Energy System Modelling
Summer Semester 2018, Lecture 7

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Table of Contents

Optimisation Revision

Optimisation problem

We have an **objective function** $f : \mathbb{R}^k \to \mathbb{R}$

$$\max_x f(x)$$

$[x = (x_1, \ldots x_k)]$ subject to some **constraints** within $\mathbb{R}^k$:

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \ldots n \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \ldots m \end{array}$$

λ_i and μ_j are the **KKT multipliers** (basically Lagrange multipliers) we introduce for each constraint equation; it measures the change in the objective value of the optimal solution obtained by relaxing the constraint (shadow price).

KKT conditions

The **Karush-Kuhn-Tucker (KKT) conditions** are necessary conditions that an optimal solution x^*, μ^*, λ^* always satisfies (up to some regularity conditions):

1. **Stationarity**: For $l = 1, \dots k$

$$\frac{\partial \mathcal{L}}{\partial x_l} = \frac{\partial f}{\partial x_l} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial x_l} - \sum_j \mu_j^* \frac{\partial h_j}{\partial x_l} = 0$$

2. **Primal feasibility**:

$$g_i(x^*) = c_i$$
$$h_j(x^*) \leq d_j$$

3. **Dual feasibility**: $\mu_j^* \geq 0$

4. **Complementary slackness**: $\mu_j^*(h_j(x^*) - d_j) = 0$

min/max and signs

If the problem is a **maximisation** problem (like above), then $\mu_j^* \geq 0$ since $\mu_j = \frac{\partial \mathcal{L}}{\partial d_j}$ and if we increase d_j in the constraint $h_j(x) \leq d_j$, then the feasible space can only get bigger. Since if $X \subseteq X'$

$$\max_{x \in X} f(x) \leq \max_{x \in X'} f(x)$$

then the objective value at the optimum point can only get bigger, and thus $\mu_j^* \geq 0$. (If $d_j \to \infty$ then the constraint is no longer binding, if $d_j \to -\infty$ then the feasible space vanishes.)

If however the problem is a **minimisation** problem (e.g. cost minimisation) then we can use

$$\min_{x \in X} f(x) = -\max_{x \in X} [-f(x)]$$

We can keep our definition of the Lagrangian and almost all the KKT conditions, but we have a change of sign $\mu_j^* \leq 0$, since

$$\min_{x \in X} f(x) \geq \min_{x \in X'} f(x)$$

The λ_i^* also change sign.

Welfare maximisation revision

KKT and Welfare Maximisation 1/2

Apply KKT now to maximisation of total economic welfare:

$$\max_{\{d_b\},\{g_s\}} f(\{d_b\},\{g_s\}) = \left[\sum_b U_b(d_b) - \sum_s C_s(g_s)\right]$$

subject to the balance constraint:

$$g(\{d_b\},\{g_s\}) = \sum_b d_b - \sum_s g_s = 0 \qquad \leftrightarrow \qquad \lambda$$

and any other constraints (e.g. limits on generator capacity, etc.).

Our optimisation variables are $\{x\} = \{d_b\} \cup \{g_s\}$.

We get from stationarity:

$$0 = \frac{\partial f}{\partial d_b} - \sum_b \lambda^* \frac{\partial g}{\partial d_b} = U_b'(d_b) - \lambda^* = 0$$

$$0 = \frac{\partial f}{\partial g_s} - \sum_s \lambda^* \frac{\partial g}{\partial g_s} = -C_s'(g_s) + \lambda^* = 0$$

KKT and Welfare Maximisation 2/2

So at the optimal point of maximal total economic welfare we get the same result as if everyone maximises their own welfare separately:

$$U'_b(d_b) = \lambda^*$$
$$C'_s(g_s) = \lambda^*$$

This is the CENTRAL result of microeconomics.

If we have further inequality constraints that are binding, then these equations will receive additions with $\mu_i^* > 0$.

Optimise Single Node with Inelastic Demand, Linear Generation Costs

Supply-demand linear example: generator bids

Example from Kirschen and Strbac pages 56-58.

The following generators bid into the market for the hour between 0900 and 1000 on 20th April 2016:

Company	Quantity [MW]	Price [$/MWh]
Red	200	12
Red	50	15
Red	150	20
Green	150	16
Green	50	17
Blue	100	13
Blue	50	18

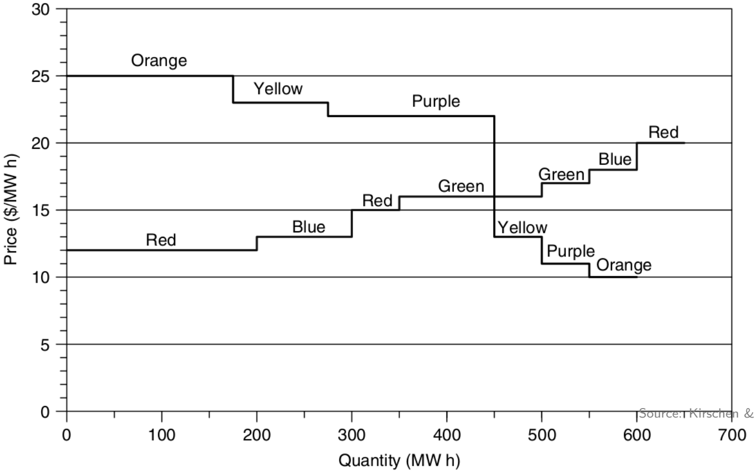
Supply-demand linear example: Consumer offers

The following consumers make offers for the same period:

Company	Quantity [MW]	Price [$/MWh]
Yellow	50	13
Yellow	100	23
Purple	50	11
Purple	150	22
Orange	50	10
Orange	200	25

Supply-demand example: Curve

If the bids and offers are stacked up in order, the supply and demand curves meet with a demand of 450 MW at a system marginal price of $\lambda = 16$ \$/MWh.



Source: Kirschen & Strbac

Supply-demand example: Revenue and Expenses

Dispatch and revenue/expense of each company:

Company	Production [MWh]	Consumption [MWh]	Revenue [$]	Expense [$]
Red	250		4000	
Blue	100		1600	
Green	100		1600	
Orange		200		3200
Yellow		100		1600
Purple		150		2400
Total	450	450	7200	7200

Simplified world: linear generation costs, inelastic demand

We will now turn to an even simpler world: all the generator cost functions are linear

$$C_s(g_s) = o_s g_s$$

and each generator has limited output $0 \leq g_s \leq G_s$.

We also fix the demand to a value D so that it does not respond to price changes (i.e. the demand is **inelastic**) up to a very high marginal utility $V >> o_s \; \forall s$, i.e.

$$U(d) = Vd$$

for $d \leq D$.

V is sometimes called the **Value Of Lost Load (VOLL)**.

Simplify representation of consumers and generators

In this case we get for our welfare maximisation:

$$\max_{d,\{g_s\}} \left[Vd - \sum_s o_s g_s \right]$$

subject to:

$$d - \sum_s g_s = 0 \qquad \leftrightarrow \qquad \lambda$$

$$d \leq D \qquad \leftrightarrow \qquad \mu$$

$$g_s \leq G_s \qquad \leftrightarrow \qquad \bar{\mu}_s$$

$$-g_s \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_s$$

Simplest example: one generator type, inelastic demand

Suppose all generators have the same marginal cost o and we represent their total dispatch by g and total capacity by G

$$\max_{d,g} [Vd - og]$$

such that:

$$\begin{aligned}
d - g = 0 &\quad \leftrightarrow \quad \lambda \\
d \leq D &\quad \leftrightarrow \quad \mu \\
g \leq G &\quad \leftrightarrow \quad \bar{\mu} \\
-g \leq 0 &\quad \leftrightarrow \quad \underline{\mu}
\end{aligned}$$

Simplest example: one generator type, inelastic demand

If $D < G$ then since $V >> o$, it will be always profitable for the generators to dispatch to satisfy the load, i.e.

$$g^* = d^* = D$$

If the demand is non-zero then since $g^* > 0$ by complementarity we have $\underline{\mu}^* = 0$. Since $D < G$ then $g^* < G$ and by complementarity we have $\bar{\mu}^* = 0$. To compute λ^* we use stationarity:

$$0 = \frac{\partial \mathcal{L}}{\partial g} = \frac{\partial f}{\partial g} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial g} - \sum_j \mu_j^* \frac{\partial h_j}{\partial g} = -o + \lambda^* - \bar{\mu}^* + \underline{\mu}^*$$

Thus $\lambda^* = o$, which is the cost per unit of supplying extra demand. The **generator sets the price**.

For the load μ^* can be non-zero because $d^* = D$:

$$0 = \frac{\partial \mathcal{L}}{\partial d} = \frac{\partial f}{\partial d} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial d} - \sum_j \mu_j^* \frac{\partial h_j}{\partial d} = V - \lambda^* - \mu^*$$

$\mu^* = V - \lambda^*$ is the marginal benefit of each increase in demand.

Simplest example: one generator type, inelastic demand

If $D > G$ then the generator will dispatch up to its maximum capacity

$$g^* = d^* = G$$

For its lower limit we have $\underline{\mu}^* = 0$. From stationarity:

$$0 = \frac{\partial \mathcal{L}}{\partial g} = \frac{\partial f}{\partial g} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial g} - \sum_j \mu_j^* \frac{\partial h_j}{\partial g} = -o + \lambda^* - \bar{\mu}^* + \underline{\mu}^*$$

Thus $\lambda^* = o + \bar{\mu}^*$. To find λ^* we have to look at the demand:

$$0 = \frac{\partial \mathcal{L}}{\partial d} = \frac{\partial f}{\partial d} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial d} - \sum_j \mu_j^* \frac{\partial h_j}{\partial d} = V - \lambda^* - \mu^*$$

Since $d^* < D$, $\mu^* = 0$, $\lambda^* = V$ and thus $\bar{\mu}^* = V - o$, the marginal benefit of extending the generator The **demand sets the price**.

Next simplest example: several generators, fixed demand

Suppose we have several generators with dispatch g_s and strictly ordered operating costs o_s such that $o_s < o_{s+1}$. We now maximise

$$\max_{\{d,g_s\}} \left[Vd - \sum_s o_s g_s \right]$$

such that

$$\begin{aligned}
d - \sum_s g_s = 0 \qquad &\leftrightarrow \qquad \lambda \\
d \leq D \qquad &\leftrightarrow \qquad \mu \\
g_s \leq G_s \qquad &\leftrightarrow \qquad \bar{\mu}_s \\
-g_s \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_s
\end{aligned}$$

Next simplest example: several generators, fixed demand

Stationarity gives us for each s:

$$0 = -o_s + \lambda^* - \bar{\mu}_s^* + \underline{\mu}_s^*$$

and from complementarity we get

$$\bar{\mu}_s(g_s^* - G_s) = 0$$
$$\underline{\mu}_s g_s^* = 0$$

We can see by inspection that we will dispatch the cheapest generation first. Suppose that we have enough generation for the demand, i.e. $D < \sum_s G_s$. [If $D > \sum_s G_s$ we have the same situation as for a single generator, i.e. $\lambda^* = V$, so that the demand sets the price.]

Find m such that $\sum_{s=1}^{m-1} G_s < D < \sum_{s=1}^{m} G_s$.

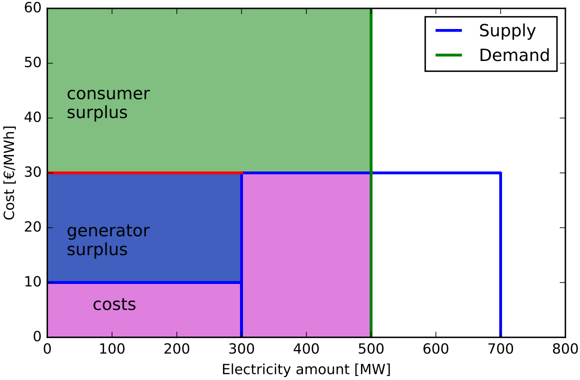
For $s \leq m-1$ we have $g_s^* = G_s$, $\underline{\mu}_s^* = 0$, $\bar{\mu}_s^* = \lambda^* - o_s$.

For $s = m$ we have $g_m^* = D - \sum_{s=1}^{m-1} G_s$ to cover what's left of the demand. Since $0 < g_m^* < G_m$ we have $\underline{\mu}_m^* = \bar{\mu}_m^* = 0$ and thus $\lambda^* = o_m$.

Next simplest example: several generators, fixed demand

Specific example of two generators with $G_1 = 300$ MW, $G_2 = 400$ MW, $o_1 = 10$ €/MWh, $o_2 = 30$ €/MWh and $D = 500$ MW.

In this case $m = 2$, $g_1^* = G_1 = 300$ MW, $g_2^* = d - G_1 = 200$ MW, $\lambda^* = o_2$, $\underline{\mu}_i = 0$, $\bar{\mu}_2 = 0$ and $\bar{\mu}_1 = o_2 - o_1$.



From welfare maximisation to cost minimisation

For the case $D > \sum_s G_s$ we can instead imagine that the demand is rigidly fixed to D and that instead we have a dummy generator with dispatch $g_d = D - \sum_s G_s$ that represents **load shedding**.

In this case we can substitute $d = D - g_d$ to get

$$\max_{\{g_d, g_s\}} \left[VD - Vg_d - \sum_s o_s g_s \right]$$

such that

$$\begin{aligned} D - g_d - \sum_s g_s = 0 \qquad &\leftrightarrow \qquad \lambda \\ g_s \leq G_s \qquad &\leftrightarrow \qquad \bar{\mu}_s \\ -g_s \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_s \end{aligned}$$

Since VD is a constant, we can use $\max_{x \in X} [-f(x)] = -\min_{x \in X} f(x)$ to recast this as a minimisation of the total generator costs, absorbing g_d into the set $\{g_s\}$. The constant VD is dropped.

From welfare maximisation to cost minimisation

We have turned the maximisation of total welfare into **cost minimisation**:

$$\min_{\{g_s\}} \sum_s o_s g_s$$

such that:

$$\begin{aligned}
\sum_s g_s - d = 0 \qquad &\leftrightarrow \qquad \lambda \\
g_s \leq G_s \qquad &\leftrightarrow \qquad \bar{\mu}_s \\
-g_s \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_s
\end{aligned}$$

The most expensive generator has $o_s = V$ and $G_s = \infty$ and represents **load shedding**.

We've replaced the symbol D with d for simplicity going forward.

NB: Because the signs of the KKT multipliers change when we go from maximisation to minimisation, we've also changed the sign of the balance constraint to keep the marginal price λ positive.

Optimise nodes in a network

Welfare optimisation for several nodes in a network

Now let's suppose we have several nodes i with different loads and different generators, with flows f_ℓ in the network lines.

Now we have additional optimisation variables f_ℓ AND additional constraints for welfare maximisation:

$$\max_{\{d_{i,b}\},\{g_{i,s}\},\{f_\ell\}} \left[\sum_{i,b} U_{i,b}(d_{i,b}) - \sum_{i,s} C_{i,s}(g_{i,s})\right]$$

such that demand is met either by generation or by the network at each node i

$$\sum_b d_{i,b} - \sum_s g_{i,s} + \sum_\ell K_{i\ell} f_\ell = 0 \qquad \leftrightarrow \qquad \lambda_i$$

and generator constraints are satisified

$$g_{i,s} \leq G_{i,s} \qquad \leftrightarrow \qquad \bar{\mu}_{i,s}$$

$$-g_{i,s} \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{i,s}$$

Cost minimisation at several nodes in a network

For cost minimisation we have a fixed load d_i at each node, and absorb load-shedding above a value V into a dummy generator.

Now we minimise over $g_{i,s}$ and f_ℓ:

$$\min_{\{g_{i,s}\},\{f_\ell\}} \sum_{i,s} o_{i,s} g_{i,s}$$

such that demand is met either by generation or by the network at each node i

$$\sum_s g_{i,s} - d_i = \sum_\ell K_{i\ell} f_\ell \qquad \leftrightarrow \qquad \lambda_i$$

and generator constraints are satisified

$$g_{i,s} \leq G_{i,s} \qquad \leftrightarrow \qquad \bar{\mu}_{i,s}$$
$$-g_{i,s} \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{i,s}$$

Several generators at different nodes in a network

In addition we have constraints on the line flows.

First, they have to satisfy Kirchoff's Voltage Law around each closed cycle c:

$$\sum_{c} C_{\ell c} x_{\ell} f_{\ell} = 0 \qquad \leftrightarrow \qquad \lambda_c$$

and in addition the flows cannot overload the thermal limits, $|f_\ell| \leq F_\ell$

$$\begin{aligned} f_\ell \leq F_\ell & \qquad \leftrightarrow \qquad \bar{\mu}_\ell \\ -f_\ell \leq F_\ell & \qquad \leftrightarrow \qquad \underline{\mu}_\ell \end{aligned}$$

Simplest example: two nodes connected by a single line

At node 1 we have demand of $d_1 = 100$ MW and a generator with costs $o_1 = 10$ €/MWh and a capacity of $G_1 = 300$ MW.

At node 2 we have demand of $d_2 = 100$ MW and a generator with costs $o_1 = 20$ €/MWh and a capacity of $G_2 = 300$ MW.

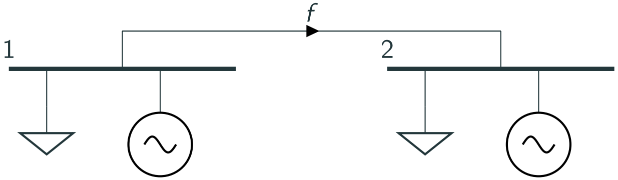
What happens if the capacity of the line connecting them is $F = 0$?

What about $F = 50$ MW?

What about $F = \infty$?

See example on board.

Simplest example: two nodes connected by a single line



d_1 $\quad$ g_1 $\qquad\qquad$ d_2 $\quad$ g_2

$g_1 - d_1 = f \quad \leftrightarrow \quad \lambda_1 \qquad\qquad g_2 - d_1 = -f \quad \leftrightarrow \quad \lambda_2$

Objective function is $\min_{g_1,g_2,f} [o_1 g_1 + o_2 g_2]$ subject to:

$$\begin{aligned}
g_1 \leq G_1 &\quad \leftrightarrow \quad \bar{\mu}_1 \\
-g_1 \leq 0 &\quad \leftrightarrow \quad \underline{\mu}_1 \\
g_2 \leq G_2 &\quad \leftrightarrow \quad \bar{\mu}_2 \\
-g_2 \leq 0 &\quad \leftrightarrow \quad \underline{\mu}_2 \\
f \leq F &\quad \leftrightarrow \quad \bar{\mu} \\
-f \leq F &\quad \leftrightarrow \quad \underline{\mu}
\end{aligned}$$

Two nodes: Case $F = 0$

For the case $F = 0$ the nodes are like two separated islands, $f^* = 0$.

The generator on each island provides the demand separately, so:

$$g_1^* = d_1 \qquad \text{and} \qquad g_2^* = d_2$$

Neither generator has any binding constraints, since in each case the demand (100 MW) is less than the generator capacity (300 MW), so

$$\bar{\mu}_1^* = \underline{\mu}_1^* = \bar{\mu}_2^* = \underline{\mu}_2^* = 0$$

From stationarity for each site we get

$$0 = \frac{\partial \mathcal{L}}{\partial g_i} = o_i - \lambda_i^* - \bar{\mu}_i^* + \underline{\mu}_i^*$$

Thus we have at each site $\lambda_i^* = o_i$.

Two nodes: Case $F = 50$ MW

For the case F =50 MW the cheaper node 1 will export to the more expensive node 2 as much as the restricted capacity F allows:

$$f^* = F = 50 \text{ MW}$$

Generator 1 covers 50 MW of the demand from node 2:

$$g_1^* = d_1 + f^* \qquad \text{and} \qquad g_2^* = d_2 - f^*$$

Neither generator has any binding constraints, so

$$\bar{\mu}_1^* = \underline{\mu}_1^* = \bar{\mu}_2^* = \underline{\mu}_2^* = 0$$

and thus we have again different prices at each $\lambda_i = o_i$. For the flow:

$$0 = \frac{\partial \mathcal{L}}{\partial f} = 0 + \lambda_1^* - \lambda_2^* - \bar{\mu}^* + \underline{\mu}^*$$

Only the upper limit is binding, so we get $\underline{\mu}^* = 0$ and

$$\bar{\mu}^* = \lambda_1^* - \lambda_2^* = -10 \text{EUR/MWh}$$

Two nodes: Case $F = \infty$

For the case $F = \infty$ we have unrestricted capacity, so it is like merging the two nodes to one node. Now all the demand is covered by the cheapest node:

$$f^* = d_2 = 100 \text{ MW}$$

Generator 1 covers all the demand:

$$g_1^* = d_1 + d_2 \qquad \text{and} \qquad g_2^* = 0$$

Only generator 2 has a non-zero KKT multiplier, so at node 1 we have $\lambda_1^* = o_1$ and at node 2 we have:

$$\underline{\mu}_2^* = \lambda_2^* - o_2$$

From KKT for the flow f we have no constraints so

$$0 = \frac{\partial \mathcal{L}}{\partial f} = 0 + \lambda_1^* - \lambda_2^* - \bar{\mu}^* + \underline{\mu}^*$$

i.e. $\lambda_1^* = \lambda_2^*$. We have price equalisation, as if it were a single node.

Congestion rent

Due to the congestion of the transmission line, the marginal cost of producing electricity can be different at node 1 and node 2. The competitive price at node 2 is higher than at node 1 – this corresponds to **locational marginal pricing**, or **nodal pricing**.

Since consumers pay and generators get paid the price in their local market, in case of congestion there is a difference between the total payment of consumers and the total revenue of producers – this is the **merchandising surplus** or **congestion rent**, collected by the market operator. For each line it is given by the price difference in both regions times the amount of power flow between them:

$$\text{Congestion rent} = \Delta\lambda \times f$$

Congestion rent: Two node example

Returning to our two node example:

Case	Demand pays [€/h]	Generator gets [€/h]	$\lambda_2^* - \lambda_1^*$ [€/MWh]	flow f [MW]	Cong. rent [€/h]
$F = 0$	3000	3000	0	0	0
$F = 50$	3000	2500	10	50	500
$F = \infty$	2000	2000	0	100	0

To get a congestion rent, we need congestion to cause a price difference between the nodes, as well as a non-zero flow between the nodes.

Congestion rent

In this example we saw that the sum of what consumers pay does not always equal the sum of generator revenue.

In fact if we take the balance constraint and sum it weighted by the market price at each node we find

$$\sum_i \lambda_i^* d_i - \sum_i \lambda_i^* \sum_s g_{i,s}^* = -\sum_i \lambda_i^* \sum_\ell K_{i\ell} f_\ell^*$$

The quantity for each ℓ

$$-f_\ell^* \sum_i K_{i\ell} \lambda_i^* = f_\ell (\lambda_{\text{end}}^* - \lambda_{\text{start}}^*)$$

is called the **congestion rent** and is the money the network operator receives for transferring power from a low price node (start) to a high price node (end), 'buy it low, sell it high'.

It is zero if: a) the flow is zero or b) the price difference is zero.

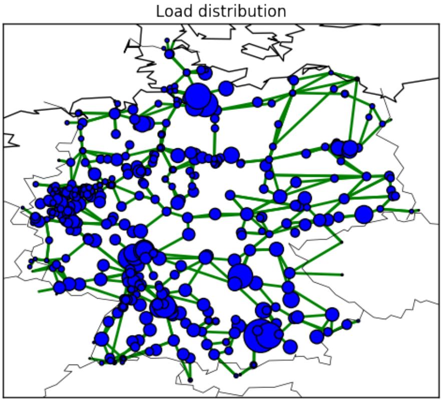
The European Market

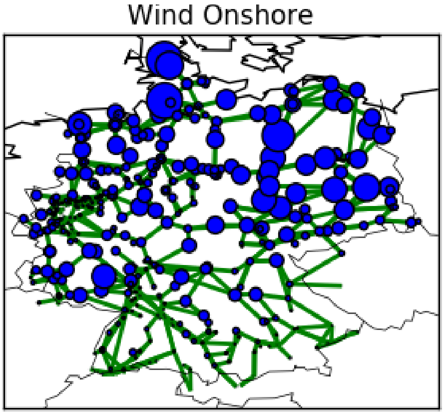
Existing bidding zones

- Bids for German electricity take place in a **giant bidding zone** encompassing both Austria and Germany (Austria will be separated from October 2018)
- This means that transmission constraints are only visible to the market at the **borders** to the other national zones
- Internal transmission constraints are **ignored** - market bids are handled as if they do not exist
- Only KCL enforced - KVL impossible

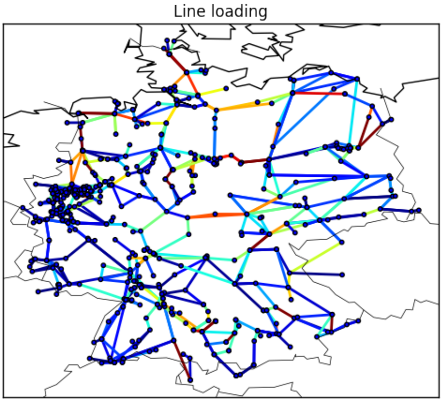
The Problem

Renewables are not always located near demand centres, as in this example from Germany.





The Problem



- This leads to **overloaded lines** in the middle of Germany, which cannot transport all the wind energy from North Germany to the load in South Germany
- It also overloads lines in neighbouring countries due to **loop flows** (unplanned physical flows 'according to least resistance' which do not correspond to traded flows)
- It also **blocks imports and exports** with neighbouring countries, e.g. Denmark

Solution 1: Redispatch after energy market clearing

These problems are **not visible** in the day-ahead electricity market, which treats the whole of Germany and Austria as a single bidding zone. It dispatches wind in North Germany as if there was no internal congestion...

To ensure that the physical limits of transmission are not exceeded, the network operator must **'re-dispatch'** power stations and **curtail** (Einspeisemanagement) renewables to restore order. This is **costly** (0.8 redispatch + 0.6 RE-compensation = 1.4 billion EUR in 2017 - although exceptional circumstances in 1st quarter) and results in **lost CO_2-free generation** (5.5 TWh curtailment of RE and CHP in 2017).

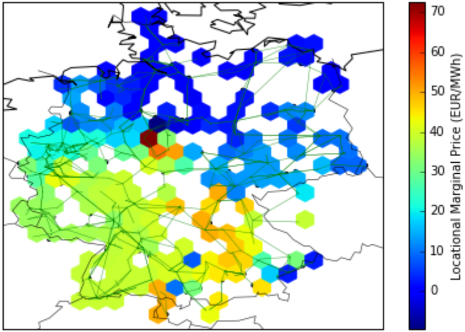
International redispatch is sometimes also required (Multilateral Remedial Actions = MRA).

Furthermore, there are **no market incentives** to reinforce the North-South grid, to locate more power stations in South Germany or to build storage / P2X in North Germany.

Solution 2: Smaller bidding zones to "see" congested boundaries

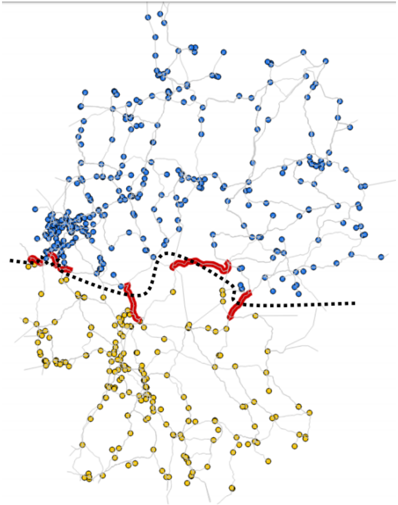
- In Scandinavia they have solved this by introducing **smaller bidding zones**
- Now congestion at the boundaries between zones is taken into account in the **implicit auctions** of the market
- This is also done in Italy (again, a long country), where prices for small consumers are **uniformised** for fairness

Solution 3: Nodal pricing



- The ultimate solution, as used in the US and other markets, is **nodal pricing**, which exposes all transmission congestion
- Considered too complex and subject to market power to be used in Europe, but this is questionable...
- Here we see clearly why many argue for a North-South German split

First step: Split Germany North-South

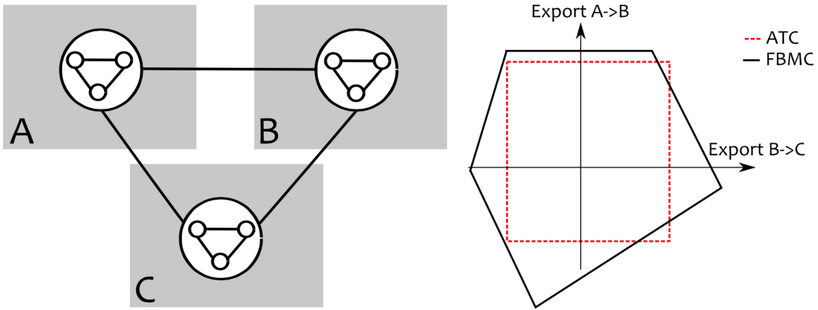


- Initial price difference could average up to 12 EUR/MWh
- Prices would converge with more network expansion
- Redispatch costs reduced by 39% in 2025, 58% in 2035 (assuming NEP 2030 transmission projects get built)
- Politically difficult, may require, like Italy, uniformised price on consumer side

Source: Fraunholz & Hladik, 2018

Solution 1.5: Flow-based market coupling

Flow-based market coupling can be used in zonal markets to see precise individual line constraints, instead of “boxing” the feasible space like ATC/NTC schemes do.



(a) Zonal market model. (b) Flow domain.

Figure 4. In the FMBC method, only one equivalent node per zone is considered, but all (critical) lines are taken into account. In this simple grid, the zonal network consists of 3 nodes and 12 lines. The FBMC flow domain is larger than the ATC flow domain as the physical characteristics of the grid are better represented in the FBMC method.

Source: Van den Bergh, Boury, Delarue

Storage Optimisation

Storage equations

Now, like the network case where we add different nodes i with different loads, for storage we have to consider different time periods t.

Label conventional generators by s, storage by r and now minimise

$$\min_{\{g_{i,s,t}\},\{g_{i,r,t,\text{store}}\},\{g_{i,r,t,\text{dispatch}}\},\{f_{\ell,t}\}} \left[\sum_{i,s,t} o_{i,s} g_{i,s,t} + \sum_{i,r,t} o_{i,r,\text{store}}\, g_{i,r,t,\text{store}} + \sum_{i,r,t} o_{i,r,\text{dispatch}}\, g_{i,r,t,\text{dispatch}}\right]$$

The power balance constraints are now (cf. Lecture 4) for each node i and time t that the demand is met either by generation, storage or network flows:

$$\sum_s g_{i,s,t} + \sum_r (g_{i,r,t,\text{dispatch}} - g_{i,r,t,\text{store}}) - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t} \qquad \leftrightarrow \qquad \lambda_{i,t}$$

Storage equations

We have constraints on normal generators

$$0 \leq g_{i,s,t} \leq G_{i,s}$$

and on the storage

$$0 \leq g_{i,r,t,\mathrm{dispatch}} \leq G_{i,r,\mathrm{dispatch}}$$
$$0 \leq g_{i,r,t,\mathrm{store}} \leq G_{i,r,\mathrm{store}}$$

The energy level of the storage is given by

$$e_{i,r,t} = \eta_0 e_{i,r,t-1} + \eta_1 g_{i,r,t,\mathrm{store}} - \eta_2^{-1} g_{i,r,t,\mathrm{dispatch}}$$

and limited by

$$0 \leq e_{i,r,t} \leq E_{i,r}$$

Idea of storage

Storage does 'buy it low, sell it high' arbitrage, like network, but in time rather than space, i.e. between cheap times (e.g. with lots of zero-marginal-cost renewables) and expensive times (e.g. with high demand, low renewables and expensive conventional generators).

Storage plus network equations

Finally for the flows we repeat the constraints for each time t.

We have KVL for each cycle c and time t

$$\sum_c C_{\ell c} x_\ell f_{\ell,t} = 0 \qquad \leftrightarrow \qquad \lambda_{c,t}$$

and in addition the flows cannot overload the thermal limits, $|f_{\ell,t}| \leq F_\ell$

$$\begin{aligned} f_{\ell,t} \leq F_\ell &\qquad \leftrightarrow \qquad \bar{\mu}_{\ell,t} \\ -f_{\ell,t} \leq -F_\ell &\qquad \leftrightarrow \qquad \underline{\mu}_{\ell,t} \end{aligned}$$

Preview: Investment optimisation

Preview for next time:

Next time we will also optimise **investment** in the **capacities** of generators, storage and network lines, to maximise **long-run efficiency**.

We will promote the capacities $G_{i,s}$, $G_{i,r,*}$, $E_{i,r}$ and F_ℓ to optimisation variables.