Energy System Modelling
Summer Semester 2018, Lecture 8

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Optimisation Energy System Operation: Network

Generators at a single node

Last time we saw that if the demand is inelastic and fixed, welfare maximisation is equivalent to a generation cost minimisation problem:

$$\min_{\{g_s\}} \sum_s o_s g_s$$

such that:

$$\begin{aligned} \sum_s g_s - d = 0 \qquad &\leftrightarrow \qquad \lambda \\ g_s \leq G_s \qquad &\leftrightarrow \qquad \bar{\mu}_s \\ -g_s \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_s \end{aligned}$$

Several generators at different nodes in a network

Now let's suppose we have several nodes i with different loads and different generators, with flows f_ℓ in the network lines (we use the linear power flow approximation).

Now we have additional optimisation variables f_ℓ AND additional constraints:

$$\min_{\{g_{i,s}\},\{f_\ell\}} \sum_{i,s} o_{i,s} g_{i,s}$$

such that demand is met either by generation or by the network at each node i

$$\sum_s g_{i,s} - d_i = \sum_\ell K_{i\ell} f_\ell \qquad \leftrightarrow \qquad \lambda_i$$

and generator constraints are satisified

$$\begin{aligned} g_{i,s} &\leq G_{i,s} \qquad &\leftrightarrow \qquad &\bar{\mu}_{i,s} \\ -g_{i,s} &\leq 0 \qquad &\leftrightarrow \qquad &\underline{\mu}_{i,s} \end{aligned}$$

Several generators at different nodes in a network

In addition we have constraints on the line flows.

First, they have to satisfy Kirchoff's Voltage Law (KVL) for each cycle c

$$\sum_c C_{\ell c} x_\ell f_\ell = 0 \qquad \leftrightarrow \qquad \lambda_c$$

In addition the flows cannot overload the thermal limits, $|f_\ell| \leq F_\ell$

$$\begin{aligned} f_\ell &\leq F_\ell & \leftrightarrow \qquad & \bar{\mu}_\ell \\ -f_\ell &\leq F_\ell & \leftrightarrow \qquad & \underline{\mu}_\ell \end{aligned}$$

Simplest example: two nodes connected by a single line

At node 1 we have demand of $d_1 = 100$ MW and a generator with costs $o_1 = 10$ €/MWh and a capacity of $G_1 = 300$ MW.

At node 2 we have demand of $d_2 = 100$ MW and a generator with costs $o_1 = 20$ €/MWh and a capacity of $G_2 = 300$ MW.

What happens if the capacity of the line connecting them is $F_\ell = 0$?

What about $F_\ell = 50$ MW?

What about $F_\ell = \infty$?

Congestion rent

In this example we saw that the sum of what consumers pay does not always equal the sum of generator revenue.

In fact if we take the balance constraint and sum it weighted by the market price at each node we find

$$\sum_i \lambda_i^* d_i - \sum_i \lambda_i^* \sum_s g_{i,s}^* = -\sum_i \lambda_i^* \sum_\ell K_{i\ell} f_\ell^*$$

The quantity for each ℓ

$$-f_\ell^* \sum_i K_{i\ell} \lambda_i^* = f_\ell(\lambda_{\mathrm{end}}^* - \lambda_{\mathrm{start}}^*)$$

is called the **congestion rent** and is the money the network operator receives for transferring power from a low price node (start) to a high price node (end), 'buy it low, sell it high'.

It is zero if: a) the flow is zero or b) the price difference is zero.

Storage Optimisation

Storage equations

Now, like the network case where we add different nodes i with different loads, for storage we have to consider different time periods t.

Label conventional generators by s, storage by r and now minimise

$$\min_{\{g_{i,s,t}\},\{g_{i,r,t,\text{store}}\},\{g_{i,r,t,\text{dispatch}}\},\{f_{\ell,t}\}} \left[\sum_{i,s,t} o_{i,s} g_{i,s,t} + \sum_{i,r,t} o_{i,r,\text{store}}\, g_{i,r,t,\text{store}} + \sum_{i,r,t} o_{i,r,\text{dispatch}}\, g_{i,r,t,\text{dispatch}}\right]$$

The power balance constraints are now (cf. Lecture 4) for each node i and time t that the demand is met either by generation, storage or network flows:

$$\sum_s g_{i,s,t} + \sum_r (g_{i,r,t,\text{dispatch}} - g_{i,r,t,\text{store}}) - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t} \qquad \leftrightarrow \qquad \lambda_{i,t}$$

Storage equations

We have constraints on normal generators

$$0 \leq g_{i,s,t} \leq G_{i,s}$$

and on the storage

$$0 \leq g_{i,r,t,\text{dispatch}} \leq G_{i,r,\text{dispatch}}$$
$$0 \leq g_{i,r,t,\text{store}} \leq G_{i,r,\text{store}}$$

The energy level of the storage is given by

$$e_{i,r,t} = \eta_0 e_{i,r,t-1} + \eta_1 g_{i,r,t,\text{store}} - \eta_2^{-1} g_{i,r,t,\text{dispatch}}$$

and limited by

$$0 \leq e_{i,r,t} \leq E_{i,r}$$

Storage equations

Finally for the flows we repeat the constraints for each time t.

We have KVL for the flows, therefore for each cycle c and time t

$$\sum_{c} C_{\ell c} x_\ell f_{\ell,t} = 0 \qquad \leftrightarrow \qquad \lambda_{c,t}$$

and in addition the flows cannot overload the thermal limits, $|f_{\ell,t}| \leq F_\ell$

$$\begin{aligned} f_{\ell,t} &\leq F_\ell \qquad &\leftrightarrow \qquad &\bar{\mu}_{\ell,t} \\ -f_{\ell,t} &\leq F_\ell \qquad &\leftrightarrow \qquad &\underline{\mu}_{\ell,t} \end{aligned}$$

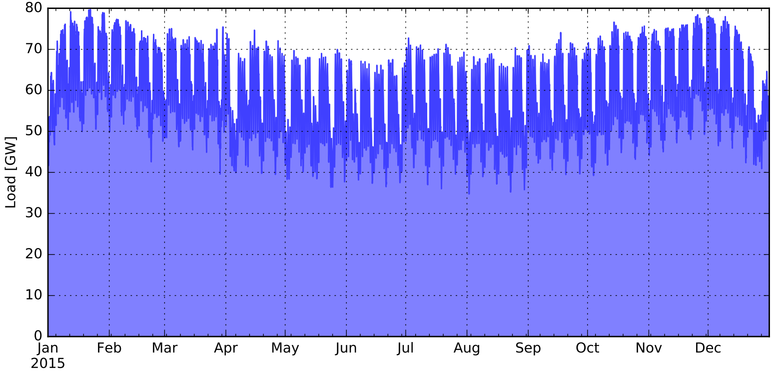
Idea of storage

Storage does ‘buy it low, sell it high’ arbitrage, like network, but in time rather than space, i.e. between cheap times (e.g. with lots of zero-marginal-cost renewables) and expensive times (e.g. with high demand, low renewables and expensive conventional generators).

Duration Curves and Capacity Factors: Examples from Germany in 2015

Load curve

Here's the electrical demand (load) in Germany in 2015:

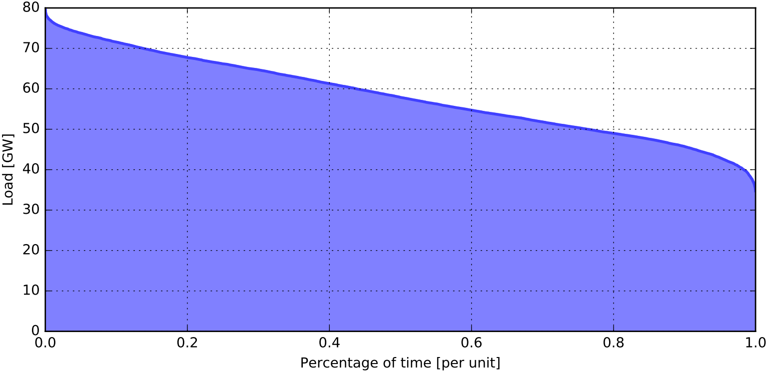


To understand this curve better and its implications for the market, it's useful to stack the hours of the year from left to right in order of the amount of load.

Load duration curve

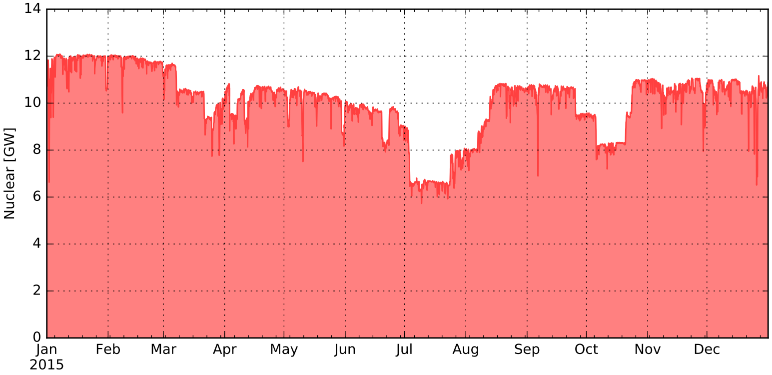
This re-ordering is called a **duration curve**.

For the load it's the **load duration curve**.



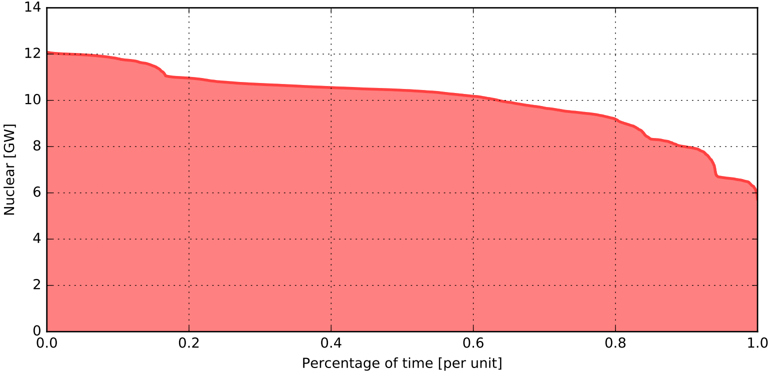
Nuclear curve

Can do the same for nuclear output:



Nuclear duration curve

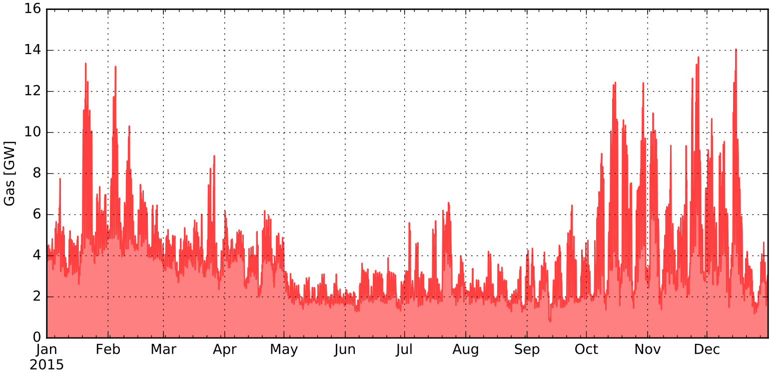
Duration curve is pretty flat, because it is economic to run nuclear almost all the time as **baseload plant**:



The equivalent fraction of time that the plants run at full capacity over the year is the **capacity factor** - nuclear has a high capacity factor, usually around 70-90%.

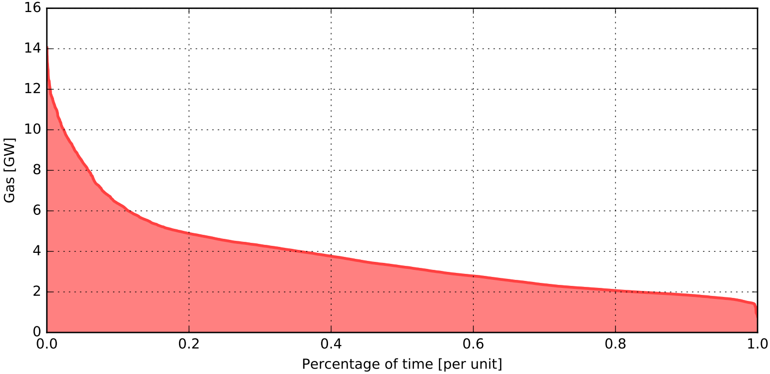
Gas curve

Can do the same for gas output:



Gas duration curve

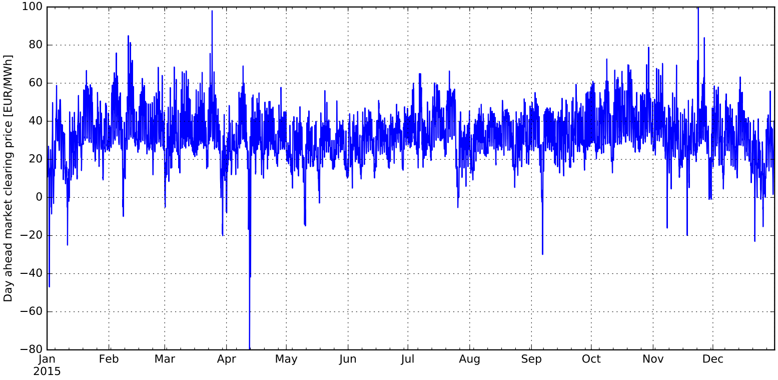
Duration curve is partially flat (for heat-driven CHP) and partially peaked (for **peaking plant**):



The capacity factor for gas is much lower - more like 20%.

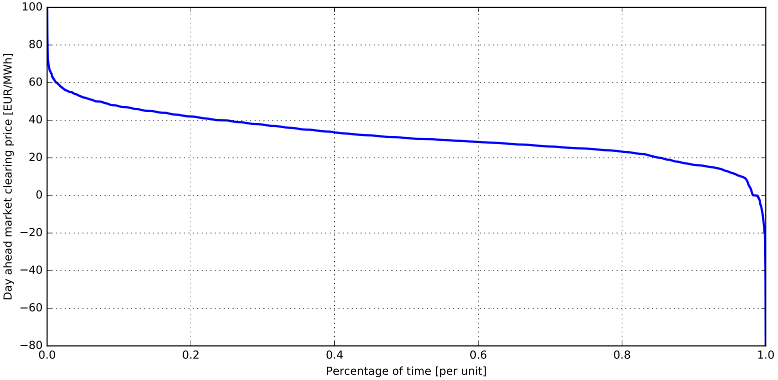
Price curve

Can do the same for price during the year:



Price duration curve

Price duration curve:



Question

Now we are in a position to consider the questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find the price and load duration curves very useful.

Investment Optimisation: Generation

Investment optimisation

Now we also optimise **investment** in the **capacities** of generators, storage and network lines, to maximise **long-run efficiency**.

We will promote the capacities $G_{i,s}$, $G_{i,r,*}$, $E_{i,r}$ and F_ℓ to optimisation variables.

For generation investment, we want to answer the following questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find price and load duration curves very useful.

Definition of long-run efficiency

Up until now we have considered **short-run** equilibria that ensure **short-run** efficiency (static), i.e. they make the best use of presently available productive resources.

Long-run efficiency (dynamic) requires in addition the optimal investment in productive capacity.

Concretely: given a set of options, costs and constraints for different generators (nuclear/gas/wind/solar) what is the optimal generation portfolio for maximising long-run welfare?

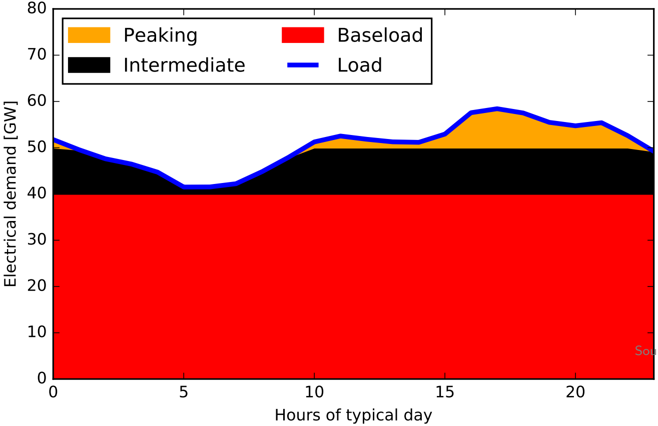
From an indivdual generators' perspective: how best should I invest in extra capacity?

We will show again that with perfect competition and no barriers to entry, the system-optimal situation can be reached by individuals following their own profit.

Baseload versus Peaking Plant

Load (= Electrical Demand) is low during night; in Northern Europe in the winter, the peak is in the evening.

To meet this load profile, cheap **baseload** generation runs the whole time; more expensive **peaking plant** covers the difference.



Source: Tom Brown

Different types of generators

Fuel/Prime mover	Marginal cost	Capital cost	Controllable	Predictable days ahead	CO2
Oil	V. High	Low	Yes	Yes	Medium
Gas OCGT	High	Low	Yes	Yes	Medium
Gas CCGT	Medium	Medium	Yes	Yes	Medium
Hard Coal	Medium	Lowish	Yes	Yes	High
Brown Coal	Low	Medium	Yes	Yes	High
Nuclear	V. Low	High	Partly	Yes	Zero
Hydro dam	Zero	High	Yes	Yes	Zero
Wind/Solar	Zero	High	Down	No	Zero

System-optimal generator capacities and dispatch

Suppose we have generators labelled by s at a single node with **marginal costs** o_s arising from each unit of production $g_{s,t}$ and **capital costs** c_s that arise from fixed costs regardless of the rate of production (such as the investment in building capacity G_s). For a variety of demand values d_t in representative situation t we optimise the total system costs

$$\min_{\{g_{s,t}\},\{G_s\}} \left[\sum_s c_s G_s + \sum_{s,t} o_s g_{s,t} \right]$$

such that

$$\begin{aligned} \sum_s g_{s,t} = d_t \qquad & \leftrightarrow \qquad \lambda_t \\ -g_{s,t} \leq 0 \qquad & \leftrightarrow \qquad \underline{\mu}_{s,t} \\ g_{s,t} - G_s \leq 0 \qquad & \leftrightarrow \qquad \bar{\mu}_{s,t} \end{aligned}$$

We will also allow load-shedding with a ‘dummy’ generator $s = S$, $o_S = V$ (Value of Lost Load), $c_S = 0$ (the capacity to shed load doesn’t cost anything, so can be as big as d_t if necessary).

System-optimal generator capacities and dispatch

Stationarity gives us for each s and t:

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = o_s - \lambda_t^* - \bar{\mu}_{s,t}^* + \underline{\mu}_{s,t}^*$$

and for each s:

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = c_s + \sum_t \bar{\mu}_{s,t}^*$$

and from complementarity we get

$$\begin{aligned} \bar{\mu}_{s,t}^*(g_{s,t}^* - G_s^*) &= 0 \\ \underline{\mu}_{s,t}^* g_{s,t}^* &= 0 \end{aligned}$$

and dual feasibility (for minimisation) $\bar{\mu}_{s,t}^*, \underline{\mu}_{s,t}^* \leq 0$.

System-optimal generator capacities and dispatch

The solution for the dispatch $g^*_{s,t}$ is exactly the same as without capacity optimisation. For each t, find m such that $\sum_{s=1}^{m-1} G_s < d_t < \sum_{s=1}^{m} G_s$.

For $s < m$ we have $g^*_{s,t} = G^*_s$, $\underline{\mu}^*_{s,t} = 0$, $\bar{\mu}^*_{s,t} = o_s - \lambda^*_t \leq 0$.

For $s = m$ we have $g^*_{m,t} = d_t - \sum_{s=1}^{m-1} G^*_s$ to cover what's left of the demand. Since $0 < g^*_{m,t} < G_m$ we have $\underline{\mu}^*_{m,t} = \bar{\mu}^*_{m,t} = 0$ and therefore $\lambda^*_t = o_m$.

For $s > m$ we have $g^*_{s,t} = 0$, $\underline{\mu}^*_{s,t} = \lambda^*_t - o_s \leq 0$, $\bar{\mu}^*_{s,t} = 0$.

What about the G^*_s?

System-optimal generator capacities and dispatch

The G_s^* are determined implicitly based on the interactions between costs and prices.

From stationarity we had the relation

$$c_s = -\sum_t \bar{\mu}_{s,t}^*$$

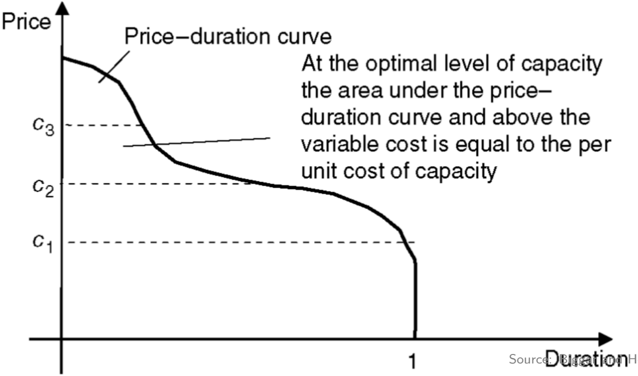
The $\bar{\mu}_{s,t}^*$ were only non-zero with $\lambda_t^* > o_s$ so we can re-write this as

$$c_s = \sum_{t|\lambda_t^* > o_s} \left(\lambda_t^* - o_s\right)$$

'Increase capacity until marginal increase in profit equals the cost of extra capacity.'

Multiple price duration

The optimal mix of generation is where, for each generation type, the area under the price–duration curve and above the variable cost of that generation type is equal to the fixed cost of adding capacity of that generation type. (In the graphic c_s is o_s in our notation.)



Source: Biggar and Hesamzadeh, 2014

Multiple generators with inelastic demand

Assume again we have $o_1 \leq o_2 \leq \cdots \leq o_S = V$ and $K_p = \sum_{s=1}^{p} G_s$ then:

$$\lambda_t = \begin{cases} V & \text{for } d_t > K_{S-1} \\ o_s & \text{if } K_{s-1} < d_t \leq K_s, \qquad\qquad \text{for } s = 1, \dots S-1 \end{cases}$$

Looking at the area under the price duration curve but above the variable cost, we then find:

$$c_s = (V - o_s)P(d > K_{S-1}) + \sum_{j=s+1}^{S-1} (o_j - o_s)P(K_{j-1} < d \leq K_j)$$

Screening curve

These equations can be rewritten recursively using the substitution $\theta_s = P(d > K_s)$:

$$c_{S-1} + \theta_{S-1} o_{S-1} = V \theta_{S-1}$$

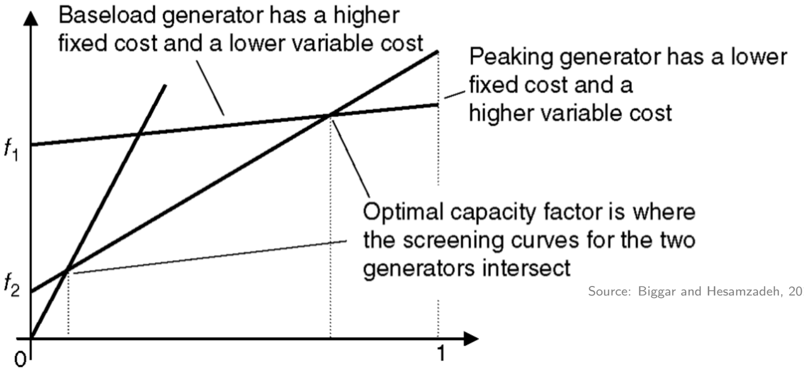
$$c_s + \theta_s o_s = c_{s+1} + \theta_s o_{s+1} \qquad \forall s = 1, \ldots S-2$$

The first equation can be solved to find θ_{S-1}, then the other equations can be solved recursively to find the remaining θ_s. The θ_s correspond to the optimal **capacity factors** of each type of generator, which correspond to the fraction of time the generator runs at full power.

Screening curve

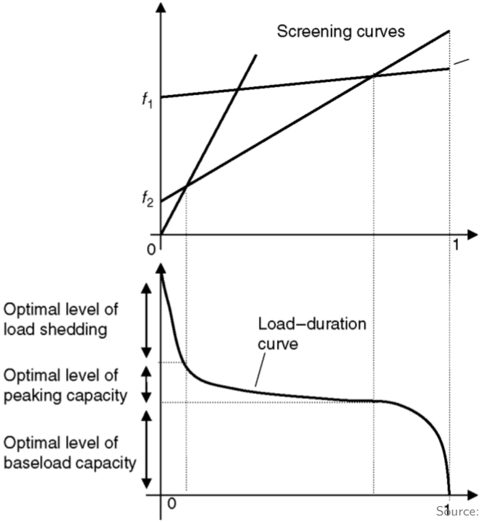
The costs as a function of the capacity factors can be drawn together as a **screening curve** (more expensive options are *screened* from the optimal inner polygon).

The intersection points determine the optimal capacity factors and hence, using the load duration curve, the optimal capacities of each generator type. (In the graphic f_s is c_s in our notation.)



Source: Biggar and Hesamzadeh, 2014

Screening curve versus Load duration



Source: Biggar and Hesamzadeh, 2014

Example: 2 generation technologies and load shedding

Suppose that electrical demand is inelastic with a demand-duration curve given by $d(x) = 1000 - 1000x$ for $0 \leq x \leq 1$. Suppose that there are two different types of generation with variable costs of 2 and 12 €/MWh respectively, together with load-shedding at a cost of 1012 €/MWh. The fixed costs of the two generation types are 15 and 10 €/MWh respectively. See the below table for a summary of the costs.

Generator	o_s [€/MWh]	c_s [€/MW/h]
A	2	15
B	12	10
LS	1012	0

Example: 2 generation technologies and load shedding

1. What is the interpretation of the demand-duration curve?
2. Below which capacity factor x_1 is it cheaper to run Generator B rather than to run Generator A?
3. Below which capacity factor x_0 is it cheaper to shed load than to run Generator B?
4. Plot the costs as a function of x and mark these intersection points on a screening curve.
5. Find the optimal capacities of Generators A and B in this market.

Example: 2 generation technologies and load shedding

For the solution see the flipchart photos at
`https://nworbmot.org/courses/esm-2018/board/`.

To find x_1, solve for the intersection of Generator A's cost curve with Generator B's cost curve as a function of capacity factor:

$$c_A + x_1 o_A = c_B + x_1 o_B$$

This gives $x_1 = 0.5$. At this point the demand is $d(0.5) = 500$ MW therefore

$$G_A = 500 \text{ MW}$$

To find x_0, solver for where Generator B crosses the load-shedding line:

$$c_B + x_0 o_B = c_{LS} + x_0 o_{LS}$$

This gives $x_0 = 0.01$. At this point the demand is $d(0.5) = 990$ MW so:

$$G_A + G_B = 990 \text{ MW}$$

i.e. $G_B = 490$ MW and $G_{LS} = 10$ MW.

Investment Optimisation: Transmission

Investment optimisation: transmission

As before, our approach to the question of **"What is the optimal amount of transmission"** is determined by the most efficient long-term solution, i.e. the infrastructure investement that maximising social welfare over the long-run.

Promote F_ℓ to an optimisation variable with capital cost c_ℓ.

In brief: Exactly as with generation dispatch and investment, we continue to invest in transmission until the marginal benefit of extra transmission (i.e. extra congestion rent for extra capacity) is equal to the marginal cost of extra transmission. This determines the optimal investment level.