Technische Universität Berlin

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Lecture 3: Input-Output Analysis

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Table of Contents

1. Input-Output Tables
2. Leontief Multipliers
3. Uses and Criticisms

Input-Output Tables

- Energy balances reveal the **flow of energy** through the economy, but not how sectors interact. For example: an increase in industry production could trigger an increase in transport demand, which triggers more steel production, etc.
- **Input-Output Tables** reveal how goods and services, including energy, are exchanged between different parts of the economy.
- They reveal which **inputs** are required by which economic sectors in order to produce that sectors' **outputs**.
- Input-output tables typically value goods **monetarily**, but they can be used for energy flows too.
- Each **row** represents the **sales of a product** to each sector and to final consumers.
- Each **column** represents the **costs of production factors** for each sector consisting of purchases from economic sectors and primary inputs.

Example input-output table (monetary units)

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
Oil, gas, coal	**0.09**	**0.07**	**0.18**	**2.86**	**4.18**	**5.72**	**13.10**
Electricity	**0.01**	**0.28**	**0.09**	**1.18**	**2.22**	**3.67**	**7.45**
Agriculture	0.00	0.00	0.90	11.54	1.33	3.31	17.08
Industry, constr.	0.01	0.61	3.82	45.08	26.02	143.42	218.96
Services	0.06	0.82	1.98	20.01	38.48	159.62	220.97
Imports	**8.16**	**1.24**	1.29	62.15	15.18		88.02
Depreciations	0.98	1.26	2.98	11.23	17.26		33.71
Interest, profits	0.26	0.62	3.07	9.49	32.92		46.40
Wages, salaries	0.83	1.25	3.51	44.89	72.72		123.20
Indirect taxes/subsidies	2.70	1.30	-0.74	10.53	10.66		24.50
Input X_j	13.10	7.45	17.08	218.96	220.97	315.74	793.30

Source: Zweifel, Praktiknjo & Erdmann (2017)

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$N = 5$ sectors labelled here by $i = 1, \dots 5$: oil, gas, coal; electricity; agriculture; industry; services.

Total output of each sector X_i (row sum) is equal to the total input of each sector (column sum).

First quadrant: intermediate use of products

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
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The first quadrant is the $N \times N$ matrix which shows the input of sector i to sector j, X_{ij}, i.e. the **intermediate use of the product** by other sectors.

Example: entry X_{24} shows an input of 1.18 electricity to industry.

Diagonal entries show self-consumption, e.g. electricity industry needs $X_{22} = 0.28$ electricity to run.

Second quadrant: final use of products

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
Oil, gas, coal	**0.09**	**0.07**	**0.18**	**2.86**	**4.18**	**5.72**	**13.10**
Electricity	**0.01**	**0.28**	**0.09**	**1.18**	**2.22**	**3.67**	**7.45**
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The second quadrant shows the final use of products, which could be to final demand from consumers F_i, or also to exports.

Thus summing up the rows we get from the first and second quadrants:

$$X_i = \sum_{j=1}^{N} X_{ij} + F_i$$

Third quadrant: other production factors

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
Oil, gas, coal	**0.09**	**0.07**	**0.18**	**2.86**	**4.18**	**5.72**	**13.10**
Electricity	**0.01**	**0.28**	**0.09**	**1.18**	**2.22**	**3.67**	**7.45**
Agriculture	0.00	0.00	0.90	11.54	1.33	3.31	17.08
Industry, constr.	0.01	0.61	3.82	45.08	26.02	143.42	218.96
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Indirect taxes/subsidies	2.70	1.30	-0.74	10.53	10.66		24.50
Input X_j	13.10	7.45	17.08	218.96	220.97	315.74	793.30

The third quadrant shows the costs of other production factors beyond inputs from the other factors, i.e. imports and other costs.

The columns thus sum up to the input X_j, which equals the output for each sector.

Leontief Multipliers

Linearisation

Our fundamental equation for the input-output analysis:

$$X_i = \sum_{j=1}^{N} X_{ij} + F_i$$

is a snapshot of the interactions in the economy.

What happens if the demand F_i changes?

To analyse this, we make a further assumption that the inputs i to sector j, X_{ij}, depend **linearly** on the output of sector j, i.e.

$$X_{ij} = a_{ij} X_j$$

a_{ij} is an $N \times N$ matrix.

Leontief multipliers

Now our fundamental equation becomes:

$$X_i = \sum_{j=1}^{N} a_{ij} X_j + F_i$$

The first term is matrix multiplication, so we can write this more compactly:

$$X = a \cdot X + F$$

For a give final demand F, we can invert this to find the output X:

$$X = (1 - a)^{-1} \cdot F$$

The $N \times N$ factors $f_{ij} = (1 - a)^{-1}_{ij}$ are called **Leontief multipliers**. With all indices:

$$X_i = \sum_j (1 - a)^{-1}_{ij} F_j = \sum_j f_{ij} F_j$$

The Leontief multiplier f_{ij} tells us that if the final demand in sector j rises by 1 unit, then the output of sector i goes up by f_{ij} units.

Leontief multipliers from our example

	Oil, gas	Electricity	Agriculture	Industry	Services
Oil, gas, coal	1.007	0.015	0.019	0.021	0.026
Electricity	0.001	1.041	0.010	0.009	0.014
Agriculture	0.000	0.008	1.075	0.074	0.018
Industry, constr.	0.002	0.133	0.332	1.304	0.190
Services	0.006	0.155	0.189	0.156	1.236
Total	1.016	1.352	1.625	1.564	1.484

Note that the diagonal entries are always greater than one, $f_{jj} > 1$.

If electricity demand increases by one unit, electricity output has to increase by 1.041 units, since there is additional electricity demand in the power plant to run it. There are also additional inputs from the service sector of 0.155 and from industry of 0.133, so that in total output increases by 1.352 units.

Recursive explanation

If higher electricity demand triggers extra electricity demand, won't that extra electricity trigger extra extra demand?

Yes, but this is accounted for in the formula:

$$X = (1 - a)^{-1} \cdot F$$

The inverse can be expanded (like a Taylor series):

$$(1 - a)^{-1} = 1 + a + a^2 + a^3 + \cdots$$

so that

$$X = (1 - a)^{-1} \cdot F = (1 + a + a^2 + a^3 + \cdots) \cdot F$$

$a \cdot F$ is the first approximation of the extra electricity demand, $a^2 \cdot F$ is the extra demand from the extra electricity demand etc.

It is **convergent**.

Uses and Criticisms

Uses

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There are many ways that input-output tables can be used and extended:

- They represent how changes in demand cascade through different sectors of the economy.
- They can be extended to analyse energy demand in each sector.
- They can be extended with other data, such as CO_2 and other pollutant impacts, or employment impacts for each sector.
- They can be used for **Life Cycle Analysis (LCA)**, e.g. impact of production and dismantling of a solar PV panel on its CO_2 balance.

Energy input-output

Energy input-output table for our example (in energy units):

	Oil, gas	Electricity	Agriculture	Industry	Services	Final consumption F_i	Output X_i
Oil, gas	8.2	4.7	11.6	157.3	149.6	206.2	537.6
Electricity	0.6	20.6	3.1	56.4	46.2	130.4	257.3
Imports	571.1	56.1					627.2

From this and the output of each sector X_j we can calculate the **direct energy coefficients** e_{kj} for each energy source $k = 1, \ldots M$ ($M = 2$ in out example):

$$e_{kj} = \frac{\text{energy supply from energy source } k \text{ to sector } j}{X_j}$$

e_{kj} has units energy per money. The **aggregated energy coefficient** per sector sums over the energy sources $e_j = \sum_{k=1}^{M} e_{kj}$ and represents the total energy per unit output of sector j.

There are also **indirect energy factors** to account for energy use in other sectors, which can be determined with the Leontief factors

$$\hat{e}_{kj} = \sum_{i=1}^{N} e_{ki} f_{ij}$$

Criticisms

- Static, linear representation of economy
- Cannot represent non-linear relations in the economy, only linear $X_{ij} = a_{ij} X_j$
- Demand is exogenous and completely inelastic, i.e. it doesn't react to prices
- Cannot represent limits of capacities, bottlenecks, scarcity, etc. (e.g. switching from nuclear to gas above a certain demand level)

This means it's only useful for short-term analysis.

For more comprehensive analysis we will need better models!