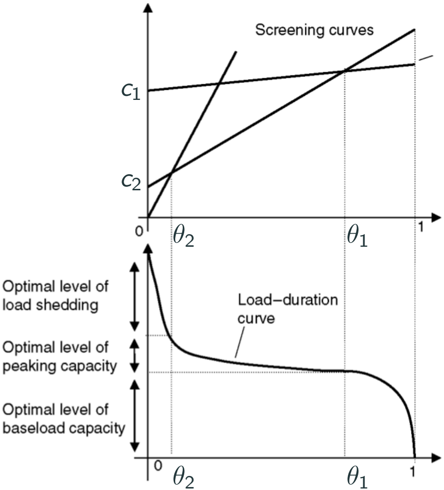
$$\min_{\{g_{s,t}, G_s\}} \sum_{s,t} o_s g_{s,t} + \sum_s c_s G_s$$

with constraints

$$\sum_s g_{s,t} = d_t \qquad \leftrightarrow \qquad \lambda_t$$

$$g_{s,t} \leq G_s \qquad \leftrightarrow \qquad \bar{\mu}_{s,t}$$

$$-g_{s,t} \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{s,t}$$



Multi-horizon investment

Dynamic multi-horizon investment is concerned with the **changing capacities of investments** in the energy system over many years or even **decades**.

At which point in time should we invest in renewables/gas/storage?

We consider several time horizons, typically years, in which plants can be dismantled or built.

Why are we concerned with changes over decades?

Multi-horizon investment

Dynamic multi-horizon investment is concerned with the **changing capacities of investments** in the energy system over many years or even **decades**.

At which point in time should we invest in renewables/gas/storage?

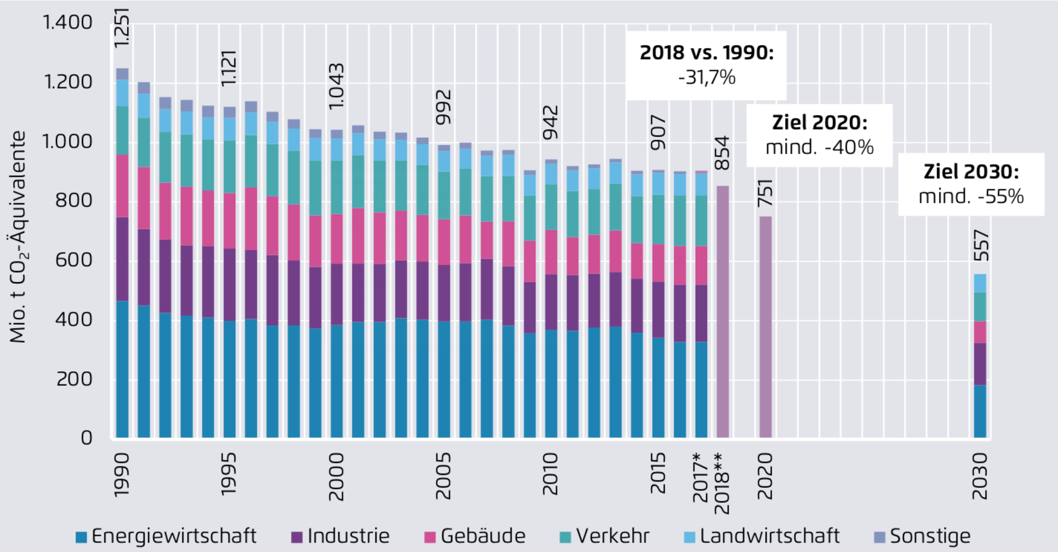
We consider several time horizons, typically years, in which plants can be dismantled or built.

Why are we concerned with changes over decades?

Since many aspects of the energy system change over decades, e.g.:

- **Energy consumption** (particularly in developing countries)
- **Resource scarcity** (scarcity of oil, cobalt, rare earth metals, etc.)
- **Political targets** (e.g. reduction of greenhouse gas emissions)
- **Technology maturity, costs and other parameters** (e.g. efficiency)
- **Economic growth**
- **Behavioural change** (car sharing, less flying, online gaming, etc.)

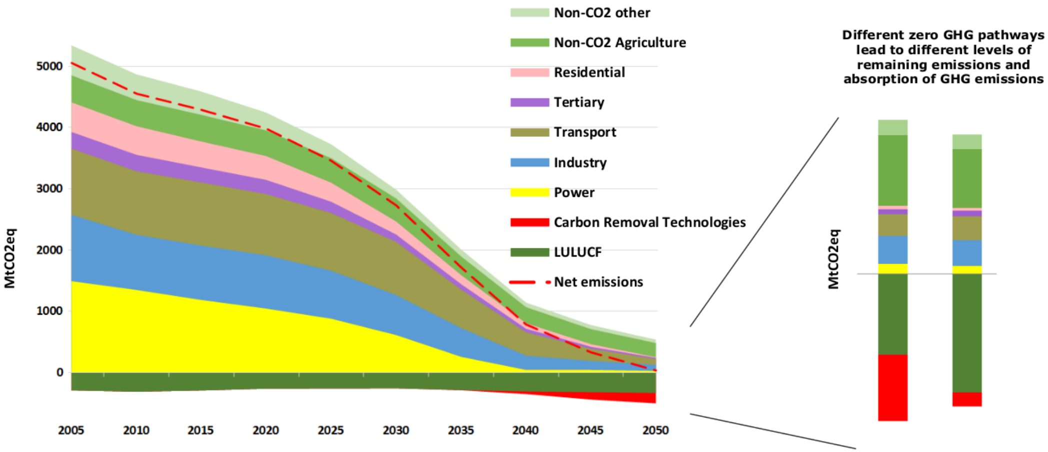
Example: political targets



Source: Agora Energiewende

Example: Net-Zero Emissions by 2050

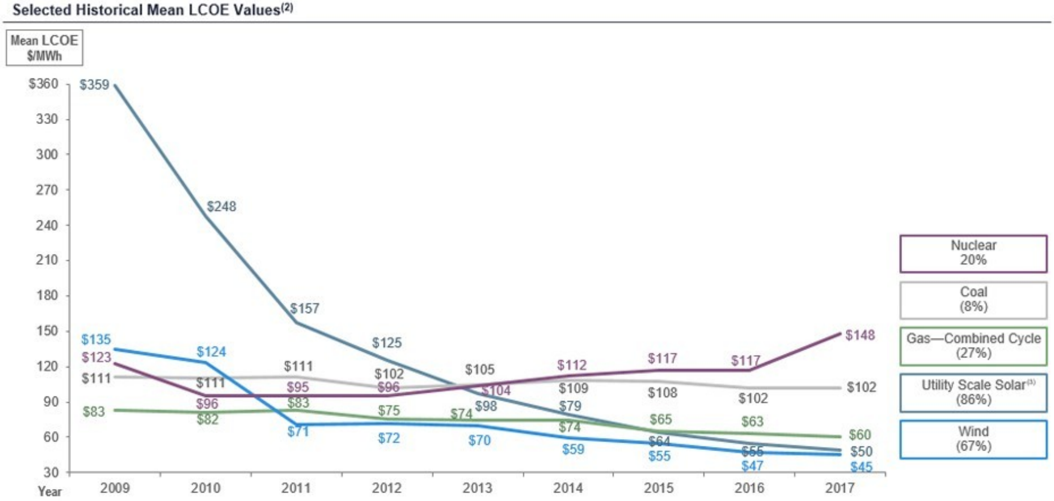
Paris-compliant 1.5° C scenarios from European Commission - **net-zero GHG in EU by 2050**



Source: European Commission 'Clean Planet for All', 2018

Example: Cost Developments of Renewable Energy

LCOE = **Levelised Cost of Energy** = Total Costs / Energy Output



Source: **Lazard's LCOE Analysis V11**

Multi-horizon investment: Theoretical formulation

Discounted Total Costs

We will consider the total costs over **multiple years** $a = 1, \ldots A$.

How do we compare costs in 2020 to those in 2040?

Discounted Total Costs

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We will consider the total costs over **multiple years** $a = 1, \ldots A$.

How do we compare costs in 2020 to those in 2040?

The totals costs are expressed in their **present value** using the **discount rate** r, to allow comparison between different years.

For costs (or income) in year a we **discount** the costs with a factor

$$\frac{1}{(1+r)^a}$$

because we could have invested until this year a with return r.

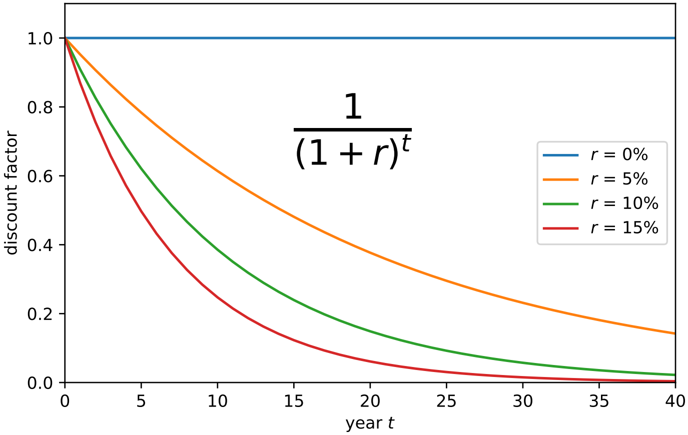
Costs in the future are **worth less** from today's point of view.

For rate r we optimised the **discounted total costs**

$$\sum_{a=1}^{A} \frac{1}{(1+r)^a} \{\text{Total costs in year } a\}$$

Warning: Discounting over long time periods

Over long time periods the discounting can have a very large effect....



- Long-term benefits aren't seen, e.g. long production life of nuclear power plants or benefits of long-lived efficiency measures
- Long-term costs are also suppressed, e.g. decommissioning, waste disposal, climate damages
- This is a **controversial topic!**

Example of Electricity System until 2050

We optimise the discounted total costs over 30 years from 2021 to 2050

$$\min_{\{g_{s,t,a},Q_{s,a},G_{s,a}\}} \sum_{a=1}^{A} \frac{1}{(1+r)^a} \left\{ \sum_{s,t} o_{s,a} g_{s,t,a} + \sum_{s,b|b\leq a<b+L_s} c_{s,b} Q_{s,b} \right\}$$

Here $Q_{s,a}$ is the new capacity built in year a and $G_{s,a}$ is the total capacity available in year a, L_s is the lifetime. $Q_{s,a}$ may also have fixed values for $a < 1$ to represent existing capacity. $Q_{s,a}$ and $G_{s,a}$ are related by

$$G_{s,a} = \sum_{b=1}^{L_s} Q_{s,a-b}$$

The old constraints apply for each year a

$$\sum_s g_{s,t,a} = d_{s,a} \quad \leftrightarrow \quad \lambda_{t,a}$$

$$g_{s,t,a} \leq G_{s,a} \quad \leftrightarrow \quad \bar{\mu}_{s,t,a}$$

$$-g_{s,t,a} \leq 0 \quad \leftrightarrow \quad \underline{\mu}_{s,t,a}$$

Global constraints

With a long-term perspective we can now set exciting constraints.

For example, we can restrict total **emissions** over the period:

$$\sum_{s,t,a} e_i g_{s,t,a} \leq \mathrm{CAP}_{\mathrm{CO}_2}$$

where e_s is the specific emissions of technology s (tonnes of CO_2 per $\mathrm{MWh}_{\mathrm{el}}$).

Or limit **resource consumption** for a technology s:

$$\sum_{t,a} g_{s,t,a} \leq \mathrm{CAP}_s$$

Learning effects

Technology costs sink with accumulated manufacturing experience, particularly for new immature technologies.

We promote $c_{s,a}$ to an optimisation variable that depends on the cumulative generator capacity.

A simple **one-factor learning model** for the costs is

$$c_{s,a} = c_{s,0} \left(\sum_{b=1}^{a} Q_{s,b} \right)^{-\gamma_s}$$

where $c_{s,0}$ is the initial cost, $Q_{s,b}$ is the capacity produced in year b and γ_s is the **learning parameter**.

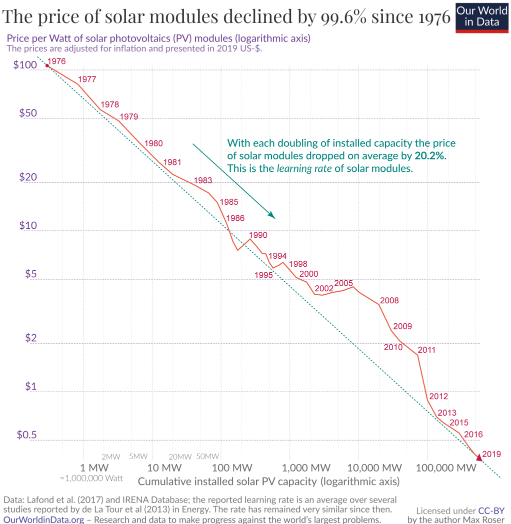
The **learning rate** LR is the reduction in cost for every doubling of production

$$LR_s = 1 - 2^{-\gamma_s}$$

Example for photovoltaics: $\gamma = 0.33 \implies$ if cumulative production doubles, the costs reduce by 20% (**Swanson's Law**).

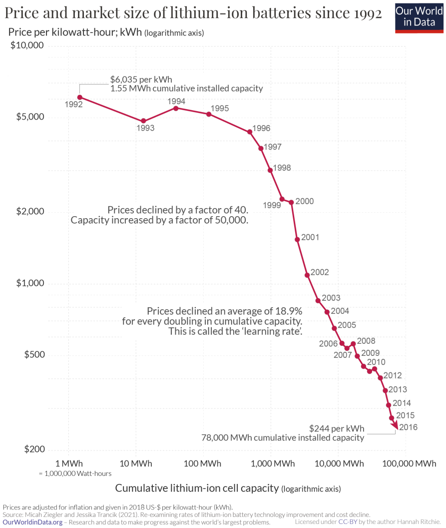
Swanson’s Law for photovoltaic modules

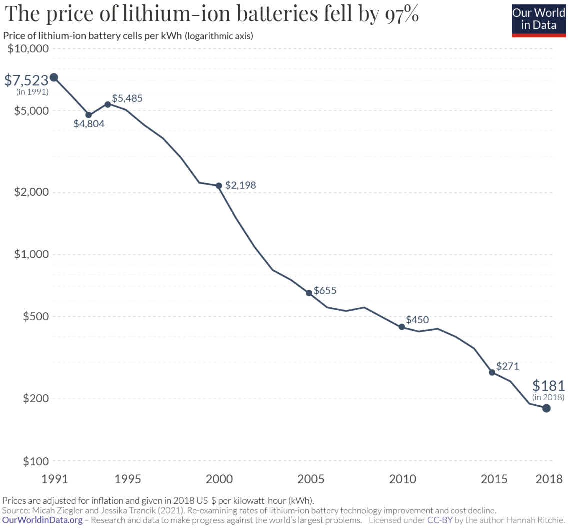
The underlying dynamic is a fast decay in costs with deployment (**learning-by-doing**).



Source: Our World in Data

Learning also seen for Lithium ion batteries

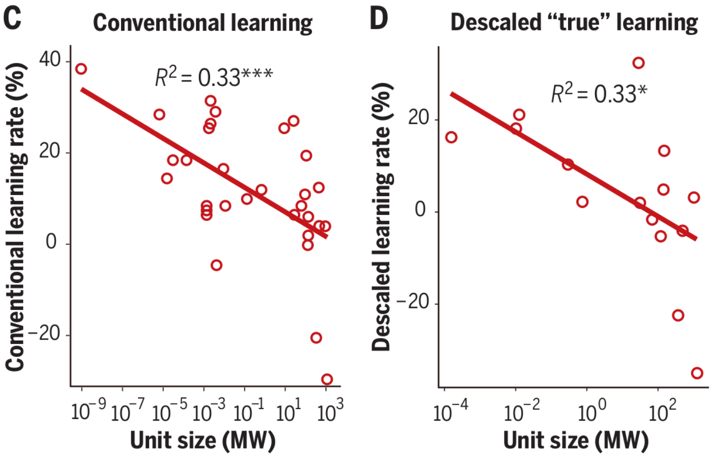




Source: Our World in Data

Learning tends to correlated with unit size

'Conventional learning rate' conflates two drivers of cost reduction: unit scale economies (more capacity per unit) and experience (more units). 'Descaled learning rate', % cost reduction per doubling of cumulative numbers of units, strips out effects of unit scale economies.



Source: **Wilson et al (2020)**

More complicated learning models

In the literature there are more sophisticated learning models than the one-factor model, e.g.

- **Multi-component learning models**: different parts of the cost experience different learning rates, e.g. some parts of the cost do not experience learning, such as fixed material and labour costs, call it $c_{s,\mathrm{base}}$. Only the remainder experiences learning:

$$c_{s,a} = c_{s,\mathrm{base}} + (c_{s,0} - c_{s,\mathrm{base}}) \left(\sum_{b=1}^{a} Q_{s,b} \right)^{-\gamma_s}$$

  In the case of PV, $c_{s,\mathrm{base}}$ would include e.g. the labour costs of installation.

- **Multi-factor learning models**: the cost depends not just on the cumulative capacity, but on other factors such as knowledge stock KS through research and development

$$c_{s,a} = c_{s,0} \left(\sum_{b=1}^{a} Q_{s,b} \right)^{-\gamma_{s,1}} \left(\sum_{b=1}^{a} KS_{s,b} \right)^{-\gamma_{s,2}}$$

Multi-horizon investment: Simplified example

Simplified example

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https://nworbmot.org/courses/esm-2020/lectures/notebooks/dynamic_investment.ipynb
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Time period: 2021 until 2070. Discount rate: $r = 0.05$.

Constant electricity demand $d_{t,a} = d = 100$ GW.

At the start of the simulation there is already 100 GW of 20-year-old coal plants.

3 generation technologies are available that are dispatchable (for Concentrating Solar Power (CSP) need good direct solar insolation, e.g. New Mexico or Morocco).

Tech	Capital costs (€MW^{-1} a^{-1})	Marg. costs (€$\text{MWh}_{\text{el}}^{-1}$)	LCOE (€$\text{MWh}_{\text{el}}^{-1}$)	Cap factor	Emissions ($\text{tCO}_2\text{MWh}_{\text{el}}^{-1}$)	Lifetime years
Coal	30*8760	20	50	1	1	40
Nuclear	65*8760	10	75	1	0	40
CSP	150*8760	0	150	1	0	30

Simplified example

Since each technology can generate continuously and the demand is constant, we assume $g_{s,t,a}$ is constant for all t

$$g_{s,t,a} = g_{s,a} \leq G_{s,a}$$

This simplifies the optimisation problem considerably:

$$\min_{\{g_{s,t,a},Q_{s,a},G_{s,a}\}} \sum_{a=1}^{A} \frac{1}{(1+r)^a} \left\{ \sum_s o_{s,a} g_{s,a} \cdot 8760 + \sum_{s,b|b \leq a < b+L_s} c_{s,b} Q_{s,b} \right\}$$

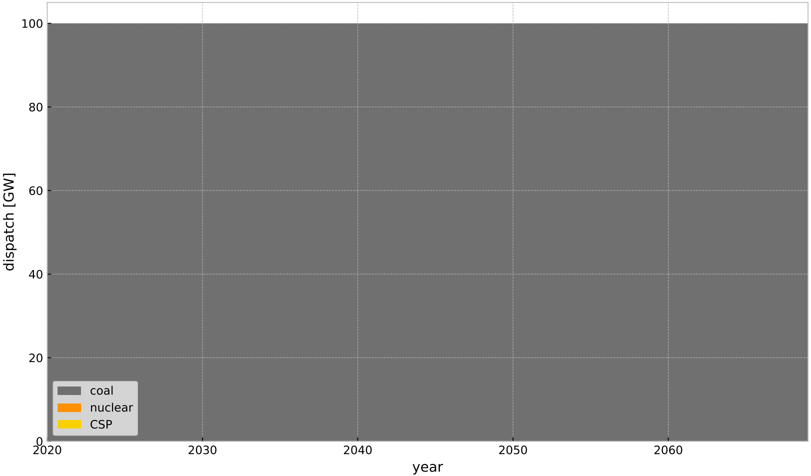
with constraints for each year a

$$\sum_s g_{s,a} = d$$

Vanilla Version: No CO_2 budget, no learning, no discounting

Only new coal is built, since it's cheapest.

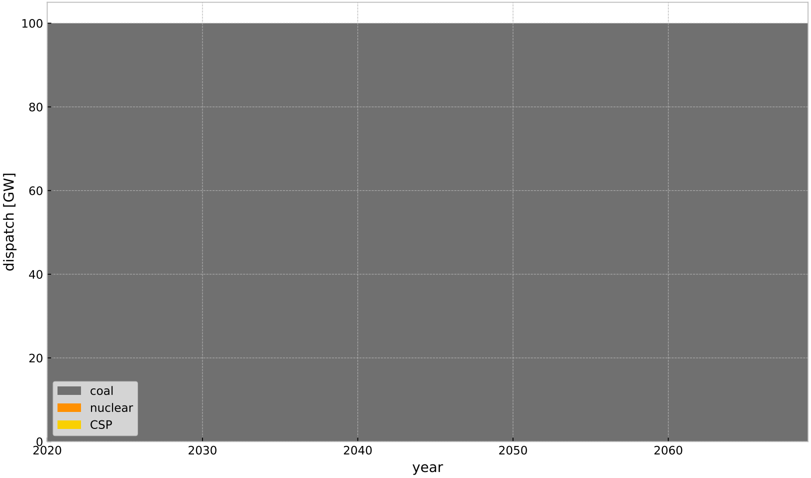
Total costs without discounting: $50€/\text{MWh} \cdot 8760 \cdot 100\ \text{GW} \cdot 50\ \text{years} = 2190\ \text{billion}\ €$



Vanilla Version: No CO_2 budget, no learning, discounting

Only coal is built, since it's cheapest.

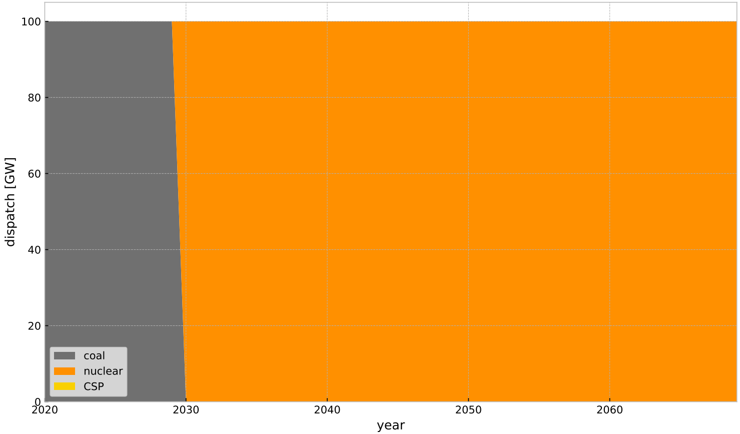
Total costs with discount rate 5%: 840 billion €



CO_2 budget, no learning, discounting

Limit CO_2 to 20% of coal emissions. Nuclear takes over before coal lifetimes are finished. Why is it built only later in the period (even when no existing plants assumed)? (Hint: discounting)

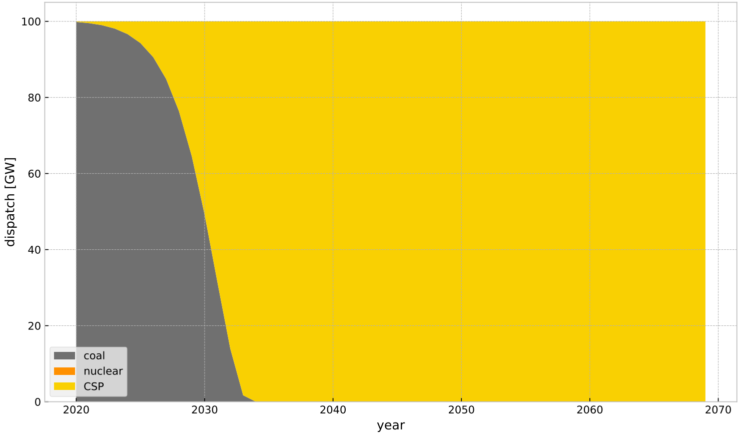
Total costs with discount rate 5%: 1147 billion €



CO_2 budget, learning for CSP, discounting

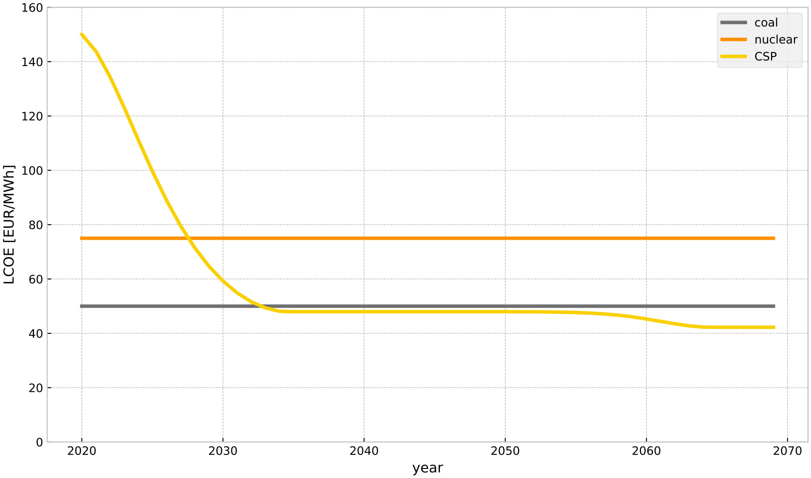
Limit CO_2 to 20% of coal emissions. CSP has learning rate 20%, $\gamma = 0.33$, and a base long-term potential LCOE of 20 €/MWh that represents material and labour costs.

Total costs with discount rate 5%: 1032 billion €



CO_2 budget, learning for CSP, discounting

LCOE needs subsidy initially to push down learning curve, since it is more expensive than incumbent technologies. But from 2034 onwards it is the most competitive technology.



Lessons from this example

- Non-linear effects such as learning-by-doing make the results hard to predict
- It may be cost-effective in the long-run to subsidise technologies that are uncompetitive today
- Depending on how subsidy and policy is arranged, there could be **path dependencies**

To improve the realism of this example we need to:

- Include more technologies, spatial resolution
- Consider more representative times per year to capture the variability of renewables and load