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Revising Ohm’s Law
**Ohm’s Law**: The potential difference (voltage) $V_1 - V_2$ across an ideal conductor is proportional to the current through it $I$. The constant of proportionality is called the **resistance**, $R$. Ohm's Law is thus:

$$V_1 - V_2 = I \times R$$
Analogy DC circuits to linear power flow

The equations for DC circuits and linear power flow in AC circuits are analogous:

\[ I = \frac{V_i - V_j}{R} \quad \leftrightarrow \quad f_\ell = \frac{\theta_i - \theta_j}{x_\ell} \]

if we make the following identification:

<table>
<thead>
<tr>
<th>Current flow ( I )</th>
<th>Active power flow ( f_\ell )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential/voltage ( V_i )</td>
<td>Voltage angle ( \theta_i )</td>
</tr>
<tr>
<td>Resistance ( R )</td>
<td>Reactance ( X )</td>
</tr>
</tbody>
</table>

The simplifications that lead to the linear power flow were explained in the previous lecture.
Kirchhoff’s Current Law (KCL)

KCL enforces energy conservation at each vertex (the power imbalance equals what goes out minus what comes in).
KCL isn’t enough to determine the flow as soon as there are **closed cycles** in the network. For this we need Ohm’s law in combination with KVL: voltage differences around each cycle add up to zero.

For equal reactances for each edge:

NB: For directed graph, sign determines direction of flow.
Computing the Linear Power Flow
Suppose we have \( N \) nodes labelled by \( i \), and \( L \) edges labelled by \( \ell \) forming a directed graph \( G \).

Suppose at each node we have a **power imbalance** \( p_i \) \((p_i > 0\) means its generating more than it consumes and \( p_i < 0 \) means it is consuming more than it\).

Since we cannot create or destroy energy (and we’re ignoring losses):

\[
\sum_i p_i = 0
\]

**Question**: How do the flows \( f_\ell \) in the network relate to the nodal power imbalances?

**Answer**: According to the impedances (generalisation of resistance for oscillating voltage/current) and the corresponding voltages.
KCL says (in this linear setting) that the nodal power imbalance at node $i$ is equal to the sum of direct flows arriving at the node. This can be expressed compactly with the incidence matrix

$$p_i = \sum_{\ell} K_{i\ell} f_{\ell} \quad \forall i$$
Kirchhoff’s Voltage Law (KVL)

KVL says that the sum of voltage differences across edges for any closed cycle must add up to zero.

If the voltage at any node is given by $\theta_i$ (this is in fact the voltage angle - more next week) then the voltage difference across edge $\ell$ is

$$\sum_i K_{i\ell} \theta_i$$

And Kirchhoff’s law can be expressed using the cycle matrix encoding of independent cycles

$$\sum_{\ell} C_{\ell c} \sum_i K_{i\ell} \theta_i = 0 \quad \forall c$$

[Automatic, since we already said $KC = 0.$]
Kirchhoff’s Voltage Law (KVL)

If we express the flow on each line in terms of the voltage angle (a relative of $V = IR$) then for a line $\ell$ with reactance $x_\ell$

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

KVL now becomes

$$\sum_{\ell} C_{\ell c} x_\ell f_\ell = 0 \quad \forall c$$
Solving the equations

If we combine

\[ f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i \]

with Kirchhoff’s Current Law we get

\[ p_i = \sum_\ell K_{i\ell} f_\ell = \sum_\ell K_{i\ell} \frac{1}{x_\ell} \sum_j K_{j\ell} \theta_j \]

This is a **weighted Laplacian**. If we write \( B_{k\ell} \) for the diagonal matrix with \( B_{\ell\ell} = \frac{1}{x_\ell} \) then

\[ L = KBK^t \]

and we get a **discrete Poisson equation** for the \( \theta_i \) sourced by the \( p_i \)

\[ p_i = \sum_j L_{ij} \theta_j \]

We can solve this for the \( \theta_i \) and thus find the flows.
Solving the equations

Given $p_i$ at every node, we want to find the flows $f_\ell$. We have the equations

\[ p_i = \sum_j L_{ij} \theta_j \]
\[ f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i \]

Basic idea: invert $L$ to get $\theta_i$ in terms of $p_i$

\[ \theta_i = \sum_k (L^{-1})_{ik} p_k \]

then insert to get the flows as a linear function of the power injections $p_i$

\[ f_\ell = \frac{1}{x_\ell} \sum_{i,k} K_{i\ell} (L^{-1})_{ik} p_k = \sum_k \text{PTDF}_{\ell k} p_k \]

called the **Power Transfer Distribution Factors** (PTDF).
There is one small catch: $L$ is **not invertible** since we saw last time it has (for a connected network) one zero eigenvalue, with eigenvector $(1, 1, \ldots, 1)$, since by construction $\sum_j L_{ij} = 0$.

This is related to a gauge freedom to add a constant to all voltage angles

$$\theta_i \rightarrow \theta_i + c$$

(corresponding to the zero eigenvector of $L$) which does not affect physical quantities:

$$p_i = \sum_j L_{ij}(\theta_j + c) = \sum_j L_{ij}(\theta_j)$$

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i + c) = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i)$$

Typically we choose a **slack** or **reference bus** such that $\theta_1 = 0$. 
Inverting Laplacian $L$

Two solutions:

1. Since $\theta_1 = 0$ and $p_1$ is not independent of the other power injections ($\sum_{i=1}^{N} p_i = 0$ implies $p_1 = -\sum_{i=2}^{N} p_i$), we can ignore these elements and invert the lower-right $(N - 1) \times (N - 1)$ part of $L$ (which doesn’t have zero eigenvalues) to find the remaining $\{\theta_i\}_{i=2,...,N}$ in terms of the $\{p_i\}_{i=2,...,N}$.

2. Use the Moore-Penrose pseudo-inverse.

Write $L$ in terms of its basis of orthonormal eigenvectors $e_i^n$ ($\sum_j L_{ij}e_j^n = \lambda_n e_i^n$, $\sum_i e_i^n e_i^n = 1$ and $\sum_i e_i^n e_i^m = 0$ if $n \neq m$):

$$L_{ij} = \sum_n \lambda_n e_i^n e_j^n$$

then the Moore-Penrose pseudo-inverse is:

$$L_{ij}^\dagger = \sum_{n|\lambda_n \neq 0} \frac{1}{\lambda_n} e_i^n e_j^n$$
4-node example

\[
K_{i\ell} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
-1 & 1 & 1 & 0 \\
0 & -1 & 0 & 1 \\
0 & 0 & -1 & -1 \\
\end{pmatrix}
\]

\[
L_{ij} = \begin{pmatrix}
1 & -1 & 0 & 0 \\
-1 & 3 & -1 & -1 \\
0 & -1 & 2 & -1 \\
0 & -1 & -1 & 2 \\
\end{pmatrix}
\]

\[
PTDF_{\ell i} = \begin{pmatrix}
0 & -1 & -1 & -1 \\
0 & 0 & -2/3 & -1/3 \\
0 & 0 & -1/3 & -2/3 \\
0 & 0 & 1/3 & -1/3 \\
\end{pmatrix}
\]
PTDF as sensitivity

Can also ‘experimentally’ determine the Power Transfer Distribution Factors (PTDF) by choosing a slack bus (in this case bus 1).

Each column (labelled by $i$) is then the resulting line flows if we have a simple power transfer from bus $i$ to the slack $p_i = 1$ and $p_1 = -1$.

\[
\text{PTDF}_{\ell i} = \begin{pmatrix}
0 & -1 & -1 & -1 \\
0 & 0 & -2/3 & -1/3 \\
0 & 0 & -1/3 & -2/3 \\
0 & 0 & 1/3 & -1/3 \\
\end{pmatrix}
\]
Consequences of limiting power transfers
Line loading limits

You cannot pass infinite current through a transmission line.

As it warms, it sags, then it will become damaged and/or hit a building/tree and cause a short-circuit. For this reasons there are always **thermal limits** on current transfer. There may also be limits on the amount of power or current based on concerns about **voltage stability** or **general stability**.

Typically each line has a well-defined **line loading limit** on the amount of current or power that can flow through it:

\[ |f_\ell| \leq F_\ell \]

where here \( F_\ell \) is the maximum power capacity of the transmission lines.

These limits prevent the transfer of renewable energy or other power sources.
Adjusting generator dispatch to avoid overloading

To avoid overloading the power lines, we must adjust our generator output (or the demand) so that the power imbalances do not overload the network.

We will now generalise and adjust our notation.

From lecture 2 we had for a single node:

\[-p_t = m_t - b_t + c_t = d_t - Ww_t - Ss_t - b_t + c_t = 0\]

where \(p_t\) was the nodal power balance, \(m_t\) was the mismatch (load \(d_t\) minus wind \(Ww_t\) and solar \(Ss_t\)), \(b_t\) was the backup power and \(c_t\) was curtailment.

We generalised this to multiple nodes labelled by \(i\)

\[-p_{i,t} = m_{i,t} - b_{i,t} + c_{i,t} = d_{i,t} - W_{i,wi_{i,t}} - S_{i,si_{i,t}} - b_{i,t} + c_{i,t}\]

where now we don’t enforce \(p_{i,t} = 0\) but \(\sum_i p_{i,t} = 0\) for all \(t\).
Adjusting generator dispatch to avoid overloading

Now we write the dispatch of all generators at node $i$ (wind, solar, backup) labelled by technology $s$ as $g_{i,s,t}$ ($i$ labels node, $s$ technology and $t$ time) so that we have a relation between load $d_{i,t}$, generation $g_{i,s,t}$ and network flows $f_{\ell,t}$

$$p_{i,t} = \sum_s g_{i,s,t} - d_{i,t} = \sum_{\ell} K_{i\ell} f_{\ell,t}$$

Where $s$ runs over the wind, solar and backup capacity generators (e.g. hydro or natural gas) at the node.

A dispatchable generator’s $g_{i,s,t}$ output can be controlled within the limits of its power capacity $G_{i,s}$

$$0 \leq g_{i,s,t} \leq G_{i,s}$$
Variable generation constraints

For a renewable generator we have time series of availability
$0 \leq G_{i,s,t} \leq 1$ (the $s_t$ and $w_t$ before; $W$ and $S$ are the capacity $G_{i,s}$):

$$0 \leq g_{i,s,t} \leq G_{i,s,t} G_{i,s} \leq G_{i,s}$$

Curtailment corresponds to the case where $g_{i,s,t} < G_{i,s,t} G_{i,s}$:
See https://pypsa.org/examples/scigrid-lopf-then-pf.html.
Consider backup energy in a simplified European grid:
Germany needed backup generation for 31% of total load:

Europe needed Backup generation for only 24% of the total load:
European transmission versus backup energy

Transmission needs across a fully renewable European power system by Rodriguez, Becker, Andresen, Heide, Greiner, Renewable Energy, 2014
Principles of electricity storage
Basic idea of storage

Networks were used to shift power imbalances between different places, i.e. in space. Electricity storage can shift power in time.
Storage consistency

Storage units, such as batteries or hydrogen storage, can both dispatch power within a certain capacity:

\[ 0 \leq g_{i,s,t,\text{dispatch}} \leq G_{i,s,\text{dispatch}} \]

and consume power to store energy:

\[ 0 \leq g_{i,s,t,\text{store}} \leq G_{i,s,\text{store}} \]

The total power can then be written:

\[ g_{i,s,t} = g_{i,s,t,\text{dispatch}} - g_{i,s,t,\text{store}} \]

There is also a limit on the total energy \( e_{i,s,t} \) at each time

\[ 0 \leq e_{i,s,t} = e_{i,s,0} - \sum_{t'=1}^{t} g_{i,s,t'} \leq E_{i,s} \]

where \( E_{i,s} \) is the energy capacity (in MWh). Or in iterative form

\[ 0 \leq e_{i,s,t} = e_{i,s,t-1} + g_{i,s,t,\text{store}} - g_{i,s,t,\text{dispatch}} \leq E_{i,s} \]
Consider a single node with a constant demand

\[ d(t) = D \]

and a renewable wind generator with a capacity \( G = 2D \) and an availability time series

\[ G(t) = \frac{1}{2} \left( 1 + \sin \left( \frac{2\pi}{T} t \right) \right) \]

so that it oscillates with period \( T \) and on average produces enough energy for the demand

\[ \langle G(t)G \rangle = D \]
Our mismatch is now

\[ m(t) = d(t) - GG(t) = -D \sin \left( \frac{2\pi}{T} t \right) \]

For \( D = 1, \ T = 2\pi \):
To balance this, we need a storage unit with power profile $g_s(t)$ such that

$$0 = p(t) = m(t) - g_s(t) = d(t) - GG(t) - g_s(t)$$

i.e.

$$g_s(t) = m(t) = -D \sin \left( \frac{2\pi}{T} t \right)$$

This will have power capacities $G_{s,\text{store}} = G_{s,\text{dispatch}} = D.$
Storage Energy

How much energy capacity $E_s$ do we need? The energy profile is:

$$e_s(t) = \int_0^t (-g_s(t'))dt' = D \int_0^t \sin \left( \frac{2\pi}{T} t' \right) dt' = \frac{T D}{2\pi} \left[ 1 - \cos \left( \frac{2\pi}{T} t \right) \right]$$

so $E_s = \max_t \{e_s(t)\} = \frac{T D}{\pi}$. Faster oscillations, i.e. shorter periods, $\Rightarrow$ less energy capacity. So for $D = 1$, $T = 2\pi$, maximum is $E_s = 2$:
Storage Energy: concrete examples

How does our formula $E_s = \frac{TD}{\pi}$ look for different generation technologies with simplified sinusoidal profiles?

Consider a simplified demand of $D = 1$ MW.

<table>
<thead>
<tr>
<th>quantity</th>
<th>symbol</th>
<th>units</th>
<th>solar</th>
<th>wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation capacity</td>
<td>$G$</td>
<td>MW</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>storage power capacity</td>
<td>$G_s$</td>
<td>MW</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>period</td>
<td>$T$</td>
<td>h</td>
<td>24</td>
<td>7 \cdot 24</td>
</tr>
<tr>
<td>storage energy capacity</td>
<td>$E_s$</td>
<td>MWh</td>
<td>7.6</td>
<td>53</td>
</tr>
</tbody>
</table>

Faster daily oscillations of solar need smaller storage capacity than weekly oscillations of wind.

NB: In reality of course solar and wind are not perfect sine waves...
Efficiency and losses

There are a few extra details to add now. First, no renewable has a perfectly regular sinusoidal profile.

Second, the iterative integration equation for the storage energy

\[ e_{i,s,t} = e_{i,s,t-1} + g_{i,s,t,\text{store}} - g_{i,s,t,\text{dispatch}} \]

needs to be amended for efficiencies \( \eta \) (corresponding to losses \( 1 - \eta \))

\[ e_{i,s,t} = \eta_0 e_{i,s,t-1} + \eta_1 g_{i,s,t,\text{store}} - \eta_2^{-1} g_{i,s,t,\text{dispatch}} \]

\( 1 - \eta_0 \) corresponds to standing losses or self-discharge, \( \eta_1 \) to the charging efficiency and \( \eta_2 \) to the discharging efficiency.
Different storage units have different parameters

We can relate the power capacity $G_s$ to the energy capacity $E_s$ with the maximum number of hours the storage unit can be charged at full power before the energy capacity is full, $E_s = \text{max-hours} \times G_s$.

<table>
<thead>
<tr>
<th></th>
<th>Battery</th>
<th>Hydrogen</th>
<th>Pumped-Hydro</th>
<th>Water Tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_0$</td>
<td>$1 - \varepsilon$</td>
<td>$1 - \varepsilon$</td>
<td>$1 - \varepsilon$</td>
<td>depends on size</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.9</td>
<td>0.75</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>0.9</td>
<td>0.58</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>max-hours</td>
<td>2-10</td>
<td>weeks</td>
<td>4-10</td>
<td>depends on size</td>
</tr>
<tr>
<td>euro per kW [$G_s$]</td>
<td>300</td>
<td>500+450</td>
<td>depends</td>
<td>low</td>
</tr>
<tr>
<td>euro per kWh [$E_s$]</td>
<td>200</td>
<td>10</td>
<td>depends</td>
<td>low</td>
</tr>
</tbody>
</table>

Parameters are roughly based on Budischak et al, 2012 with projections for 2030.
Different storage units have different use cases

Consider the cost of a storage unit with 1 kW of power capacity, and different energy capacities.

The total losses are given by the round-trip losses in and out of the storage $1 - \eta_1 \cdot \eta_2$.

<table>
<thead>
<tr>
<th></th>
<th>Battery</th>
<th>Hydrogen</th>
</tr>
</thead>
<tbody>
<tr>
<td>losses</td>
<td>$1 - 0.9^2 = 0.19$</td>
<td>$1 - 0.58 \times 0.75 = 0.57$</td>
</tr>
<tr>
<td>€ for 2 kWh</td>
<td>$300 + 2 \times 200 = 700$</td>
<td>$950 + 2 \times 10 = 970$</td>
</tr>
<tr>
<td>€ for 100 kWh</td>
<td>$300 + 100 \times 200 = 20300$</td>
<td>$950 + 100 \times 10 = 1950$</td>
</tr>
</tbody>
</table>

Battery has lower losses and is cheaper for short storage periods.

Hydrogen has higher losses but is much cheaper for long storage periods (e.g. several days).
Power-to-Gas (P2G) describes concepts to use electricity to electrolyse water to hydrogen $\text{H}_2$ (and oxygen $\text{O}_2$). We can combine hydrogen with carbon oxides to get hydrocarbons such as methane $\text{CH}_4$ (main component of natural gas) or liquid fuels $\text{C}_n\text{H}_m$.

These can be used for hard-to-defossilise sectors:

- **dense fuels** for transport (planes, ships)
- **steel-making**
- **chemicals industry**
- **high-temperature heat**
- **heat for buildings**
- **backup energy** for cold low-wind winter
Power-to-Gas (P2G)

Gases and liquids are easy to store and transport than electricity.

Storage capacity of the German natural gas network in terms of energy: ca 200 TWh. In addition, losses in the gas network are small.

(NB: Volumetric energy density of hydrogen, i.e. MWh/m$^3$, is around three times lower than natural gas.)

Pipelines can carry many GW underground, out of sight.
Power to Gas Concept

**Figure 1:** Buildipedia
German Gas Grid

Figure 2: Buildipedia

Figure 3: Buildipedia
Electrolysis

Figure 3: Hyperphysics, Georgia State University
Thermodynamic Calculation Electrolysis

\[ H_2O \rightarrow H_2 + \frac{1}{2}O_2 \]

For one mole at conditions 298 K and one atmospheric pressure

<table>
<thead>
<tr>
<th></th>
<th>( H_2 )</th>
<th>( O_2 )</th>
<th>( H_2O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entropy</td>
<td>130.7</td>
<td>205.1</td>
<td>69.9</td>
</tr>
<tr>
<td>Enthalpy</td>
<td>0</td>
<td>0</td>
<td>-285.8</td>
</tr>
</tbody>
</table>

Gibbs free energy \( dG = dH - TdS \),

\[ \Delta G = 285.8kJ - 48.7kJ = 237.1kJ \]
Fuel Cell

Ideal hydrogen-oxygen fuel cell operation

Fuel energy input
\[ \Delta H = 285.83 \text{ kJ/mol} \]

Electric energy output
\[ \Delta G = 237.13 \text{ kJ/mol} \]

Electron flow:
- \( e^- \) flow from anode to cathode

Chemical reactions:
- Anode: \( H_2 \rightarrow 2H^+ + 2e^- \)
- Cathode: \( 2H^+ + 2e^- + O \rightarrow H_2O \)

Heat output:
\[ \Delta S = 48.7 \text{ kJ/mol} \]

H⁺ ions migrate across electrolyte

Water output
Again: one mole at conditions 298 K and one atmospheric pressure

\[ H_2O \rightarrow H_2 + \frac{1}{2}O_2 \]

Gibbs free energy \( dG = dH - TdS \),

\[ \Delta G = 285.8\,kJ - 48.7\,kJ = 237.1\,kJ \]

max theoretical efficiency

\[ \frac{\Delta G}{\Delta U} = 0.83 \]
Demand-Side Management (DSM)
Conceptual options to balance the power system

- Transmission grid
- Storage
- Demand-side management
- Sector coupling
From last time: basic idea of storage

Modify demand instead of generation!
Basic Idea of Demand-Side Management

Modify demand instead of generation!

![Graph showing power consumption, backup, and curtailment over time.](image)
Modification of the Demand for energy through various means such as price incentives

Charge consumers based on the true price of utilities at the time of consumption

Issues: higher utility cost for consumers, privacy
**Definition**
Demand is the total amount of a good buyers would purchase under certain conditions.

Law of demand: when the price of a good falls, the demand will rise.

A demand curve is the graphical representation of the relationship between price and quantity demanded.

Vice versa for supply: total amount of a good sellers would choose to sell under certain conditions, etc.
Definition
Degree of responseness of one variable to another

Locally: slope
Figure 5: sale/purchase day-ahead market
Different Cases of DSM
Different Cases of DSM

![Graph showing electricity demand in Germany with scheduled and realised load.](image)

Legend:
- **scheduled load**
- **realised load**
Efficiency Measures

- Permanent reduction of the demand by use of more efficient appliances
  - washing machines
  - refrigerators
  - water heaters
- Germany: Reduction of 25% of gross electrical energy by 2050 compared to 2008
Peak Shaving

- Infrastructure designed for peak demand situations
- Commercial consumers often charged based on their peak demand
• Shift electrical demand from times of deficits to times of surplusses
• provide price incentives to cause load shifting via smart meters
• different price incentive schemes possible, e.g., time of use prices, seasonal prices, etc.
Technical Aspects
Technical Aspects of Demand-Side Management

Figure 1: What makes a smart meter smart?

- Monitors and records:
  - Energy usage/demand, time of use (TOU)
  - Power quality, disturbances & events
  - Store interval data logs

- Aggregates & stores mechanical meter data logs

- Multiple communication ports & protocols
- Ethernet, IP addressable
- Programmable frameworks
- Alarm notification

Source: Adapted from GTM Research, Department of Energy
Smart Meter
Rogowski Coil

Advantages:

- low inductance, therefore sensitive to small current changes
- highly linear for a large range of currents
- open loop
- relatively low cost
- simple temperature compensation

Voltage given by

\[ v_t = - \frac{AN\mu_0}{l} i \]
Figure 4: Energy information pathways in Ontario

Source: Adapted from Independent Electricity System Operator
Modelling approach for DSM
Modelling Approach for DSM

• loads into different categories with assumed max. shifting periods (e.g., 8 hours for household applications)

• shifting charges a virtual storage buffer

\[ P_n[R_n(t)](t) = R_n(t) - L_n(t). \]  \hspace{1cm} (1)

• filling level is consequently given by

\[ E_n[R_n(t)](t) = \int_0^t P_n[R_n(t')](t')dt' \]  \hspace{1cm} (2)

• constraints by shifting periods, e.g.,

\[ E_n^+(t) = \int_t^{t+\Delta t} L_n(t')dt' \]  \hspace{1cm} (3)
Modelling Approach for DSM
Load shifting supports system integration of variable renewables, especially PV.

**Figure 7:** Kies et al., Energies, 2016
Consumer Synchronisation via Adaptive Pricing Schemes
Paper: Krause, S. et al., Econophysics of adaptive power markets: When a market does not dampen fluctuations but amplifies them, arXiv:1303.2110

![Diagram of price and demand](image-url)
demand is described via \((p_t - \text{price time series})\)

\[
d_{i,t} = \begin{cases} 
1 & \text{if } p_t \leq p_{i,t}, \\
0 & \text{if } p_t > p_{i,t}
\end{cases}
\]

and the acceptable price \((p_{i,t})\) time series of agent \(i\) evolves according to

\[
p_{i,t+1} = \begin{cases} 
\text{rand}[0, p_{i,t}] & \text{if } p_t \leq p_{i,t}, \\
\text{rand}[p_{i,t}, 1], \text{ else with prob. } f \\
p_{i,t} & \text{otherwise}
\end{cases}
\]

the parameter \(f\) describes the elasticity of the demand correlations modelled via Langevin equation

\[
p_{t+1} - p_t = -v_0(p_t - \bar{p}) + \sigma_0 \xi_t
\]
Agents synchronise → extreme peak demands. Effect also known as demand response concentration.
Figure 8: Density of highest acceptable prices (blue), total load consumed at certain prices (red)
Figure 9: Binning of events by price and demand intervals. Dashed line shows average demand. Correlated prices (left), uncorrelated prices (right)
Summary

- Demand-side management can contribute to successful power system operation
- “Daily” scale supports PV integration
- Building infrastructure for DSM is cost-intensive and causes additional energy consumption
- Synchronisation via pricing can amplify fluctuations