Energy System Modelling
Summer Semester 2018, Lecture 4

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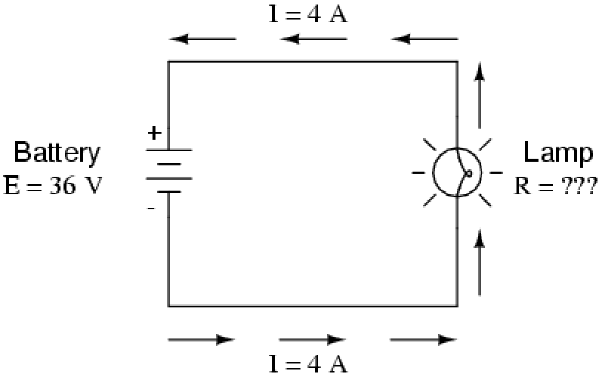
Revising Ohm's Law

Ohm's Law

Ohm's Law: The potential difference (voltage) $V_1 - V_2$ across an ideal conductor is proportional to the current through it I. The constant of proportionality is called the **resistance**, R. Ohm's Law is thus:

$$V_1 - V_2 = I\,R$$



Analogy DC circuits to linear power flow

The equations for DC circuits and linear power flow in AC circuits are analogous:

$$I = \frac{V_i - V_j}{R} \qquad \leftrightarrow \qquad f_\ell = \frac{\theta_i - \theta_j}{x_\ell}$$

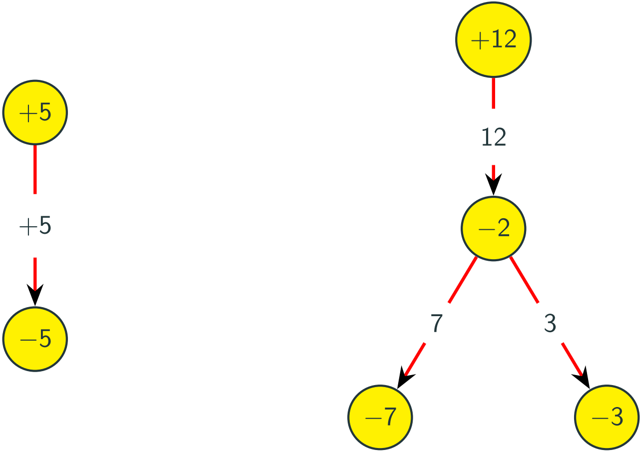
if we make the following identification:

Current flow I	$\leftrightarrow$	Active power flow f_ℓ
Potential/voltage V_i	$\leftrightarrow$	Voltage angle θ_i
Resistance R	$\leftrightarrow$	Reactance X

The simplifications that lead to the linear power flow were explained in the previous lecture.

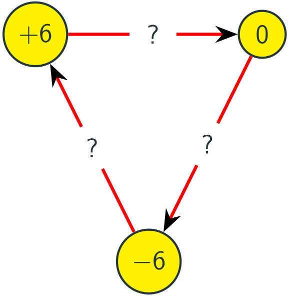
Kirchhoff's Current Law (KCL)

KCL inforces energy conservation at each vertex (the power imbalance equals what goes out minus what comes in).

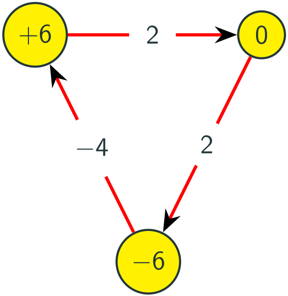


Kirchhoff's Voltage Law (KVL)

KCL isn't enough to determine the flow as soon as there are **closed cycles** in the network. For this we need Ohm's law in combination with KVL: voltage differences around each cycle add up to zero.



For equal reactances for each edge:



NB: For directed graph, sign determines direction of flow.

Computing the Linear Power Flow

Framing the load flow problem

Suppose we have N nodes labelled by i, and L edges labelled by ℓ forming a directed graph G.

Suppose at each node we have a **power imbalance** p_i ($p_i > 0$ means its generating more than it consumes and $p_i < 0$ means it is consuming more than it).

Since we cannot create or destroy energy (and we're ignoring losses):

$$\sum_i p_i = 0$$

Question: How do the flows f_ℓ in the network relate to the nodal power imbalances?

Answer: According to the impedances (generalisation of resistance for oscillating voltage/current) and the corresponding voltages.

Kirchhoff's Current Law (KCL)

KCL says (in this linear setting) that the nodal power imbalance at node i is equal to the sum of direct flows arriving at the node. This can be expressed compactly with the incidence matrix

$$p_i = \sum_\ell K_{i\ell} f_\ell \qquad\qquad \forall i$$

Kirchhoff's Voltage Law (KVL)

KVL says that the sum of voltage differences across edges for any closed cycle must add up to zero.

If the voltage at any node is given by θ_i (this is infact the voltage **angle** - more next week) then the voltage difference across edge ℓ is

$$\sum_i K_{i\ell}\theta_i$$

And Kirchhoff's law can be expressed using the cycle matrix encoding of independent cycles

$$\sum_\ell C_{\ell c}\sum_i K_{i\ell}\theta_i = 0 \qquad\qquad \forall c$$

[Automatic, since we already said KC = 0.]

Kirchhoff's Voltage Law (KVL)

If we express the flow on each line in terms of the voltage angle (a relative of $V = IR$) then for a line ℓ with reactance x_ℓ

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell}\theta_i$$

KVL now becomes

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad\qquad \forall c$$

Solving the equations

If we combine

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

with Kirchhoff's Current Law we get

$$p_i = \sum_\ell K_{i\ell} f_\ell = \sum_\ell K_{i\ell} \frac{1}{x_\ell} \sum_j K_{j\ell} \theta_j$$

This is a **weighted Laplacian**. If we write $B_{k\ell}$ for the diagonal matrix with $B_{\ell\ell} = \frac{1}{x_\ell}$ then

$$L = KBK^t$$

and we get a **discrete Poisson equation** for the θ_i sourced by the p_i

$$p_i = \sum_j L_{ij} \theta_j$$

We can solve this for the θ_i and thus find the flows.

Solving the equations

Given p_i at every node, we want to find the flows f_ℓ. We have the equations

$$p_i = \sum_j L_{ij}\theta_j$$
$$f_\ell = \frac{1}{x_\ell}\sum_i K_{i\ell}\theta_i$$

Basic idea: invert L to get θ_i in terms of p_i

$$\theta_i = \sum_k (L^{-1})_{ik} p_k$$

then insert to get the flows as a linear function of the power injections p_i

$$f_\ell = \frac{1}{x_\ell}\sum_{i,k} K_{i\ell}(L^{-1})_{ik} p_k = \sum_k \mathrm{PTDF}_{\ell k} p_k$$

Inverting Laplacian L

There is one small catch: L is **not invertible** since we saw last time it has (for a connected network) one zero eigenvalue, with eigenvector $(1, 1, \dots 1)$, since by construction $\sum_j L_{ij} = 0$.

This is related to a gauge freedom to add a constant to all voltage angles

$$\theta_i \to \theta_i + c$$

(corresponding to the zero eigenvector of L) which does not affect physical quantities:

$$p_i = \sum_j L_{ij}(\theta_j + c) = \sum_j L_{ij}(\theta_j)$$

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i + c) = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i)$$

Typically we choose a **slack** or **reference bus** such that $\theta_0 = 0$.

Inverting Laplacian L

Two solutions:

1. Set $\theta_0 = 0$, invert the lower-right $(N-1) \times (N-1)$ part of L to find the remaining $\{p_i\}_{i=1,\dots N-1}$ in terms of the $\{\theta_i\}_{i=1,\dots N-1}$, then derive p_0 from $\sum_i p_i = 0$.

2. Use the Moore-Penrose pseudo-inverse.

Write L in terms of its basis of orthonormal eigenvectors

$$L = \sum_n |\Phi_n\rangle \, \lambda_n \, \langle\Phi_n|$$

then the Moore-Penrose pseudo-inverse is:

$$L^\dagger = \sum_{n|\lambda_n \neq 0} \frac{|\Phi_n\rangle \langle\Phi_n|}{\lambda_n}$$

Check:

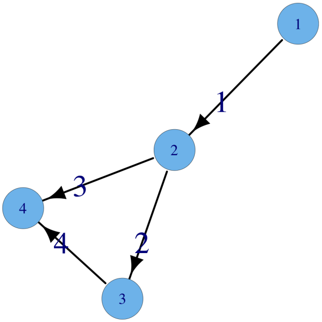
$$L^\dagger L = \sum_{n|\lambda_n \neq 0} |\Phi_n\rangle \langle\Phi_n| = I - |\Phi_0\rangle \langle\Phi_0|$$

4-node example

$$\mathbf{K}_{i\ell} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$\mathbf{L}_{ij} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$

$$\mathbf{PTDF}_{\ell i} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -2/3 & -1/3 \\ 0 & 0 & -1/3 & -2/3 \\ 0 & 0 & 1/3 & -1/3 \end{pmatrix}$$

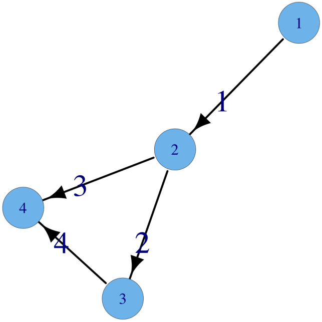


PTDF as sensitivity

Can also 'experimentally' determine the Power Transfer Distribution Factors (PTDF) by choosing a slack bus (in this case bus 1).

Each column (labelled by i) is then the resulting line flows if we have a simple power transfer from bus i to the slack $p_i = 1$ and $p_1 = -1$.

$$\mathbf{PTDF}_{\ell i} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -2/3 & -1/3 \\ 0 & 0 & -1/3 & -2/3 \\ 0 & 0 & 1/3 & -1/3 \end{pmatrix}$$



Consequences of limiting power transfers

Line loading limits

You cannot pass infinite current through a transmission line.

As it warms, it sags, then it will become damaged and/or hit a building/tree and cause a short-circuit. For this reasons there are always **thermal limits** on current transfer. There may also be limits on the amount of power or current based on concerns about **voltage stability** or **general stability**.

Typically each line has a well-defined **line loading limit** on the amount of current or power that can flow through it:

$$|f_\ell| \leq F_\ell$$

These limits prevent the transfer of renewable energy or other power sources.

Adjusting generator dispatch to avoid overloading

To avoid overloading the power lines, we must adjust our generator output (or the demand) so that the power imbalances do not overload the network.

We will now generalise and adjust our notation.

From lecture 2 we had for a single node:

$$-p_t = m_t - b_t + c_t = d_t - Ww_t - Ss_t - b_t + c_t = 0$$

where p_t was the nodal power balance, m_t was the mismatch (load d_t minus wind Ww_t and solar Ss_t), b_t was the backup power and c_t was curtailment.

We generalised this to multiple nodes labelled by i

$$-p_{i,t} = m_{i,t} - b_{i,t} + c_{i,t} = d_{i,t} - W_i w_{i,t} - S_i s_{i,t} - b_{i,t} + c_{i,t}$$

where now we don't enforce $p_{i,t} = 0$ but $\sum_i p_{i,t} = 0$ for all t.

Adjusting generator dispatch to avoid overloading

Now we write the dispatch of all generators at node i (wind, solar, backup) labelled by technology s as $g_{i,s,t}$ (i labels node, s technology and t time) so that we have a relation between load $d_{i,t}$, generation $g_{i,s,t}$ and network flows $f_{\ell,t}$

$$p_{i,t} = \sum_s g_{i,s,t} - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t}$$

Where s runs over the wind, solar and backup capacity generators (e.g. hydro or natural gas) at the node.

A dispatchable generator's $g_{i,s,t}$ output can be controlled within the limits of its power capacity $G_{i,s}$

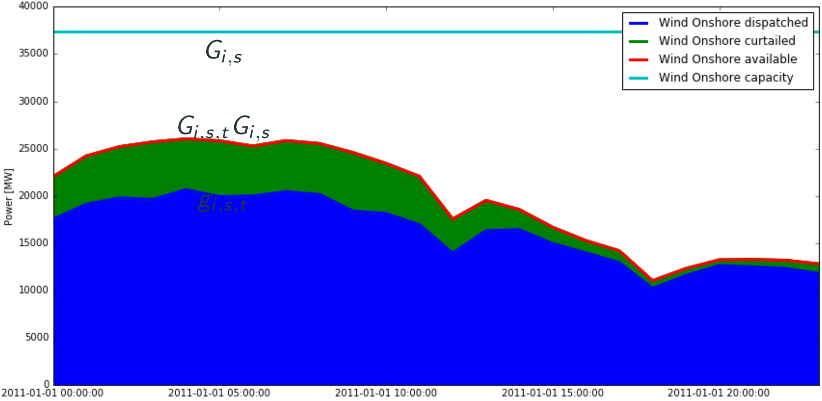
$$0 \leq g_{i,s,t} \leq G_{i,s}$$

Variable generation constraints

For a renewable generator we have time series of availability $0 \leq G_{i,s,t} \leq 1$ (the s_t and w_t before; W and S are the capacity $G_{i,s}$):

$$0 \leq g_{i,s,t} \leq G_{i,s,t} G_{i,s} \leq G_{i,s}$$

Curtailment corresponds to the case where $g_{i,s,t} < G_{i,s,t} G_{i,s}$:

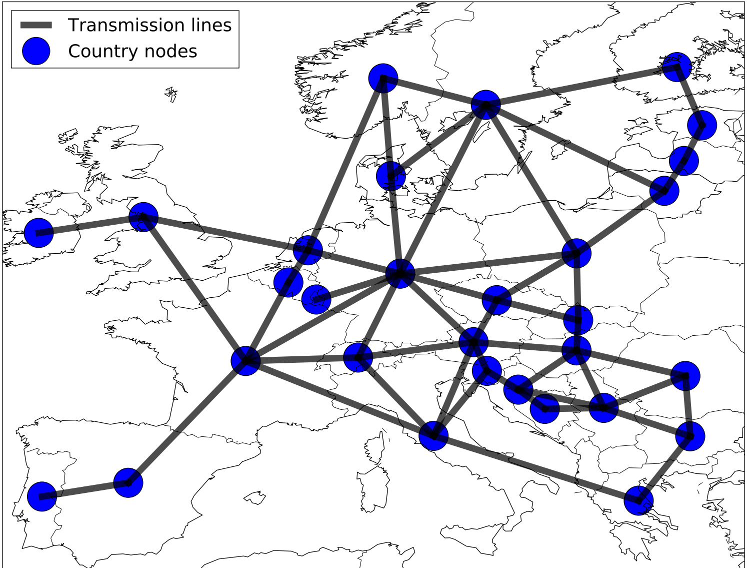


Germany curtailment example

See `https://pypsa.org/examples/scigrid-lopf-then-pf.html`.

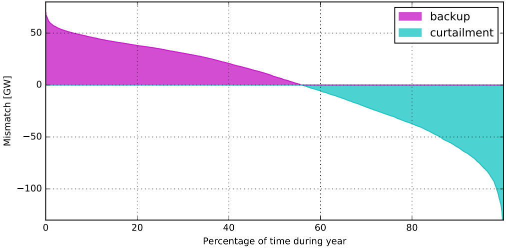
European transmission versus backup energy

Consider backup energy in a simplified European grid:

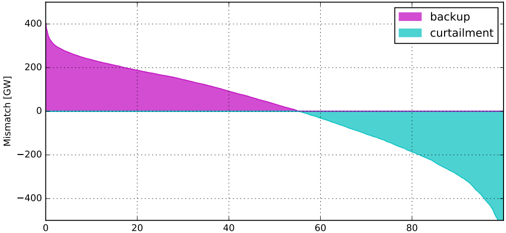


DE versus EU backup energy from last time

Germany needed backup generation for 31% of total load:

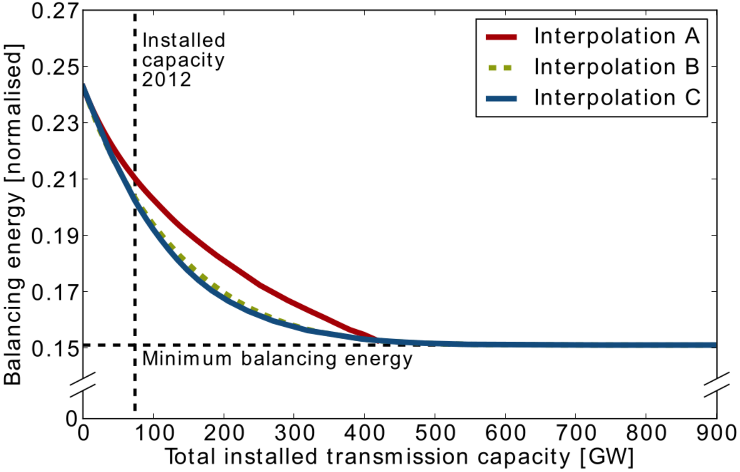


Europe needed Backup generation for only 24% of the total load:



European transmission versus backup energy

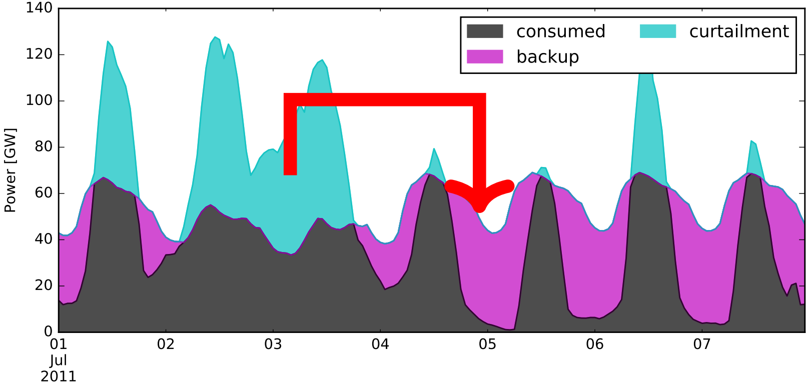
Transmission needs across a fully renewable European power system by Rodriguez, Becker, Andresen, Heide, Greiner, Renewable Energy, 2014



Principles of electricity storage

Basic idea of storage

Networks were used to shift power imbalances between different places, i.e. in **space**. Electricity storage can shift power in **time**.



Storage consistency

Storage units, such as batteries or hydrogen storage, can both dispatch power within a certain capacity:

$$0 \leq g_{i,s,t,\text{dispatch}} \leq G_{i,s,\text{dispatch}}$$

and consume power to store energy:

$$0 \leq g_{i,s,t,\text{store}} \leq G_{i,s,\text{store}}$$

The total power can then be written:

$$g_{i,s,t} = g_{i,s,t,\text{dispatch}} - g_{i,s,t,\text{store}}$$

There is also a limit on the total energy $e_{i,s,t}$ at each time

$$0 \leq e_{i,s,t} = -\int^{t} g_{i,s,t'} \, dt' \leq E_{i,s}$$

where $E_{i,s}$ is the energy capacity (in MWh). Or in iterative form

$$0 \leq e_{i,s,t} = e_{i,s,t-1} + g_{i,s,t,\text{store}} - g_{i,s,t,\text{dispatch}} \leq E_{i,s}$$

Continuous example

Consider a single node with a constant demand

$$d(t) = D$$

and a renewable wind generator with a capacity $G = 2D$ and an availability time series

$$G(t) = \frac{1}{2}\left(1 + \sin(\omega t)\right)$$

so that

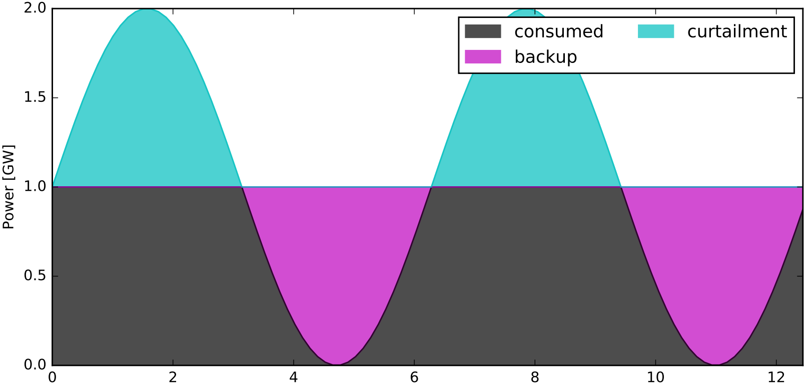
$$\langle G(t) G \rangle = D$$

Mismatch

Our mismatch is now

$$m(t) = d(t) - GG(t) = -D\sin(\omega t)$$

For $D = 1, \omega = 1$:

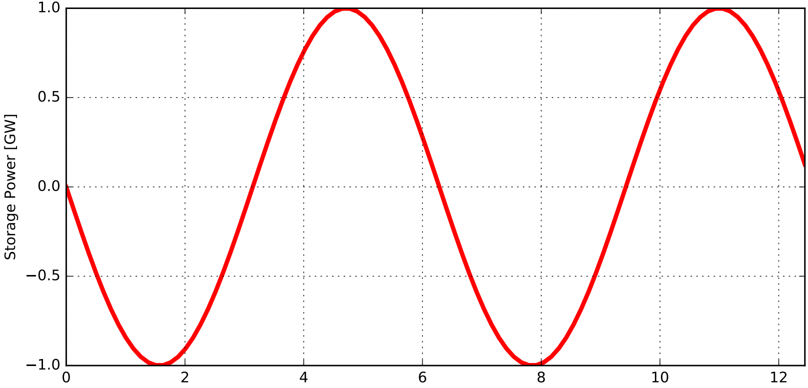


Storage Power

To balance this, we need a storage unit with a power profile to match the mismatch

$$g_s(t) = m(t) = -D\sin(\omega t)$$

This will have power capacities $G_{s,\mathrm{store}} = G_{s,\mathrm{dispatch}} = D$.

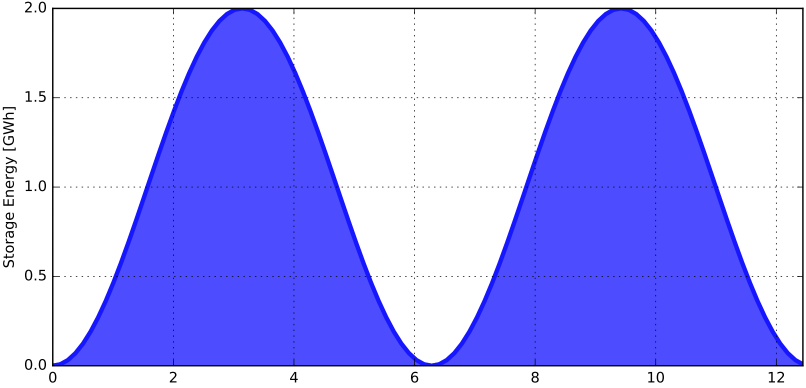


Storage Energy

How much energy capacity E_s do we need? The energy profile is:

$$e_s(t) = \int_0^t (-g_s(t'))dt' = D\int_0^t \sin(\omega t')dt' = \frac{D}{\omega}\left[1 - \cos(\omega t)\right]$$

so $E_s = \max_t\{e_s(t)\} = \frac{2D}{\omega}$. Faster oscillations $\Rightarrow$ less energy capacity.



Efficiency losses

There are a few extra details to add now. First, no renewable has a perfectly regular sinusoidal profile.

Second, the iterative integration equation for the storage energy

$$e_{i,s,t} = e_{i,s,t-1} + g_{i,s,t,\text{store}} - g_{i,s,t,\text{dispatch}}$$

needs to be amended for **efficiency losses** η

$$e_{i,s,t} = \eta_0 e_{i,s,t-1} + \eta_1 g_{i,s,t,\text{store}} - \eta_2^{-1} g_{i,s,t,\text{dispatch}}$$

Different storage units have different parameters

We can relate the power capacity G_s to the energy capacity E_s with the maximum number of hours the storage unit can be charged at full power before the energy capacity is full, $E_s = \text{max-hours} * G_s$.

	Battery	Hydrogen	Pumped-Hydro	Water Tank
η_0	$1-\varepsilon$	$1-\varepsilon$	$1-\varepsilon$	depends on size
η_1	0.9	0.75	0.9	0.9
η_2	0.9	0.58	0.9	0.9
max-hours	2-10	weeks	4-10	depends on size
euro per kW [G_s]	300	300+450	depends	low
euro per kWh [E_s]	200	10	depends	low

Parameters are roughly based on Budischak et al, 2012 with projections for 2030.

Power to Gas

Power to gas describes certain concepts to store electric energy surplusses for times of need

Surplusses are used to split water into hydrogen and oxygen. Hydrogen can then be used either directly (gas grid, transport, etc.) or converted to methane (by combination with carbon dioxide)

Storage capacity of the German gas network in terms of energy: ca 200 TWh. In addition, losses in the gas network are small.

Power to Gas Concept

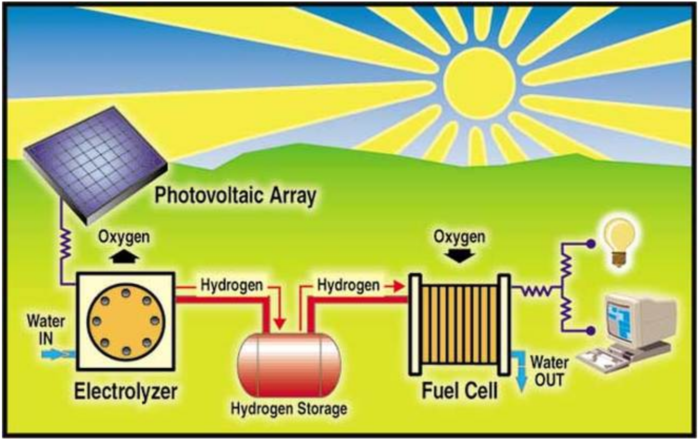
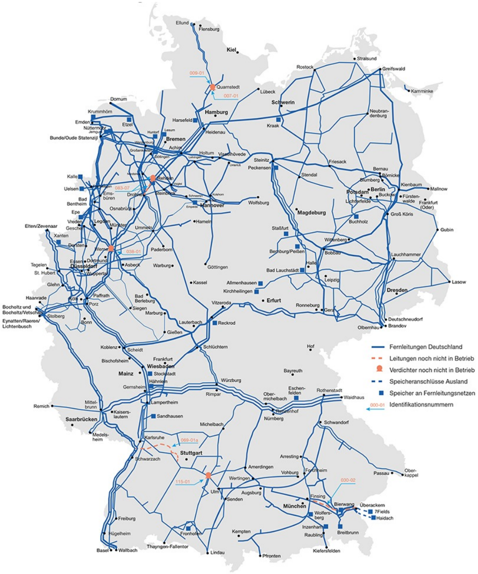


Figure 1: Buildipedia

German Gas Grid



Electrolysis

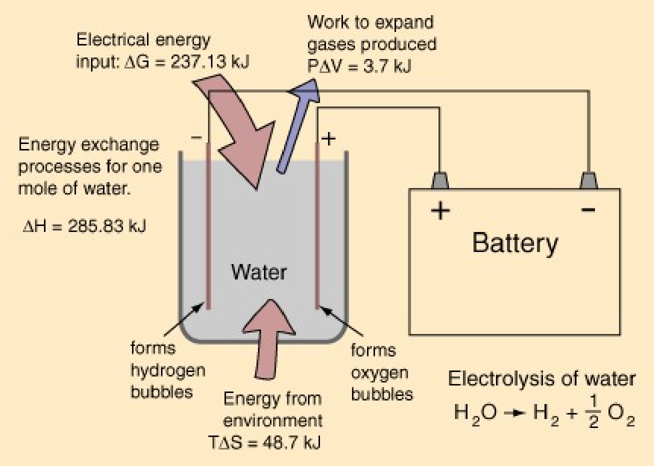


Figure 3: Hyperphysics, Georgia State University

Thermodynamic Calculation Electrolysis

$$H_2O \rightarrow H_2 + \frac{1}{2}O_2$$

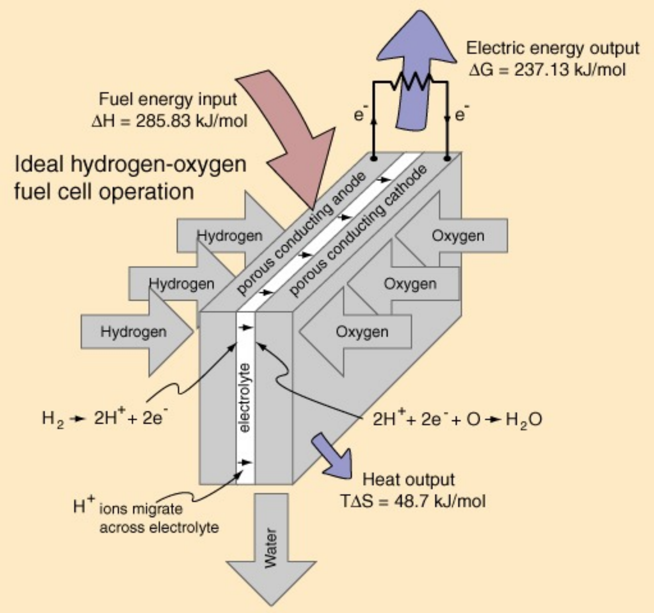
For one mole at conditions 298 K and one atmosspheric pressure

x	H_2	O_2	H_2O
Entropy [J/K]	130.7	205.1	69.9
Enthalpy [kJ]	0	0	-285.8

Gibbs free energy $dG = dH - TdS$,

$$\Delta G = 285.8kJ - 48.7kJ = 237.1kJ$$

Fuel Cell



Electric energy output
ΔG = 237.13 kJ/mol
Fuel energy input
ΔH = 285.83 kJ/mol
Ideal hydrogen-oxygen
fuel cell operation
e-
e-
porous conducting anode
porous conducting cathode
Hydrogen
Hydrogen
Hydrogen
Oxygen
Oxygen
Oxygen
electrolyte
H2 → 2H+ + 2e-
2H+ + 2e- + O → H2O
Heat output
TΔS = 48.7 kJ/mol
H+ ions migrate
across electrolyte
Water

Thermodynamics of Fuel Cell

Again: one mole at conditions 298 K and one atmosspheric pressure

$$H_2O \to H_2 + \frac{1}{2}O_2$$

Gibbs free energy $dG = dH - TdS$,

$$\Delta G = 285.8kJ - 48.7kJ = 237.1kJ$$

max theoretical efficiency

$$\frac{\Delta G}{\Delta U} = 0.83$$

Demand-Side Management (DSM)

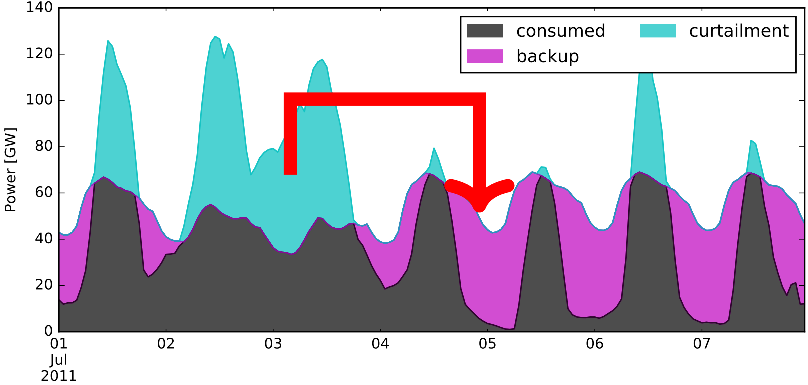
Recall from Previous Lectures

Conceptual options to balance the power system

- Transmission grid
- Storage
- **Demand-side management**
- Sector coupling

Basic Idea of Demand-Side Management

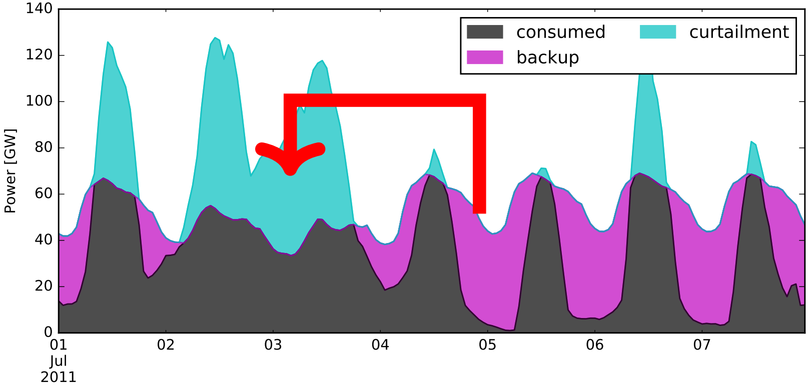
From last time: basic idea of storage



Modify demand instead of generation!

Basic Idea of Demand-Side Management

Modify demand instead of generation!



Demand-Side Management / Demand-Side Response

Modification of the Demand for energy through various means such as price incentives

Charge consumers based on the true price of utilities at the time of consumption

Issues: higher utility cost for consumers, privacy

Economics of Supply and Demand

Definition

Demand is the total amount of a good buyers would purchase under certain conditions

Law of demand: when the price of a good falls, the demand will rise.

A demand curve is the graphical representation of the relationship between price and quantity demanded

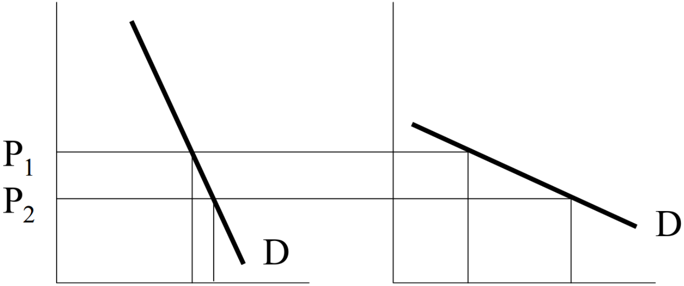
Vice versa for supply: total amount of a good sellers would choose to sell under certain conditions, etc.

Elasticity

Definition

Degree of responseness of one variable to another

Locally: slope



How Flexible is the Demand?

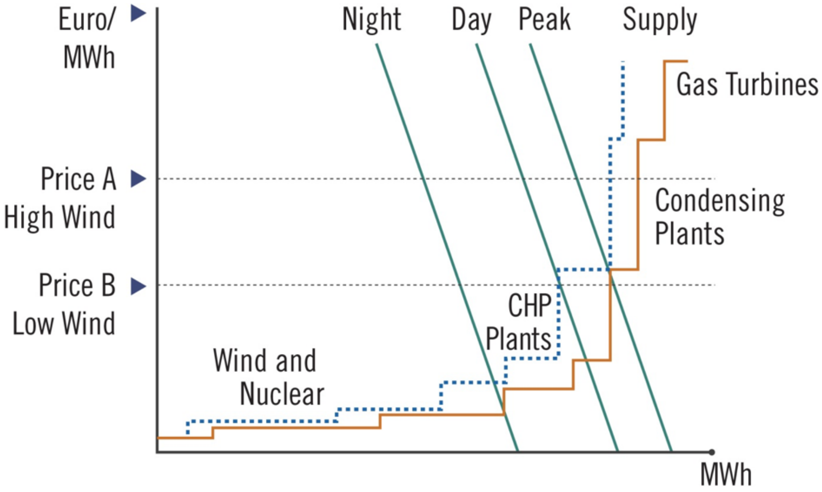
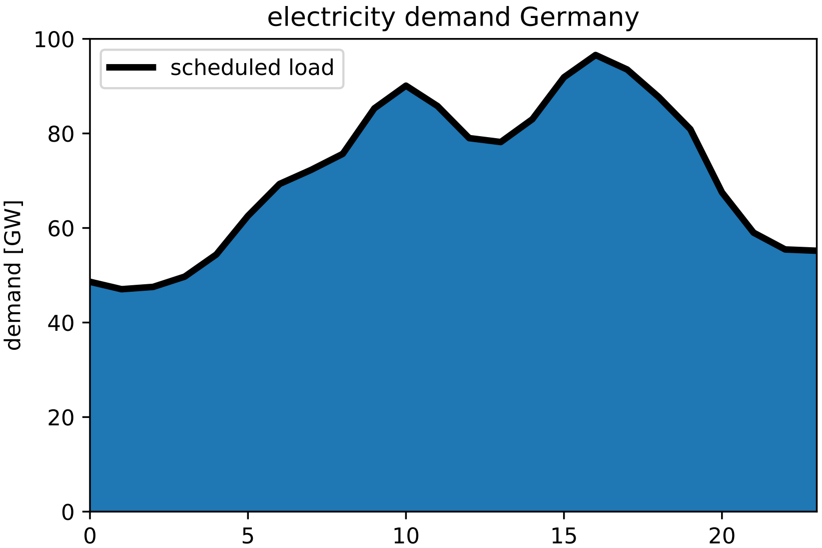


Figure 5: sale/purchase day-ahead market

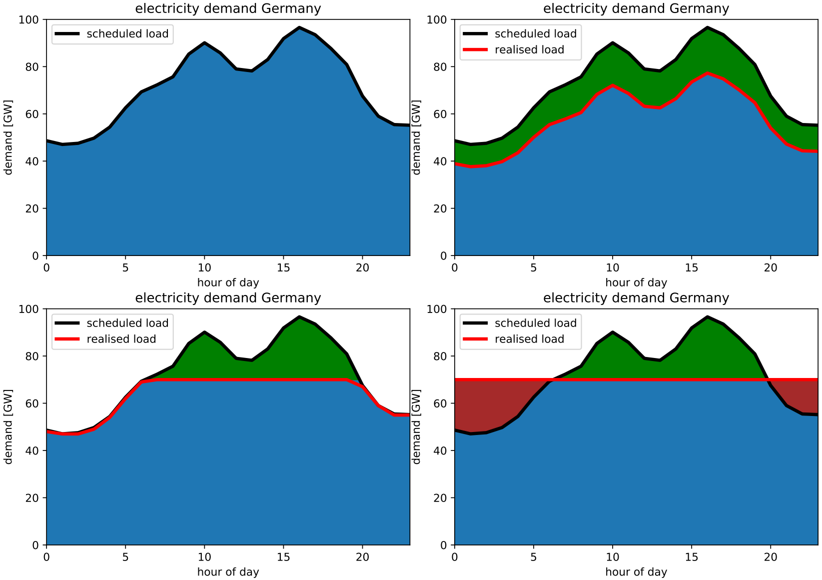
Different Cases of DSM

Electricity Demand

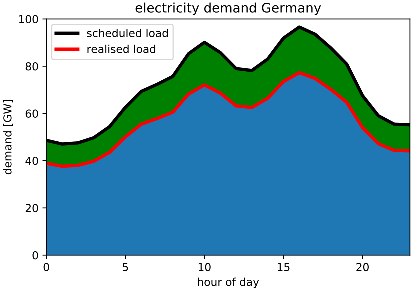
Demand curve Germany 12/01/2003



Different Cases of DSM

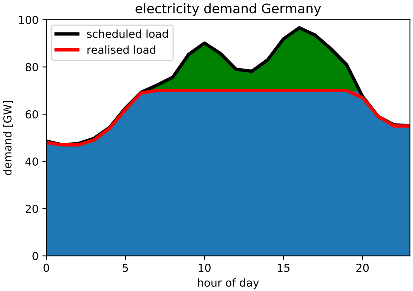


Efficiency Measures



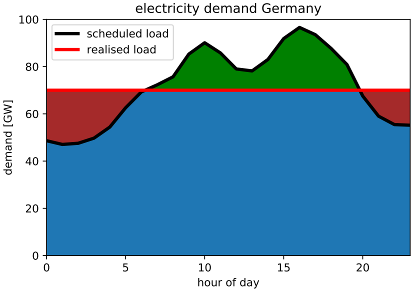
- Permanent reduction of the demand by use of more efficient appliances
  - washing machines
  - refrigerators
  - water heaters
- Germany: Reduction of 25% of gross electrical energy by 2050 compared to 2008

Peak Shaving



- Infrastructure designed for peak demand situations
- Commercial consumers often charged based on their peak demand

Load Shifting

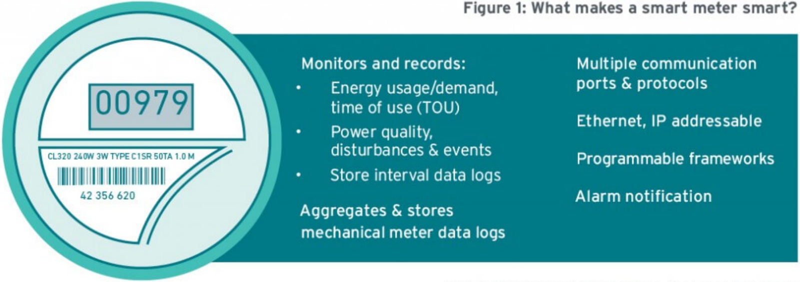


- Shift electrical demand from times of deficits to times of surplusses
- provide price incentives to cause load shifting via smart meters
- different price incentive schemes possible, e.g., time of use prices, seasonal prices, etc.

Technical Aspects

Technical Aspects of Demand-Side Management

Figure 1: What makes a smart meter smart?



Source: Adapted from GTM Research, Department of Energy

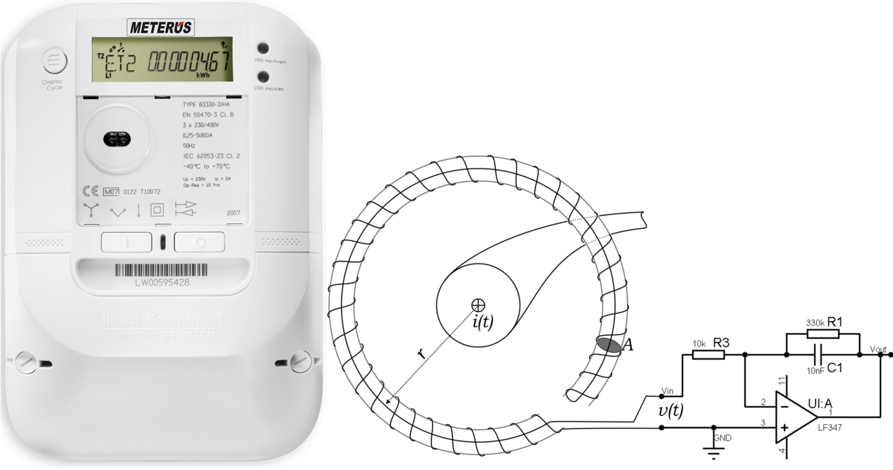


Figure 6: Wikipedia

Rogowski Coil

Advantages:

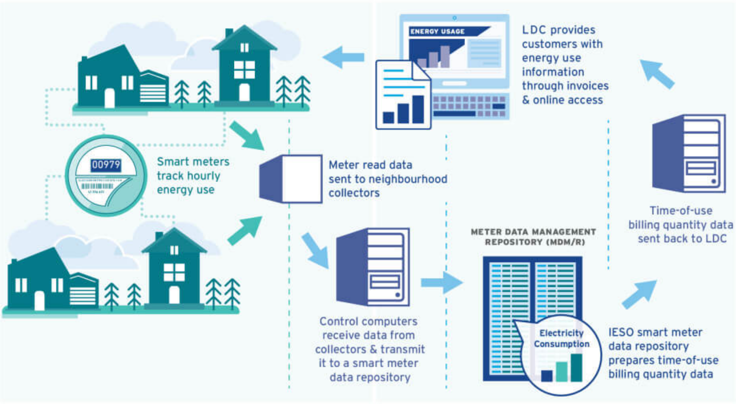
- low inductance, therefore sensitive to small current changes
- highly linear for a large range of currents
- open loop
- relatively low cost
- simple temperature compensation

Voltage given by

$$v_t = -\frac{AN\mu_0}{l}\dot{i}$$

Example

Figure 4: Energy information pathways in Ontario



Source: Adapted from Independent Electricity System Operator

Modelling approach for DSM

Modelling Approach for DSM

- loads into different categories with assumed max. shifting periods (e.g., 8 hours for household applications)
- shifting charges a virtual storage buffer

$$P_n[R_n(t)](t) = R_n(t) - L_n(t). \tag{1}$$

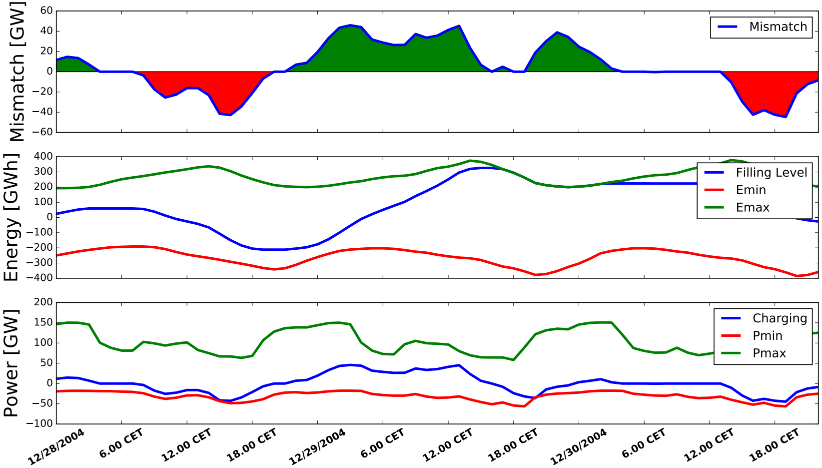
- filling level is consequently given by

$$E_n[R_n(t)](t) = \int_0^t P_n[R_n(t')](t')dt' \tag{2}$$

- constraints by shifting periods, e.g.,

$$E_n^+(t) = \int_t^{t+\Delta t} L_n(t')dt' \tag{3}$$

Modelling Approach for DSM



Modelling Approach for DSM

- Load shifting supports system integration of variable renewables, especially PV

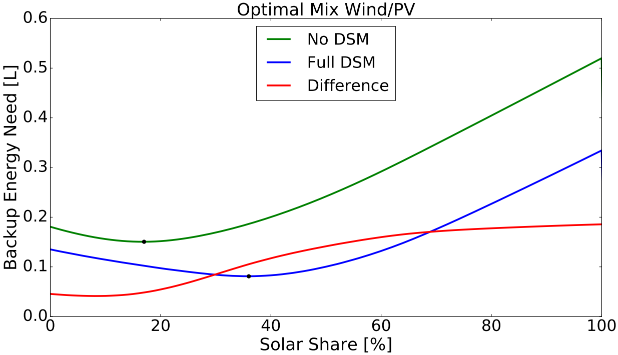
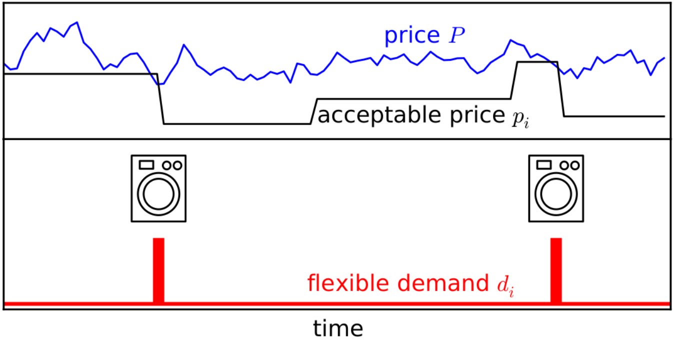


Figure 7: Kies et al., Energies, 2016

Consumer Synchronisation via Adaptive Pricing Schemes

Consumer Synchronisation via Adaptive Pricing Schemes

Paper: Krause, S. et al., Econophysics of adaptive power markets: When a market does not dampen fluctuations but amplifies them, arXiv:1303.2110



Consumer Synchronisation via Adaptive Pricing Schemes

demand is described via (p_t - price time series)

$$d_{i,t} = \begin{cases} 1 \text{ if } p_t \leq p_{i,t}, \\ 0 \text{ if } p_t > p_{i,t} \end{cases}$$

and the acceptable price ($p_{i,t}$) time series of agent i evolves according to

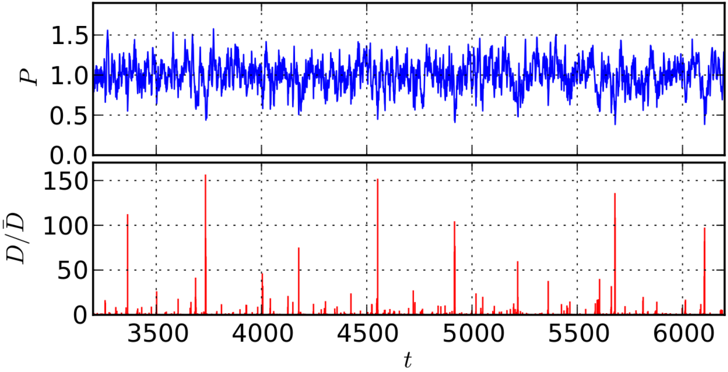
$$p_{i,t+1} = \begin{cases} \text{rand}[0, p_{i,t}] \text{ if } p_t \leq p_{i,t}, \\ \text{rand}[p_{i,t}, 1], \text{ else with prob. } f \\ p_{i,t} \text{ otherwise} \end{cases}$$

the parameter f describes the elasticity of the demand

correlations modelled via Langevin equation

$$p_{t+1} - p_t = -v_0(p_t - \bar{p}) + \sigma_0 \xi_t$$

Synchronisation



Agents synchronise → extreme peak demands. Effect also known as demand response concentration.

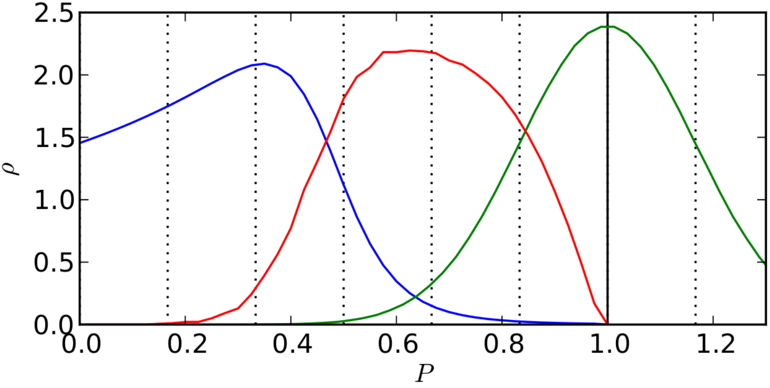


Figure 8: Density of highest acceptable prices (blue), total load consumed at certain prices (red)

Consumer Synchronisation via Adaptive Pricing Schemes

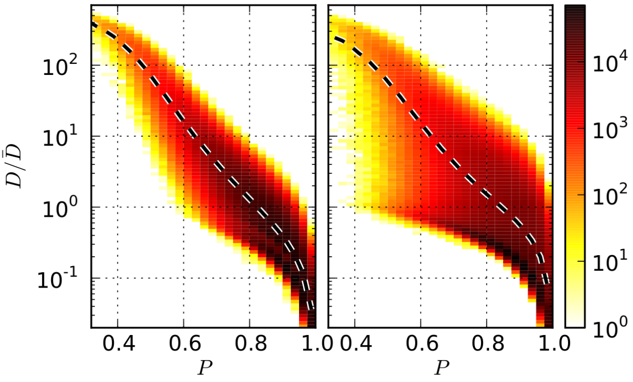


Figure 9: Binning of events by price and demand intervals. Dashed line shows average demand. Correlated prices (left), uncorrelated prices (right)

Summary

- Demand-side management can contribute to successful power system operation
- "Daily" scale supports PV integration
- Building infrastructure for DSM is cost-intensive and causes additional energy consumption
- Synchronisation via pricing can amplify fluctuations