Energy Systems, Summer Semester 2022
Lecture 3: Input-Output Analysis

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Input-Output Tables

- Energy balances reveal the **flow of energy** through the economy, but not how sectors interact. For example: an increase in industry production could trigger an increase in transport demand, which triggers more steel production, etc.
- **Input-Output Tables** reveal how goods and services, including energy, are exchanged between different parts of the economy.
- They reveal which **inputs** are required by which economic sectors in order to produce that sectors' **outputs**.
- Input-output tables typically value goods **monetarily**, but they can be used for energy flows too.
- Each **row** represents the **sales of a product** to each sector and to final consumers.
- Each **column** represents the **costs of production factors** for each sector consisting of purchases from economic sectors and primary inputs.

Example input-output table (monetary units)

All values are in monetary units (e.g. € or $).

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
Oil, gas, coal	**0.09**	**0.07**	**0.18**	**2.86**	**4.18**	**5.72**	**13.10**
Electricity	**0.01**	**0.28**	**0.09**	**1.18**	**2.22**	**3.67**	**7.45**
Agriculture	0.00	0.00	0.90	11.54	1.33	3.31	17.08
Industry, constr.	0.01	0.61	3.82	45.08	26.02	143.42	218.96
Services	0.06	0.82	1.98	20.01	38.48	159.62	220.97
Imports	**8.16**	**1.24**	1.29	62.15	15.18		88.02
Depreciations	0.98	1.26	2.98	11.23	17.26		33.71
Interest, profits	0.26	0.62	3.07	9.49	32.92		46.40
Wages, salaries	0.83	1.25	3.51	44.89	72.72		123.20
Indirect taxes/subsidies	2.70	1.30	-0.74	10.53	10.66		24.50
Input X_j	13.10	7.45	17.08	218.96	220.97	315.74	793.30

Source: Zweifel, Praktiknjo & Erdmann (2017)

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$N = 5$ economic sectors labelled here by $i = 1, \ldots 5$: oil, gas, coal; electricity; agriculture; industry; services.

Total output sales of each sector X_i (row sum) is equal to the total cost input of each sector (column sum).

Example input-output table: typical row

A **row** shows where the sales of each sector go: either to other sectors or to consumers in 'final demand'.

For example electricity is sold to all other sectors, including to the electricity sector (e.g. to run power plant processes). Most demand is in industry, services and by final demand.

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Example input-output table: typical column

A **column** shows costs of production factors for each sector.

For example costs in industry include costs for fossil fuels, electricity, other products from industry, imports of raw materials and wages.

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First quadrant: intermediate use of products

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The first quadrant is the $N \times N$ matrix which shows the input of row sector i to column sector j, X_{ij}, i.e. the **intermediate use of the product** by other sectors.

Example: entry X_{24} shows an input of 1.18 electricity to industry.

Diagonal entries show self-consumption, e.g. electricity industry needs $X_{22} = 0.28$ electricity to run.

Second quadrant: final use of products

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
Oil, gas, coal	**0.09**	**0.07**	**0.18**	**2.86**	**4.18**	**5.72**	**13.10**
Electricity	**0.01**	**0.28**	**0.09**	**1.18**	**2.22**	**3.67**	**7.45**
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The second quadrant shows the final use of products, which could be to final demand from consumers F_i, or also to exports.

Thus summing up the rows we get from the first and second quadrants:

$$X_i = \sum_{j=1}^{N} X_{ij} + F_i$$

Third quadrant: other production factors

	Oil, gas, coal	Electricity	Agriculture	Industry	Services	Final demand F_i	Output X_i
Oil, gas, coal	**0.09**	**0.07**	**0.18**	**2.86**	**4.18**	**5.72**	**13.10**
Electricity	**0.01**	**0.28**	**0.09**	**1.18**	**2.22**	**3.67**	**7.45**
Agriculture	0.00	0.00	0.90	11.54	1.33	3.31	17.08
Industry, constr.	0.01	0.61	3.82	45.08	26.02	143.42	218.96
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Input X_j	13.10	7.45	17.08	218.96	220.97	315.74	793.30

The third quadrant shows the costs of other production factors beyond inputs from the other sectors, i.e. imports and other costs.

The columns thus sum up to the input X_j, which equals the output for each sector.

Leontief Multipliers

Linearisation

Our fundamental equation for the input-output analysis:

$$X_i = \sum_{j=1}^{N} X_{ij} + F_i$$

is a snapshot of the interactions in the economy.

What happens if the demand F_i changes?

To analyse this, we make a further assumption that the inputs i to sector j, X_{ij}, depend **linearly** on the output of sector j, i.e.

$$X_{ij} = a_{ij} X_j$$

a_{ij} is an $N \times N$ matrix, found by dividing by X_j

$$a_{ij} := \frac{X_{ij}}{X_j}$$

Leontief multipliers

Now our fundamental equation becomes:

$$X_i = \sum_{j=1}^{N} a_{ij} X_j + F_i$$

The first term is matrix multiplication, so we can write this more compactly:

$$X = a \cdot X + F$$

For a give final demand F, we can invert this to find the output X (1 is identity matrix here):

$$X = (1 - a)^{-1} \cdot F$$

The $N \times N$ factors $f_{ij} = (1 - a)^{-1}_{ij}$ are called **Leontief multipliers**. With all indices:

$$X_i = \sum_{j} (1 - a)^{-1}_{ij} F_j = \sum_{j} f_{ij} F_j$$

The Leontief multiplier f_{ij} tells us that if the final demand in sector j rises by 1 unit, then the output of sector i goes up by f_{ij} units.

Leontief multipliers from our example

	Oil, gas	Electricity	Agriculture	Industry	Services
Oil, gas, coal	1.007	0.015	0.019	0.021	0.026
Electricity	0.001	1.041	0.010	0.009	0.014
Agriculture	0.000	0.008	1.075	0.074	0.018
Industry, constr.	0.002	0.133	0.332	1.304	0.190
Services	0.006	0.155	0.189	0.156	1.236
Total	1.016	1.352	1.625	1.564	1.484

The matrix entries here are the Leontief multipliers f_{ij}. Note that the diagonal entries are always greater than one, $f_{jj} > 1$.

Example: Suppose electricity demand F_2 increases by one unit.

To understand the impact on output, read the second column f_{i2}: electricity output has to increase by 1.041 units, since there is additional electricity demand in the power plant to run it, as well as additional electricity demand triggered by other inputs for electricity. There are also additional inputs from oil, gas, coal of 0.015 units, agriculture of 0.008, industry of 0.133 and service sector of 0.155, so that in total output increases by 1.352 units.

Recursive explanation

If higher electricity demand triggers extra electricity demand, won't that extra electricity trigger extra extra demand?

Yes, but this is accounted for in the formula:

$$X = (1 - a)^{-1} \cdot F$$

The inverse can be expanded (like a Taylor series):

$$(1 - a)^{-1} = 1 + a + a^2 + a^3 + \cdots$$

so that

$$X = (1 - a)^{-1} \cdot F = (1 + a + a^2 + a^3 + \cdots) \cdot F$$

$a \cdot F$ is the first approximation of the extra electricity demand, $a^2 \cdot F$ is the extra demand from the extra electricity demand etc.

It is **convergent**.

Uses and Criticisms

There are many ways that input-output tables can be used and extended:

- They represent how changes in demand cascade through different sectors of the economy.
- They can be extended to analyse energy demand in each sector.
- They can be extended with other data, such as CO_2 and other pollutant impacts, or employment impacts for each sector.
- They can be used for **Life Cycle Analysis (LCA)**, e.g. impact of production and dismantling of a solar PV panel on its CO_2 balance.

Energy input-output tables

We can also tabulate how much energy is consumed in each sector in an **energy input-output table**. For our example (in energy units):

	Oil, gas	Electricity	Agriculture	Industry	Services	Final consumption F_i	Output X_i
Oil, gas	8.2	4.7	11.6	157.3	149.6	206.2	537.6
Electricity	0.6	20.6	3.1	56.4	46.2	130.4	257.3
Imports	571.1	56.1					627.2

From this table and the output of each sector X_j in monetary units we can calculate the **direct energy coefficients** e_{kj} for each energy source $k = 1, \dots M$ ($M = 2$ in out example):

$$e_{kj} = \frac{\text{energy supply from energy source } k \text{ to sector } j}{\text{total output (monetary) } X_j}$$

e_{kj} has units of energy per money and represents energy consumed per unit production. The **aggregated energy coefficient** per sector j sums over the energy sources

$$e_j = \sum_{k=1}^{M} e_{kj}$$

This represents the total energy supply per unit output of sector j.

Direct and indirect energy coefficients

In additional to the direct energy supplies to sector j there are also indirect non-energy supplies from sectors like services that may also increase energy demand.

We can determine the **total (direct and indirect) energy factors** to account for energy use in other sectors using the Leontief factors

$$\hat{e}_{kj} = \sum_{i=1}^{N} e_{ki} f_{ij}$$

Since the Leontief diagonal values f_{jj} are usually just above 1 and other values are small, the total energy factor $\hat{e}_{kj}$ is usually dominated by the direct factor $e_{kj} f_{jj} \sim e_{kj}$. You can also understand this by expanding the matrix f in terms of a:

$$\hat{e} = e \cdot f = e \cdot (1 - a)^{-1} = e \cdot (1 + a + a^2 + a^3 + \cdots)$$

Example usage: analysis of effects of Russian gas stop

In the widely discussed paper by Bachmann et al (2022) 'What if? The Economic Effects for Germany of a Stop of Energy Imports from Russia' the authors used empirical input-output tables to help assess the impact on GDP of stopping Russian gas imports.

A Appendix to Section 2 "The macroeconomic effects of a stop of energy imports from Russia on the German economy"

We pursue a two-pronged approach for assessing the macroeconomic effects. First, we use economic theory to isolate two of the key determinants of the macroeconomic effects of cutting energy imports from Russia. These are (i) the importance of Russian imports of gas, oil and coal ("brown" energy) in production and (ii) the elasticity of substitution between these energy sources and other inputs (e.g. "green" energy).

Second, we use the multi-sector model of Baqaee and Farhi (2021)[1] to run counterfactual simulations of the macroeconomic effects of cutting energy imports from Russia. The Baqaee-Farhi model is a state-of-the-art multi-sector model with rich input-output linkages and in which energy is a critical input in production.

Source: Bachmann et al, 2022

Example usage: life cycle analysis

We say a solar panel has no direct emissions when it is producing electricity.

But what about emissions from the production of the solar panel and the disposal of the panel at then end of its life?

This is the subject of **life cycle analysis**.

Input-output tables can be used to assess the impact of production and secondary effects of electricity and material consumption (e.g. silicon for wafers, silver for contacts, aluminium from frame).

But beware: input-output table for 2020 could look quite different to 2050!

Criticisms

- Static, linear representation of economy
- Cannot represent non-linear relations in the economy, only linear $X_{ij} = a_{ij} X_j$
- Cannot represent big changes to system, e.g. decarbonisation
- Demand is exogenous and completely inelastic, i.e. it doesn't react to prices
- Cannot represent limits of capacities, bottlenecks, scarcity, etc. (e.g. switching from nuclear to gas above a certain demand level)

This means it's only useful for short-term analysis.

For more comprehensive analysis we will need better models!