Energy Systems, Summer Semester 2021

Lecture 9: Electricity Markets

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Introduction to Electricity Markets

The Economic Operation of the Electricity Sector

Given the many different ways of consuming and generating electricity:

- What is the **most efficient** way to deploy consuming and generating assets in the short-run?
- How should we invest in assets in the long-run to **maximise economic welfare**?

The operation of electricity markets is intimately related to **optimisation**.

In the past and still in many countries today, electricity was provided centrally by 'vertically-integrated' monopoly utilities that owned generating assets, the electricity networks and retailing. Given that these utilities owned all the infrastructure, it was hard for third-party generators to compete, even if they were allowed to.

From the 1980s onwards, countries began to liberalise their electricity sectors, separating generation from transmission, and allowing regulated competition for generation in **electricity markets**.

Electricity markets have several important differences compared to other commodity markets.

At every instant in time, consumption must be balanced with generation.

If you throw a switch to turn on a light, somewhere a generator will be increasing its output to compensate.

If the power is not balanced in the grid, the power supply will collapse and there will be blackouts.

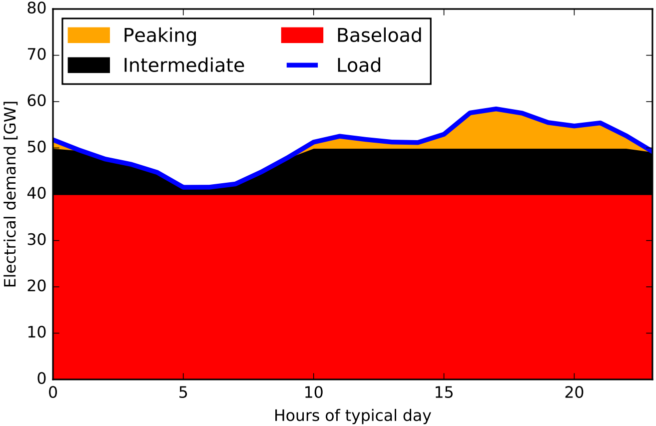
It is not possible to run an electricity market for every single second, for practical reasons (the network must be checked for stability, etc.).

So electricity is traded in blocks of time, e.g. hourly, 14:00-15:00, or quarter-hourly, 14:00-14:15, well in advance of the time when it is actually consumed (based on forecasts).

Additional markets trade in backup balancing power, which step in if the forecasts are wrong.

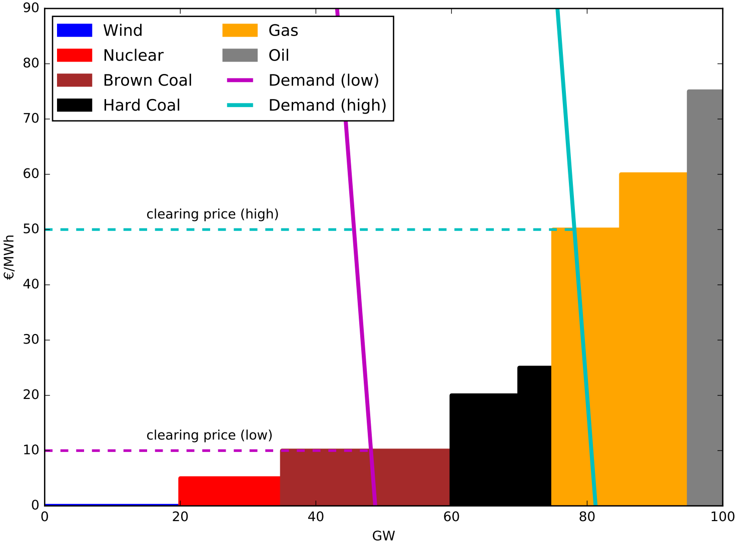
Baseload versus Peaking Plant

Load (= Electrical Demand) is low during night; in Northern Europe in the winter, the peak is in the evening. To meet this load profile, **baseload** generation with low fuel and running costs runs the whole time; more expensive **peaking plant** covers the difference.



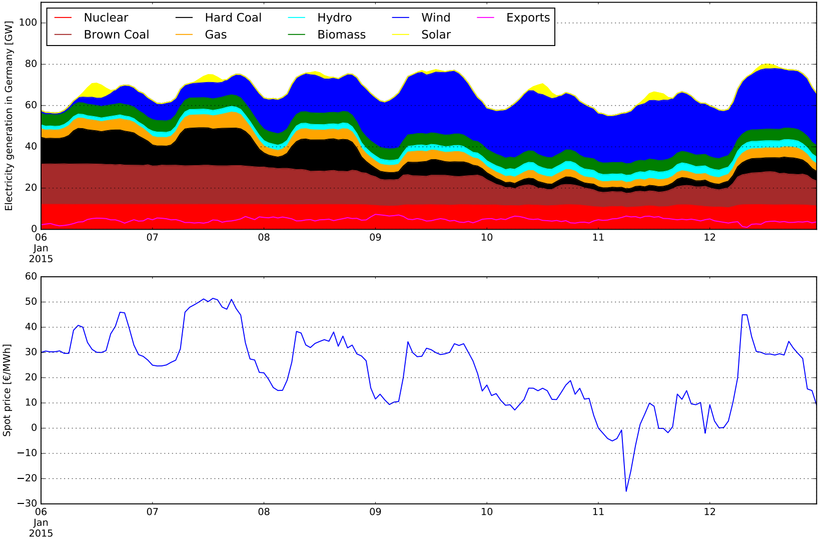
Source: Tom Brown

Effect of varying demand for fixed generation

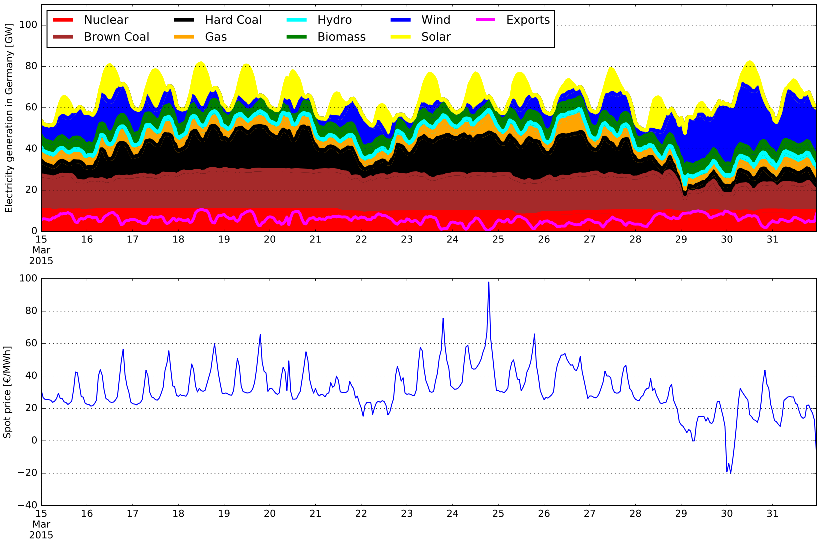


Source: Tom Brown

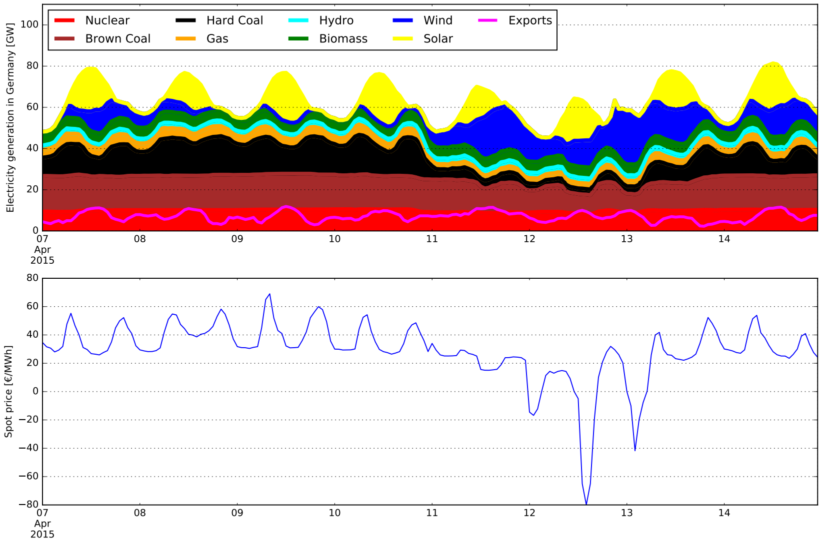
Example market 1/3



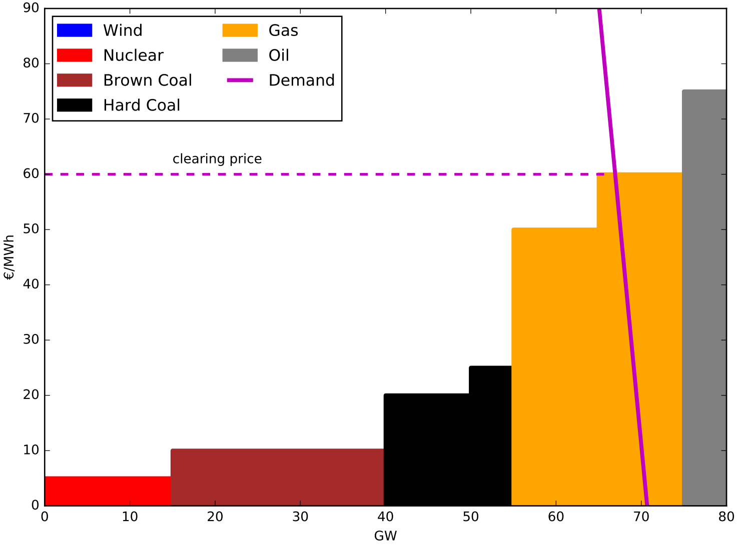
Example market 2/3



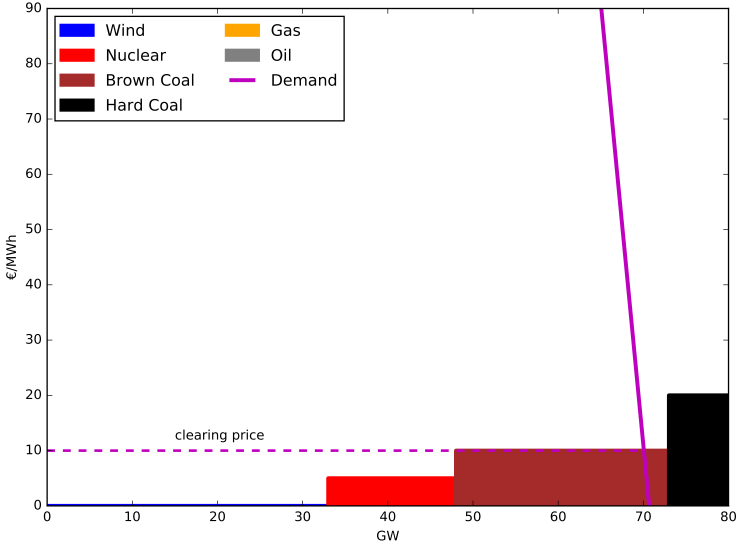
Example market 3/3



Effect of varying renewables: fixed demand, no wind

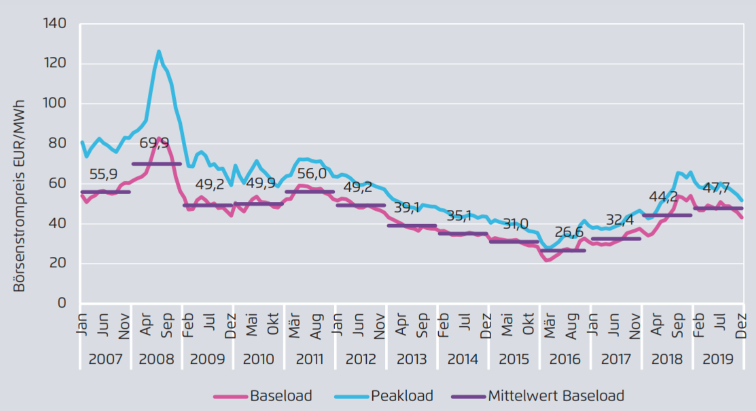


Effect of varying renewables: fixed demand, 35 GW wind



Spot market price development

As a result of so much zero-marginal-cost renewable feed-in, spot market prices steadily decreased until 2016. This is called the **Merit Order Effect**. Since then prices have been rising due to rising gas and CO_2 prices.



Source: Agora Energiewende Jahresauswertung 2019

To summarise:

- Renewables have zero marginal cost
- As a result they enter at the bottom of the merit order, reducing the price at which the market clears
- This pushes non-CHP gas and hard coal out of the market
- This is unfortunate, because among the fossil fuels, gas is the most flexible and produces lower CO_2 per MWh_{el} than e.g. lignite
- It also reduces the profits that nuclear and lignite make
- Will there be enough backup power plants for times with no wind/solar?

This has led to lots of political tension, but has been counteracted in recent years by the rising CO_2 price.

Optimisation Revision

We have an **objective function** $f : \mathbb{R}^k \to \mathbb{R}$

$$\max_x f(x)$$

$[x = (x_1, \dots x_k)]$ subject to some **constraints** within $\mathbb{R}^k$:

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \dots n \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \dots m \end{array}$$

λ_i and μ_j are the **KKT multipliers** we introduce for each constraint equation; they measure the change in the objective value of the optimal solution obtained by relaxing the constraints (for this reason they are also called **shadow prices**).

KKT conditions

The **Karush-Kuhn-Tucker (KKT) conditions** are necessary conditions that an optimal solution x^*, μ^*, λ^* always satisfies (up to some regularity conditions):

1. **Stationarity**: For $l = 1, \ldots k$

$$\frac{\partial \mathcal{L}}{\partial x_l} = \frac{\partial f}{\partial x_l} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial x_l} - \sum_j \mu_j^* \frac{\partial h_j}{\partial x_l} = 0$$

2. **Primal feasibility**:

$$g_i(x^*) = c_i$$
$$h_j(x^*) \leq d_j$$

3. **Dual feasibility**: $\mu_j^* \geq 0$

4. **Complementary slackness**: $\mu_j^*(h_j(x^*) - d_j) = 0$

If say $d_j \to d_j + \varepsilon$ then

$$f(x^*) \to f(x^*) + \mu_j^* \varepsilon$$

and similarly for $c_i \to c_i + \varepsilon$ and λ_i^*.

We will now sketch a proof of this (not in exam). The Lagrangian from last time was defined:

$$\mathcal{L}(x, \lambda, \mu) = f(x) - \sum_i \lambda_i \left[g_i(x) - c_i\right] - \sum_j \mu_j \left[h_j(x) - d_j\right]$$

Note that by primal feasibility and complementary slackness at the optimum point x^*, μ^*, λ^*:

$$\mathcal{L}(x^*, \lambda^*, \mu^*) = f(x^*)$$

Now consider the Lagrangian $\mathcal{L}^\varepsilon$ for the perturbed problem $d_j \to d_j + \varepsilon$:

$$\mathcal{L}^\varepsilon(x, \lambda, \mu) = \mathcal{L}(x, \lambda, \mu) + \mu_j \varepsilon$$

At the optimum point of the perturbed problem x^+, μ^+, λ^+ we have:

$$f(x^+) = \mathcal{L}^\varepsilon(x^+, \lambda^+, \mu^+) = \mathcal{L}(x^+, \lambda^+, \mu^+) + \mu_j^+ \varepsilon$$

Because everything is differentiable and we only have a small perturbation, we expect that $y^+ = (x^+, \mu^+, \lambda^+)$ is close to $y^* = (x^*, \mu^*, \lambda^*)$ and Taylor expand about this point:

$$\mathcal{L}(x^+, \lambda^+, \mu^+) \approx \mathcal{L}(x^*, \lambda^*, \mu^*) + \sum_l (x_l^+ - x_l^*) \frac{\partial \mathcal{L}}{\partial x_l}\bigg|_{y=y^*} + \sum_i (\lambda_i^+ - \lambda_i^*) \frac{\partial \mathcal{L}}{\partial \lambda_i}\bigg|_{y=y^*} + \sum_j (\mu_j^+ - \mu_j^*) \frac{\partial \mathcal{L}}{\partial \mu_j}\bigg|_{y=y^*}$$

From stationarity for the original problem we know that $\frac{\partial \mathcal{L}}{\partial x_l}\Big|_{y=y^*} = \frac{\partial \mathcal{L}}{\partial \lambda_i}\Big|_{y=y^*} = 0$. $\frac{\partial \mathcal{L}}{\partial \mu_j}\Big|_{y=y^*} = 0$ for binding inequalities, and for non-binding inequalities $\mu_j^* = \mu_j^+ = 0$. Thus

$$f(x^+) = \mathcal{L}(x^+, \lambda^+, \mu^+) + \mu_j^+ \varepsilon \approx \mathcal{L}(x^*, \lambda^*, \mu^*) + \mu_j^* \varepsilon = f(x^*) + \mu_j^* \varepsilon$$

Electricity Markets from Perspective of Single Generators and Consumers

Assume investments already made in generators and and consumption assets (factories, machines, etc.).

Assume all actors are price takers (i.e. nobody can exercise market power) and we have perfect competition.

How do we allocate production and consumption in the most efficient way?

I.e. we are interested in the **short-run "static" efficiency**.

(In contrast to **long-run "dynamic" efficiency** where we also consider optimal investment in assets.)

Single Generator: Cost Function

Consider now the market from the point of view of a single generator.

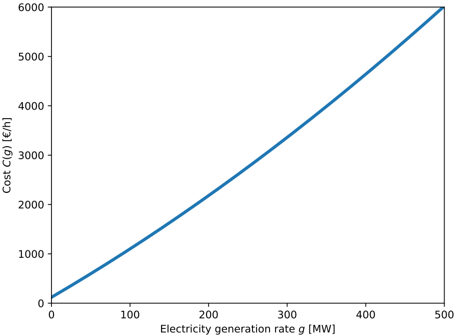
A generator has a **cost or supply function** $C(g)$ in €/h, which gives the total costs (fuel, operation and maintenance costs) for a given rate of electricity generation g MW.

Typically the generator has a higher cost for a higher rate of generation g, i.e. the first derivative is positive $C'(g) > 0$. For most generators the rate at which cost increases with rate of production itself increases as the rate of production increases, i.e. $C''(g) > 0$.

Cost Function: Example

A gas generator has a cost function which depends on the rate of electricity generation g [€/h] according to

$$C(g) = 0.005\ g^2 + 9.3\ g + 120$$



Note that the slope is always positive and becomes more positive for increasing g. The curve does not start at the origin because of startup costs, no load costs, etc.

Optimal generator behaviour

We assume that the generator is a **price-taker**, i.e. they cannot influence the price by changing the amount they generate. Suppose the market price is λ €/MWh. For a generation rate g, the **revenue** from the market is λg and the generator should adjust their generation rate g to maximise their **net generation surplus**, i.e. their **profit**:

$$\max_g \left[\lambda g - C(g)\right]$$

This optimisation problem is optimised for $g = g^*$ where by KKT stationarity we have

$$0 = \frac{\partial \mathcal{L}}{\partial g} = \frac{\partial f}{\partial g} = \lambda - \frac{dC}{dg}(g^*)$$

We'll write the derivative with a prime to get:

$$C'(g^*) \equiv \frac{dC}{dg}(g^*) = \lambda$$

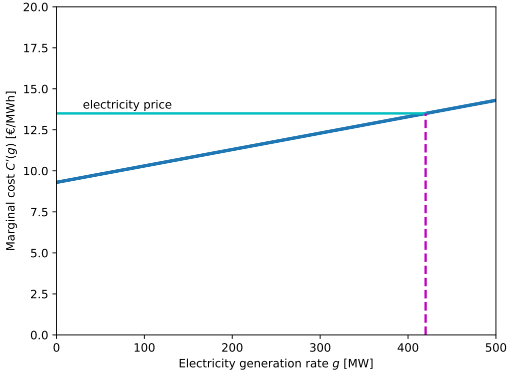
I.e. the generator increases their output until they make a net loss for any increase of generation. [Check units: $\frac{dC}{dg}$ has units $\frac{\text{€/h}}{\text{MW}}$ = €/MWh.]

Marginal cost function: Example

$C'(g)$ is known as the **marginal cost function**, which shows, for each rate of generation g what price λ the generator should be willing to supply at.

For our example the marginal cost function is given by

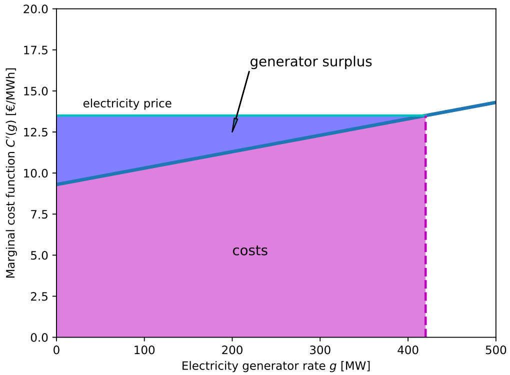
$$C'(g) = 0.01\ g + 9.3$$



Net generator surplus

The area under the curve is generator costs, which as the integral of a derivative, just gives the cost function $C(g)$ again, up to a constant.

The **net generator surplus** is the profit the generator makes by having costs below the electricity price.



Limits to generation

Note that it is quite common for generators to be limited by e.g. their capacity G, which may become a **binding constraint**, i.e. limiting factor before the price plays a role, e.g.

$$h(g) = g \leq G \leftrightarrow \mu$$

This constraint alters our KKT stationarity constraint for g to

$$0 = \frac{\partial \mathcal{L}}{\partial g} = \frac{\partial f}{\partial g} - \mu^* \frac{\partial h}{\partial g} = \lambda - C'(g^*) - \mu^*$$

Now the **shadow price** of the constraint is given by

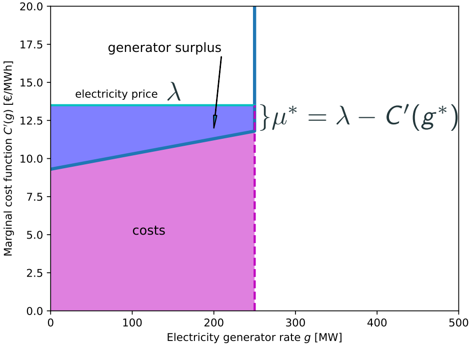
$$\mu^* = \lambda - C'(g^*)$$

If it is binding, it tells us the benefit to our objective function of an incremental increase in capacity G. It is called the **inframarginal rent**, i.e. difference between the market price λ and the marginal cost $C'(g^*)$.

Consider the constraint

$$g \leq G \leftrightarrow \mu$$

with capacity $G = 250$ MW so that for our example it is binding $g^* = G = 250$ MW. The inframarginal rent $\mu^* = \lambda - C'(g^*)$ can be marked on the graph:



Suppose for some given period a consumer consumes electricity at a rate of d MW.

Their **utility or value function** $U(d)$ in €/h is a measure of their benefit for a given consumption rate d.

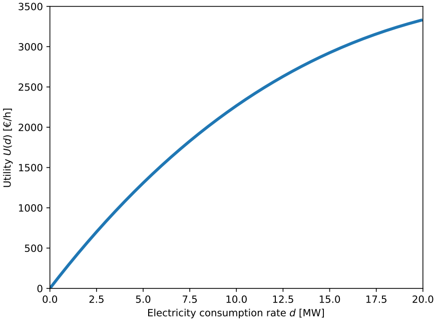
For a firm this could be the profit related to this electricity consumption from manufacturing goods.

Typical the consumer has a higher utility for higher d, i.e. the first derivative is positive $U'(d) > 0$. By assumption, the rate of value increase with consumption decreases the higher the rate of consumption, i.e. $U''(d) < 0$.

Utility: Example

A widget manufacturer has a utility function which depends on the rate of electricity consumption d [€/h] as

$$U(d) = 0.0667\ d^3 - 8\ d^2 + 300\ d$$



Note that the slope is always positive, but becomes less positive for increasing d.

Optimal consumer behaviour

We assume that the consumer is a **price-taker**, i.e. they cannot influence the price by changing the amount they consume.

Suppose the market price is λ €/MWh. The consumer should adjust their consumption rate d to maximise their **net surplus**

$$\max_d \left[U(d) - \lambda d\right]$$

This optimisation problem is optimised for $d = d^*$ where from KKT stationarity we now get

$$U'(d^*) \equiv \frac{dU}{dd}(d^*) = \lambda$$

[Check units: $\frac{dU}{dd}$ has units $\frac{\text{€/h}}{\text{MW}}$ = €/MWh.]

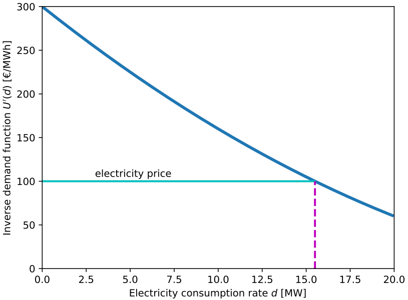
I.e. the consumer increases their consumption until they make a net loss for any increase of consumption.

$U'(d)$ is known as the **inverse demand curve** or **marginal utility curve**, which shows, for each rate of consumption d what price λ the consumer should be willing to pay.

Inverse demand function: Example

For our example the inverse demand function is given by

$$U'(d) = 0.2\ d^2 - 16\ d + 300$$



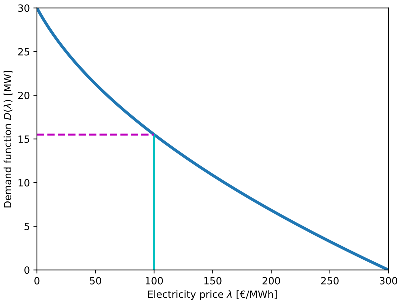
It's called the *inverse* demand function, because the demand function is the function you get from reversing the axes.

Inverse demand function: Example

The **demand function** $D(\lambda)$ gives the demand d as a function of the price λ. $D(U'(d)) = d$.

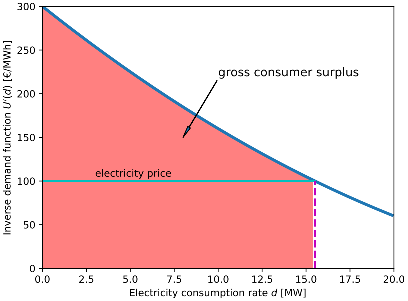
For our example the demand function is given by

$$D(\lambda) = -((\lambda + 20)/0.2)^{0.5} + 40$$



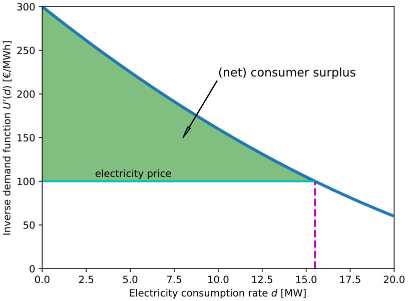
Gross consumer surplus

The area under the inverse demand curve is the **gross consumer surplus**, which as the integral of a derivative, just gives the utility function $U(d^*)$ again, up to a constant.



Net consumer surplus

The more relevant **net consumer surplus**, or just **consumer surplus** is the net gain the consumer makes by having marginal utility above the electricity price, i.e. $U(d^*) - \lambda d^*$.

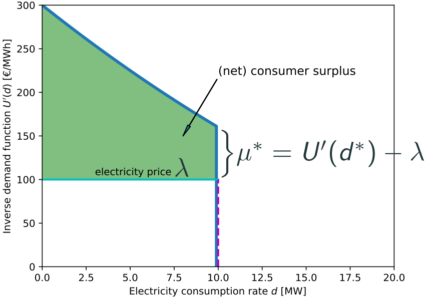


Limits to consumption

Note that it is quite common for consumption to be limited by other factors before the electricity price becomes too expensive, e.g. due to the size of electrical machinery. This gives an upper bound

$$d \leq D \leftrightarrow \mu$$

The **shadow price** μ indicates the benefit of relaxing the constraint. From KKT $\mu^* = U'(d^*) - \lambda$. In the following case the optimal consumption is at $d^* = D = 10$ MW.

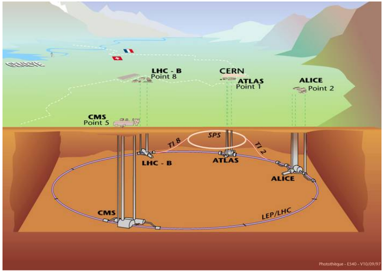


Consumers can delay their consumption

Besides changing the amount of electricity consumption, consumers can also shift their consumption in time.

For example electric storage heaters use cheap electricity at night to generate heat and then store it for daytime.

The LHC particle accelerator does not run in the winter, when prices are higher (see http://home.cern/about/engineering/powering-cern). Summer demand: 200 MW, corresponds to a third of Geneva, equal to peak demand of Rwanda (!); winter only 80 MW.



Source: CERN

Consumers can also move location

Aluminium smelting is an electricity-intensive process. Aluminium smelters will often move to locations with cheap and stable electricity supplies, such as countries with lots of hydroelectric power. For example, 73% of Iceland's total power consumption in 2010 came from aluminium smelting.

Aluminium sells on world markets for around US\$ 1500/ tonne.

Electricity consumption: 15 MWh/tonne.

At Germany consumer price of electricity of €300 / MWh, this is €4500 / tonne. Industrial consumers pay less.

If electricity is 50% of cost, then need \$750/tonne to go on electricity $\Rightarrow$ 750/15 \$/MWh = 50 \$/MWh.

Summary: Consumers and Generators

Generators: A generator has a **cost** or **supply function** $C(g)$ in €/h, which gives the costs (of fuel, etc.) for a given rate of electricity generation g MW. If the market price is λ €/MWh, the revenue is λg and the generator should adjust their generation rate g to maximise their **net generation surplus**, i.e. their profit:

$$\max_{g} [\lambda g - C(g)]$$

Consumers: Their **utility** or **value function** $U(d)$ in €/h is a measure of their benefit for a given consumption rate d. For a given price λ they adjust their consumption rate d such that their **net surplus** is maximised:

$$\max_{d} [U(d) - \lambda d]$$

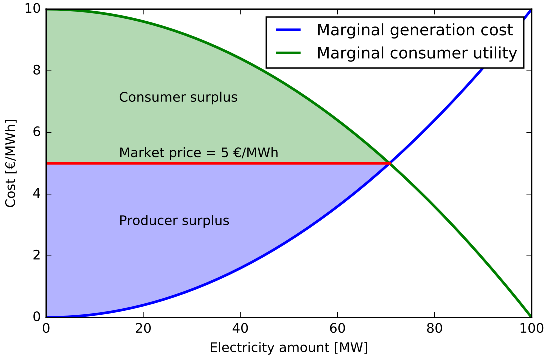
Supply and Demand at a Single Node

Setting the quantity and price

Now let's consider the case with **many** consumers and generators. We build **aggregated** marginal cost and marginal utility curves from the individual curves.

Then we maximise **total welfare**, the sum of net consumer and generator surplus of all actors.

If the price is also set from this point, then the individual optimal actions of each actor will achieve this result in a perfect decentralised market.



The result of optimisation

This is the result of maximising the **total economic welfare**, the sum of the consumer and the producer surplus for consumers b with consumption d_b and generators s generating with rate g_s:

$$\max_{\{d_b\},\{g_s\}} \left[\sum_b U_b(d_b) - \sum_s C_s(g_s) \right]$$

subject to the supply equalling the demand in the balance constraint:

$$\sum_b d_b - \sum_s g_s = 0 \qquad \leftrightarrow \qquad \lambda$$

and any other constraints (e.g. limits on generator capacity, etc.).

Market price λ is the shadow price of the balance constraint, i.e. the cost of supply an extra increment 1 MW of demand.

We will now show our main result:

Welfare-maximisation through decentralised markets

The welfare-maximising combination of production and consumption can be achieved by the decentralised profit-maximising decisions of producers and the utility-maximising decisions of consumers, provided that:

- The market price is equal to the shadow price of the overall supply-balance constraint in the welfare maximisation problem
- All producers and consumers are price-takers

Apply KKT now to maximisation of total economic welfare:

$$\max_{\{d_b\},\{g_s\}} f(\{d_b\},\{g_s\}) = \left[\sum_b U_b(d_b) - \sum_s C_s(g_s)\right]$$

subject to the balance constraint:

$$g(\{d_b\},\{g_s\}) = \sum_b d_b - \sum_s g_s = 0 \qquad \leftrightarrow \qquad \lambda$$

and any other constraints (e.g. limits on generator capacity, etc.).

Our optimisation variables are $\{x\} = \{d_b\} \cup \{g_s\}$.

We get from KKT stationarity at the optimal point:

$$0 = \frac{\partial f}{\partial d_b} - \sum_b \lambda^* \frac{\partial g}{\partial d_b} = U_b'(d_b^*) - \lambda^* = 0$$

$$0 = \frac{\partial f}{\partial g_s} - \sum_s \lambda^* \frac{\partial g}{\partial g_s} = -C_s'(g_s^*) + \lambda^* = 0$$

KKT and Welfare Maximisation 2/2

So at the optimal point of maximal total economic welfare we get the same result as if everyone maximises their own welfare separately based on the price λ^*:

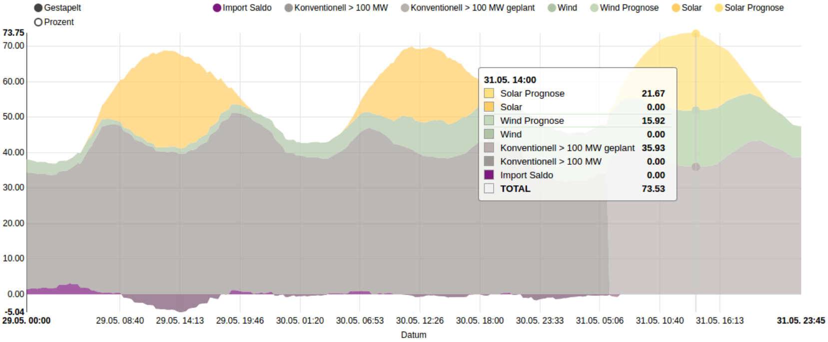
$$U_b'(d_b^*) = \lambda^*$$
$$C_s'(g_s^*) = \lambda^*$$

This is the CENTRAL result of microeconomics.

If we have further inequality constraints that are binding (e.g. capacity constraints), then these equations will receive additions with $\mu_i^* > 0$.

Power Production Real Example

At **energy-charts.de** you can see the forecast of load, wind, solar and conventional generation right now in Germany, here's an old version:



Supply-Demand Curve Real Example

At **epexspot.com** you can find the real supply-demand curves for every hour, here's an old example for Germany-Austria from 2017: