Energy System Modelling
Summer Semester 2019, Lecture 8

Dr. Tom Brown, tom.brown@kit.edu, `https://nworbmot.org/`
Karlsruhe Institute of Technology (KIT), Institute for Automation and Applied Informatics (IAI)

14th June 2019

Table of Contents

Present value and discounting

The value of money depends on time

Question 1: What would you prefer: €1000 today, or €1000 in 3 years?

The value of money depends on time

Question 1: What would you prefer: €1000 today, or €1000 in 3 years?

€1000 today can be invested in the bank with an interest rate of 5%.

After 3 years you would have

$$1000 \cdot (1 + 0.05)^3 = 1158$$

Answer 1: Best to take the money today and use the opportunity to invest!

"Money in the future is worth less than money today."

The value of money depends on time

Question 2: What would you prefer: €1000 today, or €1300 in 5 years?

The value of money depends on time

Question 2: What would you prefer: €1000 today, or €1300 in 5 years?

If you invested €1000 today, after 5 years you would have only

$$1000 \cdot (1 + 0.05)^5 = 1276$$

Answer 2: Best to wait for the €1300 in 5 years!

Present value

To allow comparison between income and outgoings in different years, we need to agree on a particular point in time to evaluate the cash flows.

The simplest and most frequently used time point: today's value, known as the **present value**.

For an **interest rate** r we multiply the income or outgoings in year t by the **discount factor**

$$\frac{1}{(1+r)^t}$$

to calculate the present value. We have **discounted** the future cash flow.

Future income or outgoings are **worth less** from today's point of view (as long as r is positive).

"Money in the future is worth less than money today."

Example: present value

For our example with interest rate 5% we can now order the options:

Income (€)	Year	Present value (€)
1000	3	$\frac{1000}{(1+0.05)^3} = 863$
1000	0	$\frac{1000}{(1+0.05)^0} = 1000$
1300	5	$\frac{1300}{(1+0.05)^5} = 1019$

Investment calculations

Motivation: Power plant investment

A company is considering investing in a photovoltaic plant on its roof. The key figures:

Size	100 kW
Investment cost	800 €kW^{-1}
Operating cost	20 €kW^{-1} a^{-1}
Feed-In Tariff	0.1 €kWh^{-1}
Full load hours	1000
Period of subsidy	20 years

The company can invest its money elsewhere for a return of 5%.

Is it worthwhile to invest in the photovoltaic plant?

Source: Fraunhofer ISE Stromgestehungskosten 2018, Wikipedia

Investment calculations

An **investment calculation** quantifies the financial costs and benefits of an investment, assuming that future income and outgoings can be predicted.

It considers

- **Capital costs** - Costs for investments and installation
- **Consumption costs** - Fuel, other materials (e.g. lubricants for wind turbine), etc.
- **Operating costs** - Maintenance, wages, insurance, management, etc.
- **Income** - depends on market price, subsidies, and production

Dynamic investment calculation

For a **dynamic investment calculation** we sum the present values of all income and outgoings over the T years of operation taking account of the interest rate r to get the **Net Present Value (NPV)**:

$$NPV = \sum_{t=0}^{T} \frac{-I_t - V_t - B_t + U_t}{(1+r)^t}$$

where I_t is the capital expenditure in year t, V_t the consumption costs (e.g. for fuel cost o_t and annual production Q_t, $V_t = o_t \cdot Q_t$), B_t the operating costs und U_t the income (e.g. average market value λ_t times annual production Q_t, $U_t = \lambda_t \cdot Q_t$).

Conclusion: If $NPV > 0$, the investment is worthwhile.

If $NPV < 0$, better to invest with a rate of return of r elsewhere.

For comparisons between different investments, a higher NPV should be preferred.

Example: Rooftop photovoltaic unit

All cash flows (costs and income) in €:

year t	0	1	2	$\cdots$	20
Capital costs I_t	80.000	0	0		0
Operating costs B_t	0	2.000	2.000		2.000
Income U_t	0	10.000	10.000		10.000
Net cash flow $U_t - I_t - B_t$	-80.000	8.000	8.000		8.000
Discount factor $\frac{1}{(1+r)^t}$	1	$\frac{1}{(1+r)}$	$\frac{1}{(1+r)^2}$		$\frac{1}{(1+r)^{20}}$

NPV simplification

If investments only occur in the first year, and the costs and income for the following years are constant, we can simplify the NPV formula:

$$NPV = -I_0 + (U - V - B) \sum_{t=1}^{T} \frac{1}{(1+r)^t}$$

The sum $\sum$ is called the **Present Value Factor** $PVF(r, T)$.

For a geometric series with $|q| < 1$ we have $\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$. For $q = (1+r)^{-1}$ we can simplify the formula

$$\begin{aligned}
PVF(r, T) &= \sum_{t=1}^{T} \frac{1}{(1+r)^t} \\
&= \left[\frac{1}{(1+r)} - \frac{1}{(1+r)^{T+1}}\right] \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \left[\frac{1}{(1+r)} - \frac{1}{(1+r)^{T+1}}\right] \frac{1}{1-(1+r)^{-1}} \\
&= \left[\frac{1}{(1+r)} - \frac{1}{(1+r)^{T+1}}\right] \frac{1+r}{1+r-1} = \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right]
\end{aligned}$$

Example: Rooftop photovoltaic unit

For our example with $r = 0.05$

$$\begin{aligned} NPV &= -80.000 + (10.000 - 2.000) \cdot \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] \\ &= -80.000 + 8.000 * 12.5 \\ &= 19698 \end{aligned}$$

Conclusion: It's worthwile to invest in the photovoltaic unit!

Example: Rooftop photovoltaic unit

For our example with $r = 0.05$

$$\begin{aligned} NPV &= -80.000 + (10.000 - 2.000) \cdot \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] \\ &= -80.000 + 8.000 * 12.5 \\ &= 19698 \end{aligned}$$

Conclusion: It's worthwile to invest in the photovoltaic unit!

NB: The calculation is very sensitive to the interest rate, e.g. with $r = 0.08$

$$\begin{aligned} NPV &= -80.000 + 8.000 * 9.8 \\ &= -1.454 \end{aligned}$$

Conclusion: The investment is not worthwhile.

Return On Investment (ROI)

The expected return or **Return On Investment (ROI)** is the required interest rate to reach the point $NPV = 0$.

In our example you can either experiment or use the Newton-Raphson algorithm to determine the ROI r

$$0 = NPV = -I_0 + (U - V - B)\sum_{t=1}^{T} \frac{1}{(1+r)^t}$$

In our example we find an ROI of $r = 7.75\%$.

German example figures for electricity production technologies in 2018

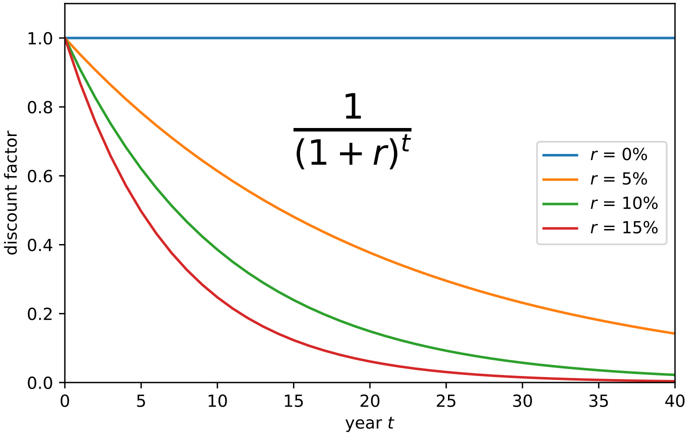
WACC is the **Weighted Average Cost of Capital** over the bank interest rate for borrowed capital (Fremdkapital) and the investor's ROI on their own investment (Eigenkapital).

	PV Dach Klein-anlagen (5-15 kWp)	PV Dach Großanlgen (100-1000 kWp)	PV Frei-fläche (ab 2000 kWp)	Wind Onshore	Wind Offshore	Biogas	Braun-kohle	Stein-kohle	GuD	GT
Lebensdauer in Jahren	25	25	25	25	25	30	40	40	30	30
Anteil Fremdkapital	80%	80%	80%	80%	70%	80%	60%	60%	60%	60%
Anteil Eigenkapital	20%	20%	20%	20%	30%	20%	40%	40%	40%	40%
Zinssatz Fremdkapital	3,5%	3,5%	3,5%	4,0%	5,5%	4,0%	5,5%	5,5%	5,5%	5,5%
Rendite Eigenkapital	5,0%	6,5%	6,5%	7,0%	10,0%	8,0%	11,0%	11,0%	10,0%	10,0%
WACC nominal	3,8%	4,1%	4,1%	4,6%	6,9%	4,8%	7,7%	7,7%	7,3%	7,3%
WACC real	1,8%	2,1%	2,1%	2,5%	4,8%	2,7%	5,6%	5,6%	5,2%	5,2%
OPEX fix [EUR/kW]	2,5% von CAPEX	2,5% von CAPEX	2,5% von CAPEX	30	100	4,0% von CAPEX	36	32	22	20
OPEX var [EUR/kWh]	0	0	0	0,005	0,005	0	0,005	0,005	0,004	0,003

Source: Fraunhofer ISE Stromgestehungskosten 2018

Warning: Discounting over long time periods

Over long time periods the discounting can have a very large effect....



- Long-term benefits aren't seen, e.g. long production life of nuclear power plants or benefits of long-lived efficiency measures
- Long-term costs are also suppressed, e.g. decommissioning, waste disposal, climate damages
- This is a **controversial topic!**

Programming example: photovoltaic plant

jupyter NPV_examples Last Checkpoint: an hour ago (unsaved changes)

Logout

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3

Run Code

PV Example

In [37]:

```
lifetime = 20 #years
discount_rate = 0.08 #per unit
size = 100 #kW
specific_cost = 800 #EUR/kW
fom = 20 #EUR/kW/a
fit = 0.1 #EUR/kWh
flh = 1000 #h/a
flows = pd.DataFrame(index=range(lifetime+1))
flows["investment"] = [-size*specific_cost] + [0]*lifetime
flows["FOM"] = [0] + [-size*fom]*lifetime
flows["income"] = [0] + [size*flh*fit]*lifetime
flows["total_flow"] = flows.sum(axis=1)
flows["discount_factor"] = [(1+discount_rate)**(-t) for t in range(lifetime+1)]
flows["discounted_total_flow"] = flows["total_flow"]*flows["discount_factor"]
```

In [38]:

```
flows.head()
```

Out[38]:

	investment	FOM	income	total_flow	discount_factor	discounted_total_flow
0	-80000	0	0.0	-80000.0	1.000000	-80000.000000
1	0	-2000	10000.0	8000.0	0.925926	7407.407407
2	0	-2000	10000.0	8000.0	0.857339	6858.710562
3	0	-2000	10000.0	8000.0	0.793832	6350.657928
4	0	-2000	10000.0	8000.0	0.735030	5880.238822

In [39]:

```
flows.sum()
```

Out[39]:

```
investment              -80000.000000
FOM                     -40000.000000
income                  200000.000000
total_flow               80000.000000
discount_factor             10.818147
discounted_total_flow    -1454.820740
dtype: float64
```

Programming example: nuclear plant

jupyter NPV_examples Last Checkpoint: an hour ago (unsaved changes)

File Edit View Insert Cell Kernel Widgets Help | Trusted | Python 3

Nuclear Example

In [56]:
```
lifetime = 40 #years
discount_rate = 0.05 #per unit
size = 3e6 #kW
specific_cost = 5000 #EUR/kW
decommissioning_cost = 1000 #EUR/kW
fom = 20 #EUR/kW/a
fuel = 10 #EUR/MWh
market_value = 50 #EUR/MWh
flh = 8000 #h/a
flows = pd.DataFrame(index=range(lifetime+1))
flows["investment"] = [-size*specific_cost] + [0]*(lifetime-1) + [-size*decommissioning_cost]
flows["FOM"] = [0] + [-size*fom]*lifetime
flows["income"] = [0] + [size*flh*market_value/1000]*lifetime
flows["total_flow"] = flows.sum(axis=1)
flows["discount_factor"] = [(1+discount_rate)**(-t) for t in range(lifetime+1)]
flows["discounted_total_flow"] = flows["total_flow"]*flows["discount_factor"]
```

In [57]:
```
flows.head()
```

Out[57]:

	investment	FOM	income	total_flow	discount_factor	discounted_total_flow
0	-1.500000e+10	0.0	0.000000e+00	-1.500000e+10	1.000000	-1.500000e+10
1	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.952381	1.085714e+09
2	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.907029	1.034014e+09
3	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.863838	9.847749e+08
4	0.000000e+00	-60000000.0	1.200000e+09	1.140000e+09	0.822702	9.378808e+08

In [59]:
```
flows.sum()
```

Out[59]:
```
investment              -1.800000e+10
FOM                     -2.400000e+09
income                   4.800000e+10
total_flow               2.760000e+10
discount_factor          1.815909e+01
discounted_total_flow    4.135221e+09
dtype: float64
```

Summary

- Future income or costs are worth less from today's point of view
- To calculate the **present value**, multiply the cash flow in year t by the **discount factor** $\frac{1}{(1+r)^t}$
- To calculate the **net present value (NPV)** for an investment, sum the present values of all income and costs
- If $NPV > 0$, the investment is worthwhile compared to investing with interest rate r
- For two different investments, a higher NPV should be preferred
- **Long-term** costs or benefits are **suppressed** by discounting

Levelised Cost Of Electricity (LCOE)

Levelised Cost Of Energy (LCOE)

You can also solve for the market value or feed-in tariff that's necessary to cover all the costs of the investment, i.e. the point where the present value of all income balances the present value of all costs. You solve for the price λ such that

$$0 = NPV = -I_0 + (\lambda Q - oQ - B)PVF(r, T)$$

(using $V = oQ$). We find:

$$\lambda = \frac{1}{Q}\left(\frac{I_0}{PVF(r, T)} + B + oQ\right) = \frac{1}{Q}\left(\frac{I_0}{PVF(r, T)} + B\right) + o$$

In our example we find a price of $\lambda = 89$ €/MWh for $i = 0.05$.

This value corresponds to the average long-term costs of the unit, since we've divided the total yearly costs by the total production Q. It is called the the **Levelised Cost Of Energy (LCOE)**.

It is also called the **Long-Run Marginal Cost (LMRC)**, since we've added to the short-run marginal cost o an annualised contribution to the capital cost and the operating costs.

Check: The higher I_0 or B are, the higher the LCOE. The higher Q is, the lower the LCOE.

Annuity

The **annuity** is the annualised investment cost $a = \frac{I_0}{PVF(r,T)}$ and $a(r,T) = \frac{1}{PVF(r,T)}$ is the **annuity factor**, which spreads the capital costs I_0 evenly over the operational years of the investment (like a mortgage for a house).

For a loan I_0 from the bank, the bank is compensated for the **opportunity cost** of investing elsewhere at a rate of r by an annual fixed sum a so that the NPV for the bank is zero

$$0 = NPV = -I_0 + \sum_{t=0}^{T} \frac{a}{(1+r)^t} = -I_0 + PVF(r,T)\frac{I_0}{PVF(r,T)}$$

The formula for the annuity factor is derived from that for the PVF:

$$a(r,T) = \frac{1}{PVF(r,T)} = \frac{r}{1-(1+r)^{-T}}$$

Examples of annuity factor

AF = Annuity Factor, $a(r, T)$

Lifetime T years	Discount Rate r %	AF $a(r, T)$ per unit
20	0	0.05
20	5	0.08
20	10	0.12
20	20	0.21
40	0	0.025
40	5	0.06
40	10	0.10
40	20	0.20

Things to notice:

- AF reduce to $1/T$ in limit $r \to 0$
- AF climbs steeply with r
- For long lifetimes, AF is similar to short lifetimes for high r - in reality investors try to pay off investments faster than lifetime
- In reality, an investor would provide some capital themselves, e.g. 10-20% of the capital cost, and borrow the rest from the bank. The weighted average of the investor's desired internal rate of return and that of the bank loan is the **weighted average cost of capital** (WACC).

Parameters for different generation technologies

Here are some typical investment and operational parameters projected for 2020:

Source	Lifetime years	Capital Cost €kW^{-1}	Fix O&M €$\text{kW}^{-1}\text{a}^{-1}$	Var O&M €$\text{MWh}_{\text{el}}^{-1}$	η [%]	Fuel Cost €/MWh_{th}	Marg. Cost €/MWh_{el}
Hard Coal	40	1200	30	6	39	10	32
Gas OCGT	30	400	15	3	39	20	54
Gas CCGT	30	800	20	4	60	20	37
Nuclear	40-60	6000	0	6	33	3.3	16
Wind Onshore	25	1240	35	0		0	0
Solar PV	25	750	25	0		0	0

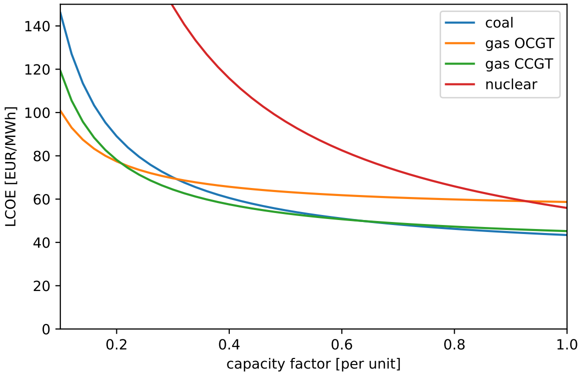
O&M $=$ Operation and Maintenance, Var. $=$ Variable, Fix. $=$ Fixed, $\eta =$ efficiency

For a plant with capacity G_s in MW and yearly production Q in MWh_{el}, we have $I_0 = 1000 \cdot G_s \cdot (\text{Capital Cost})$, $B = 1000 \cdot G_s \cdot (\text{Fix O\&M})$, $V = Q \cdot o$ where o is the marginal cost $o = (\text{Marg. Cost}) = (\text{Var O\&M}) + (\text{Fuel Cost})/\eta$.

Source: DIW Data Documentation, 2013

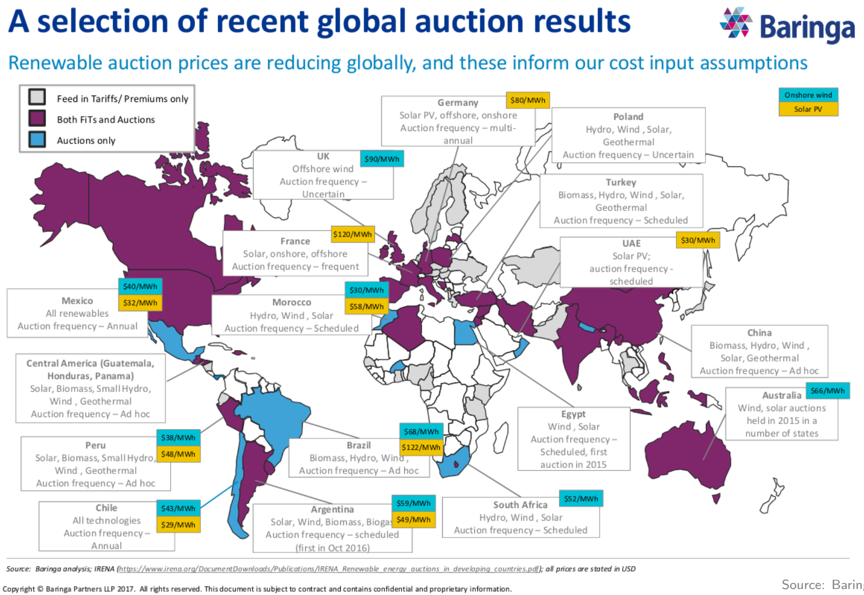
LCOE for dispatchable generators depends on capacity factor

The LCOE had the form (Marg. Cost) + (Yearly Fixed Costs)/(Yearly Production). Therefore it decreases with increasing capacity factor:



- LCOE $>$ marginal cost
- LCOE starts high then reduces as fixed costs are spread over more hours
- There are **crossing points** where some types of generators become cheaper for a given capacity factor
- NB: All generators need downtime for regular maintenance, so cf < 0.9
- NB: Carbon pricing would alter this graphic by adding to the marginal cost

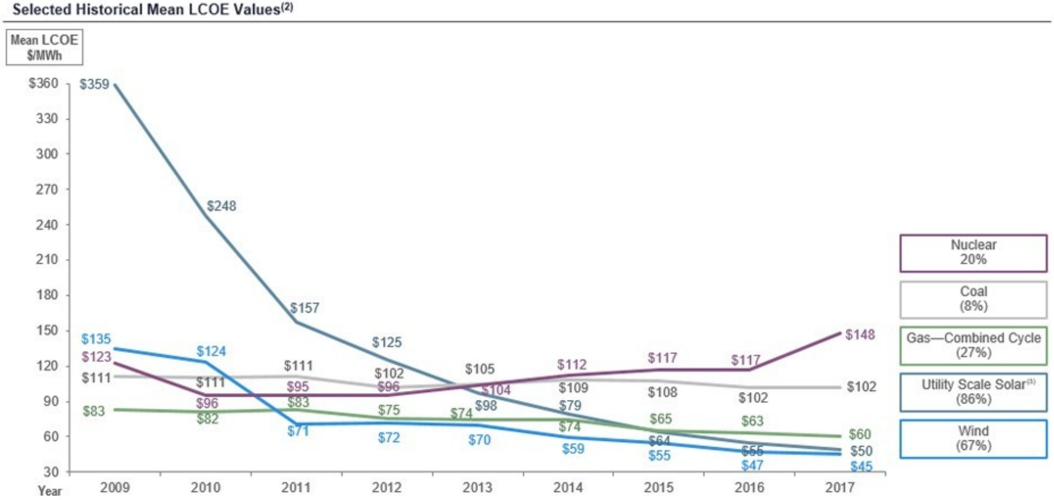
LCOE for wind and solar depends on location: worldwide auction results 2017



Source: Baringa Partners LLP 2017

Levelised Cost of Electricity Since 2009 in US

NB: Treat with care since LCOE doesn't take account of time or place of generation!

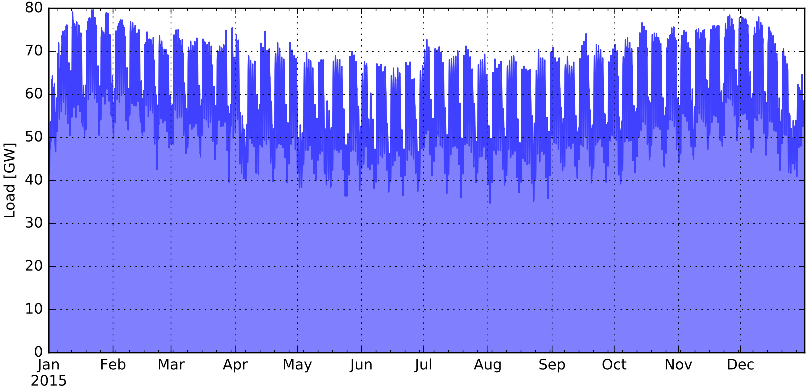


Source: Lazard's LCOE Analysis V11

Duration Curves and Capacity Factors: Examples from Germany in 2015

Load curve

Here's the electrical demand (load) in Germany in 2015:

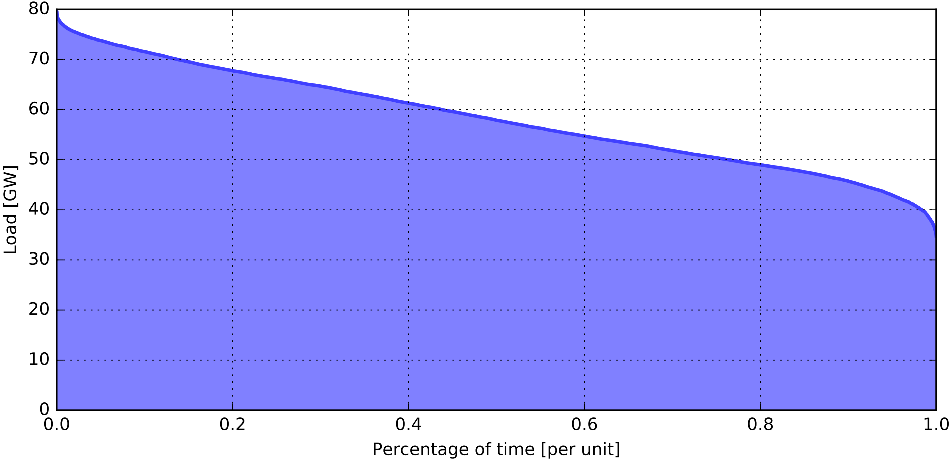


To understand this curve better and its implications for the market, it's useful to stack the hours of the year from left to right in order of the amount of load.

Load duration curve

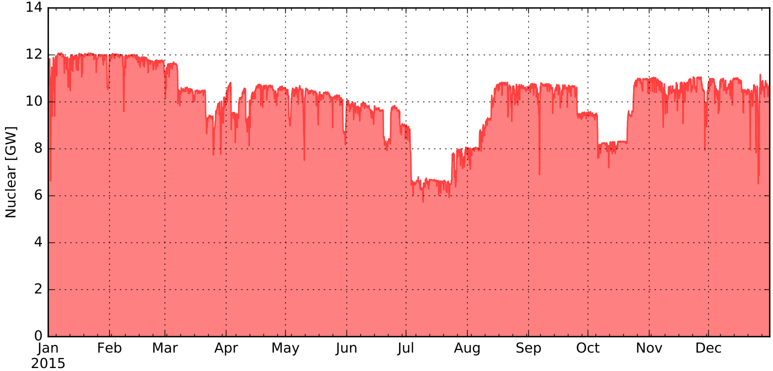
This re-ordering is called a **duration curve**.

For the load it's the **load duration curve**.



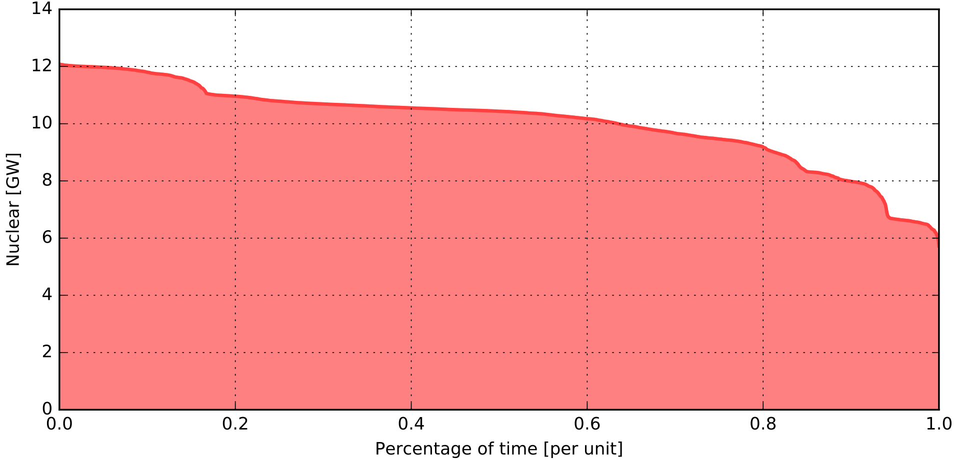
Nuclear curve

Can do the same for nuclear output:



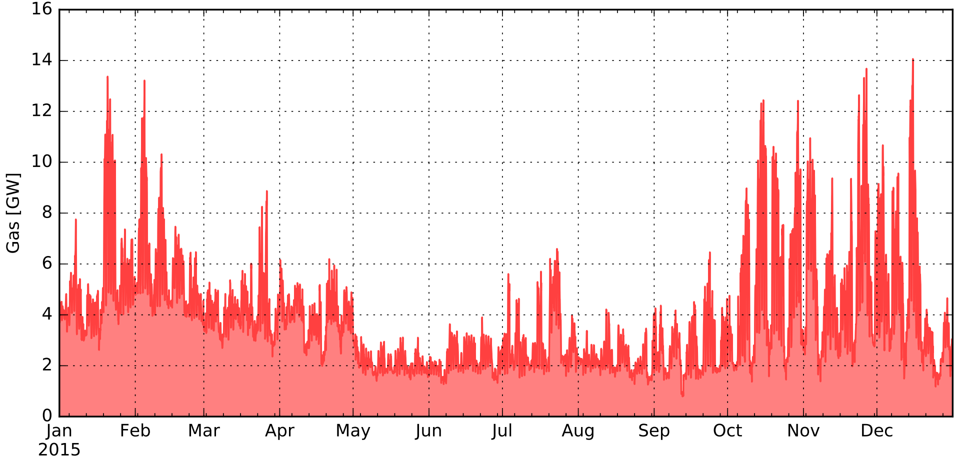
Nuclear duration curve

Duration curve is pretty flat, because it is economic to run nuclear almost all the time as **baseload plant**:



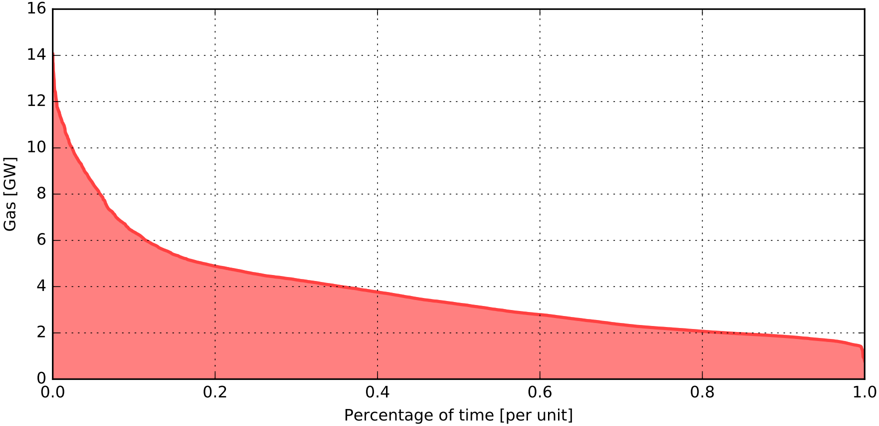
Gas curve

Can do the same for gas output:



Gas duration curve

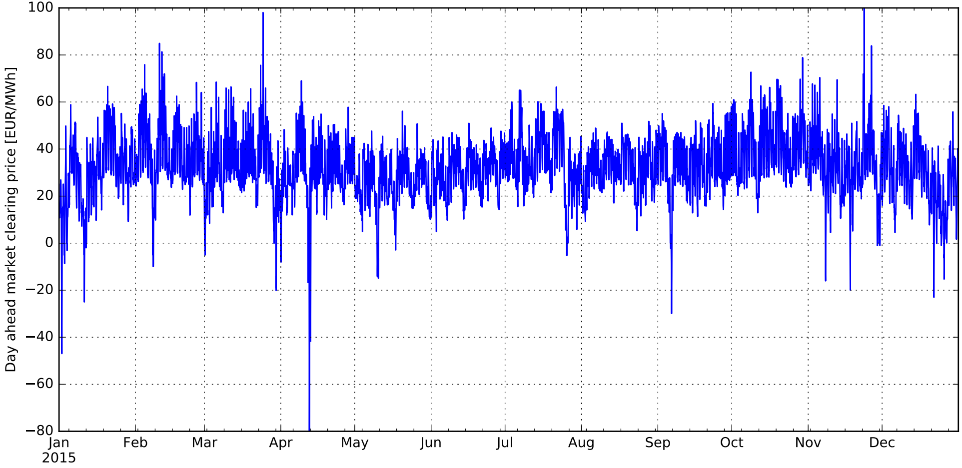
Duration curve is partially flat (for heat-driven CHP) and partially peaked (for **peaking plant**):



The capacity factor for gas is much lower - more like 20%.

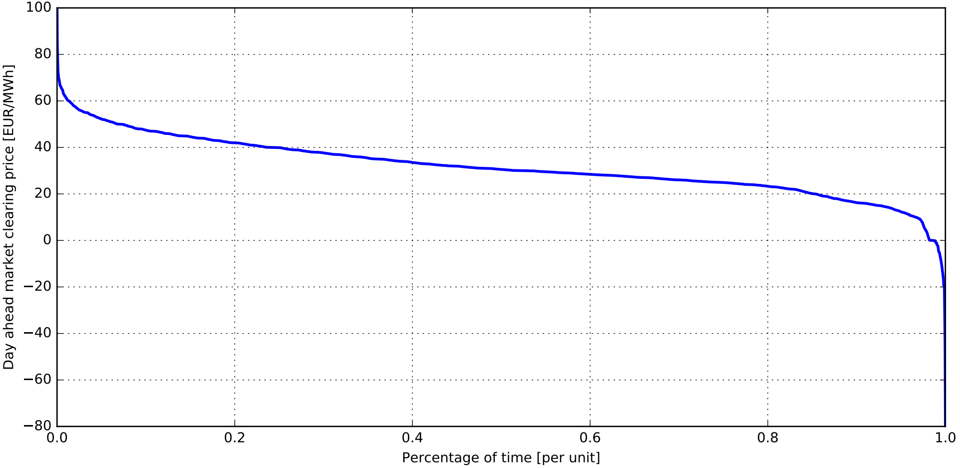
Price curve

Can do the same for price during the year:



Price duration curve

By ordering we get the **price duration curve**:



Question

Now we are in a position to consider the questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find the price and load duration curves very useful.

Investment Optimisation: Generation

Investment optimisation

Now we also optimise **investment** in the **capacities** of generators, storage and network lines for the **whole system** not just a single plant operator, to maximise **long-run efficiency**.

We will promote the capacities $G_{i,s}$, $G_{i,r,*}$, $E_{i,r}$ and F_ℓ to optimisation variables.

For generation investment, we want to answer the following questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find price and load duration curves very useful.

Definition of long-run efficiency

Up until now we have considered **short-run** equilibria that ensure **short-run** efficiency (static), i.e. they make the best use of presently available productive resources.

Long-run efficiency (dynamic) requires in addition the optimal investment in productive capacity.

Concretely: given a set of options, costs and constraints for different generators (nuclear/gas/wind/solar) what is the optimal generation portfolio for maximising long-run welfare?

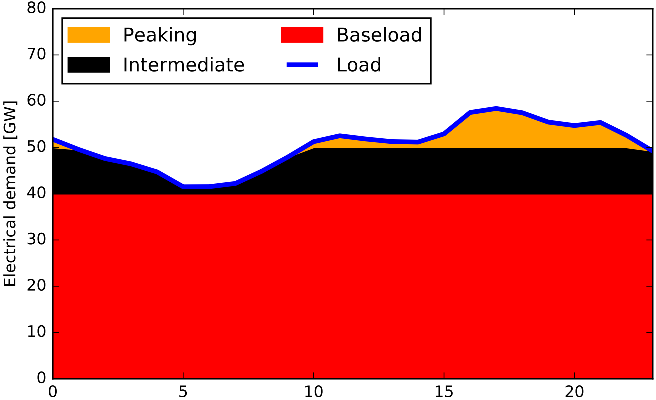
From an indivdual generators' perspective: how best should I invest in extra capacity?

We will show again that with perfect competition and no barriers to entry, the system-optimal situation can be reached by individuals following their own profit.

Baseload versus Peaking Plant

Load (= Electrical Demand) is low during night; in Northern Europe in the winter, the peak is in the evening.

To meet this load profile, cheap **baseload** generation runs the whole time; more expensive **peaking plant** covers the difference.



Source: Tom Brown

Different types of generators

Fuel/Prime mover	Marginal cost	Capital cost	Controllable	Predictable days ahead	CO2
Oil	V. High	Low	Yes	Yes	Medium
Gas OCGT	High	Low	Yes	Yes	Medium
Gas CCGT	Medium	Medium	Yes	Yes	Medium
Hard Coal	Medium	Lowish	Yes	Yes	High
Brown Coal	Low	Medium	Yes	Yes	High
Nuclear	V. Low	High	Partly	Yes	Zero
Hydro dam	Zero	High	Yes	Yes	Zero
Wind/Solar	Zero	High	Down	No	Zero

System-optimal generator capacities and dispatch

Suppose we have generators labelled by s at a single node with **marginal costs** o_s from each unit of production $g_{s,t}$ and **specific capital costs** c_s from fixed costs regardless of the rate of production (e.g. investment in building capacity G_s). For a variety of demand values d_t that occur with probability p_t ($\sum_t p_t = 1$) we optimise the total **average hourly system costs**

$$\min_{\{g_{s,t}\},\{G_s\}} \left[\sum_s c_s G_s + \sum_{s,t} p_t o_s g_{s,t}\right]$$

such that (rescaling the KKT multipliers by p_t to simplify later formulae)

$$\begin{aligned}
\sum_s g_{s,t} = d_t \qquad &\leftrightarrow \qquad p_t \lambda_t \\
-g_{s,t} \leq 0 \qquad &\leftrightarrow \qquad p_t \underline{\mu}_{s,t} \\
g_{s,t} - G_s \leq 0 \qquad &\leftrightarrow \qquad p_t \bar{\mu}_{s,t}
\end{aligned}$$

We will also allow load-shedding with a 'dummy' generator $s = S$, $o_S = V$ (Value of Lost Load), $c_S = 0$ (the capacity to shed load is assumed not to cost anything).

Beware units and scaling

We've chosen the units here so that the total objective function has units €h^{-1}, the **average hourly system costs**.

$\sum_{s,t} p_t o_s g_{s,t}$ is the expectation value of the hourly production costs. $g_{s,t}$ has units MW, o_s has units €$(\text{MWh})^{-1}$.

$c_s G_s$ is the investment cost averaged over each hour, i.e. the annuity divided by 8760, $\frac{a(r,T)I_0}{8760}$ (we can also add the fixed O&M costs B to it). G_s has units MW, c_s has units €$\text{MW}^{-1}\text{h}^{-1}$.

We could have instead optimised **average yearly system costs**, then $c_s G_s$ would simply be the annuity, and instead of weighting with p_t such that $\sum_t p_t = 1$, we replace it with a weighting w_t such that $\sum_t w_t = 8760$. In this case, the total objective would have units €a^{-1}.

System-optimal generator capacities and dispatch

Stationarity gives us for each s and t:

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = p_t \left(o_s - \lambda_t^* - \bar{\mu}_{s,t}^* + \underline{\mu}_{s,t}^* \right)$$

and for each s:

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = c_s + \sum_t p_t \bar{\mu}_{s,t}^*$$

and from complementarity we get

$$\begin{aligned} \bar{\mu}_{s,t}^*(g_{s,t}^* - G_s^*) &= 0 \\ \underline{\mu}_{s,t}^* g_{s,t}^* &= 0 \end{aligned}$$

and dual feasibility (for minimisation) $\bar{\mu}_{s,t}^*, \underline{\mu}_{s,t}^* \leq 0$.

System-optimal generator capacities and dispatch

The solution for the dispatch $g^*_{s,t}$ is exactly the same as without capacity optimisation. For each t, find the generator m where the supply curve intersects with the demand d_t, i.e. the m where $\sum_{s=1}^{m-1} G_s < d_t < \sum_{s=1}^{m} G_s$.

For $s < m$ we have $g^*_{s,t} = G^*_s$, $\underline{\mu}^*_{s,t} = 0$, $\bar{\mu}^*_{s,t} = o_s - \lambda^*_t \leq 0$.

For $s = m$ we have $g^*_{m,t} = d_t - \sum_{s=1}^{m-1} G^*_s$ to cover what's left of the demand. Since $0 < g^*_{m,t} < G_m$ we have $\underline{\mu}^*_{m,t} = \bar{\mu}^*_{m,t} = 0$ and therefore $\lambda^*_t = o_m$.

For $s > m$ we have $g^*_{s,t} = 0$, $\underline{\mu}^*_{s,t} = \lambda^*_t - o_s \leq 0$, $\bar{\mu}^*_{s,t} = 0$.

What about the G^*_s?

System-optimal generator capacities and dispatch

The G^*_s are determined implicitly based on the interactions between costs and prices.

From stationarity we had the relation

$$c_s = -\sum_t p_t \bar{\mu}^*_{s,t}$$

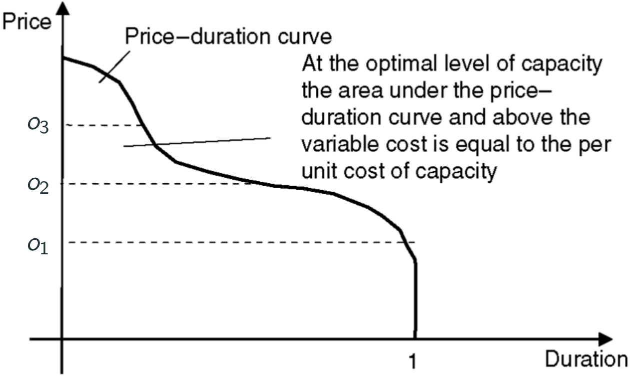
The $\bar{\mu}^*_{s,t}$ were only non-zero with $\lambda^*_t > o_s$ so we can re-write this as

$$c_s = \sum_{t|\lambda^*_t > o_s} p_t(\lambda^*_t - o_s)$$

‘Increase capacity until marginal increase in profit equals the cost of extra capacity.’

Multiple price duration

The optimal mix of generation is where, for each generation type, the area under the price–duration curve and above the variable cost of that generation type is equal to the fixed cost of adding capacity of that generation type.



Source: Biggar and Hesamzadeh, 2014

Multiple generators with inelastic demand

Assume again we have $o_1 \leq o_2 \leq \cdots \leq o_S = V$ and $K_p = \sum_{s=1}^{p} G_s$ then:

$$\lambda_t = \begin{cases} V & \text{for } d_t > K_{S-1} \\ o_s & \text{if } K_{s-1} < d_t \leq K_s, \qquad\qquad \text{for } s = 1, \ldots S-1 \end{cases}$$

Looking at the area under the price duration curve but above the variable cost, we then find:

$$c_s = (V - o_s)P(d > K_{S-1}) + \sum_{j=s+1}^{S-1} (o_j - o_s)P(K_{j-1} < d \leq K_j)$$

Screening curve

These equations can be rewritten recursively using the substitution $\theta_s = P(d > K_s)$:

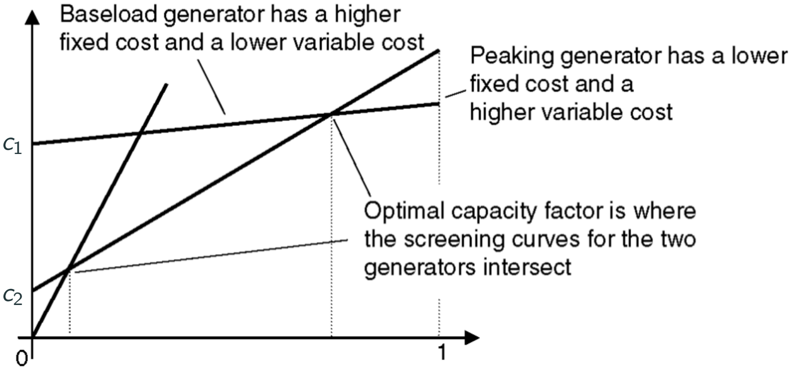
$$c_{S-1} + \theta_{S-1} o_{S-1} = V\theta_{S-1}$$
$$c_s + \theta_s o_s = c_{s+1} + \theta_s o_{s+1} \qquad \forall s = 1, \ldots S-2$$

The first equation can be solved to find θ_{S-1}, then the other equations can be solved recursively to find the remaining θ_s. The θ_s correspond to the optimal **capacity factors** of each type of generator, which correspond to the fraction of time the generator runs at full power.

Screening curve

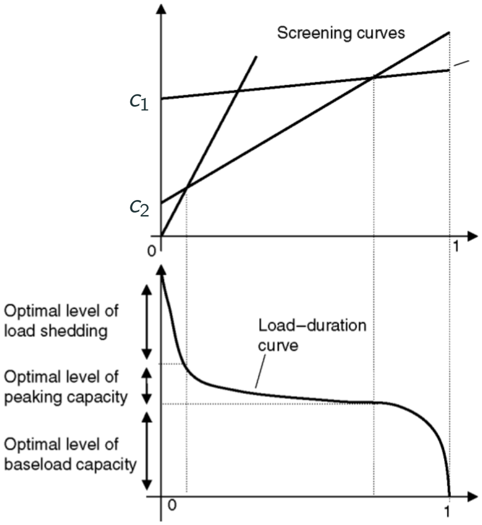
The costs as a function of the capacity factors can be drawn together as a **screening curve** (more expensive options are *screened* from the optimal inner polygon).

The intersection points determine the optimal capacity factors and hence, using the load duration curve, the optimal capacities of each generator type.



Source: Biggar and Hesamzadeh, 2014

Screening curve versus Load duration



Source: Biggar and Hesamzadeh, 2014

Example: 2 generation technologies and load shedding

Suppose that electrical demand is inelastic with a demand-duration curve given by $d(x) = 1000 - 1000x$ for $0 \leq x \leq 1$. Suppose that there are two different types of generation with variable costs of 2 and 12 €/MWh respectively, together with load-shedding at a cost of 1012 €/MWh. The fixed costs of the two generation types are 15 and 10 €/MWh respectively. See the below table for a summary of the costs.

Generator	o_s [€/MWh]	c_s [€/MW/h]
A	2	15
B	12	10
LS	1012	0

Example: 2 generation technologies and load shedding

1. What is the interpretation of the demand-duration curve?
2. Below which capacity factor x_1 is it cheaper to run Generator B rather than to run Generator A?
3. Below which capacity factor x_0 is it cheaper to shed load than to run Generator B?
4. Plot the costs as a function of x and mark these intersection points on a screening curve.
5. Find the optimal capacities of Generators A and B in this market.

Example: 2 generation technologies and load shedding

For the solution see the flipchart photos at
`https://nworbmot.org/courses/esm-2018/board/`.

To find x_1, solve for the intersection of Generator A's cost curve with Generator B's cost curve as a function of capacity factor:

$$c_A + x_1 o_A = c_B + x_1 o_B$$

This gives $x_1 = 0.5$. At this point the demand is $d(0.5) = 500$ MW therefore

$$G_A = 500 \text{ MW}$$

To find x_0, solver for where Generator B crosses the load-shedding line:

$$c_B + x_0 o_B = c_{LS} + x_0 o_{LS}$$

This gives $x_0 = 0.01$. At this point the demand is $d(0.5) = 990$ MW so:

$$G_A + G_B = 990 \text{ MW}$$

i.e. $G_B = 490$ MW and $G_{LS} = 10$ MW.

Investment Optimisation: Transmission

Investment optimisation: transmission

As before, our approach to the question of **"What is the optimal amount of transmission"** is determined by the most efficient long-term solution, i.e. the infrastructure investement that maximising social welfare over the long-run.

Promote F_ℓ to an optimisation variable with capital cost c_ℓ.

In brief: Exactly as with generation dispatch and investment, we continue to invest in transmission until the marginal benefit of extra transmission (i.e. extra congestion rent for extra capacity) is equal to the marginal cost of extra transmission. This determines the optimal investment level.

For the generator case we had $c_s = -\sum_t p_t \bar{\mu}^*_{s,t}$ where $\bar{\mu}^*_{s,t}$ were the shadow prices of the constraints $g_{s,t} \leq G_s$.

For the transmission line we have $f_\ell \leq F_\ell$ $(\bar{\mu}^*_{\ell,t})$ and $-f_\ell \leq F_\ell$ $(\underline{\mu}^*_{\ell,t})$ so we get

$$c_\ell = -\sum_t p_t \left(\bar{\mu}^*_{\ell,t} + \underline{\mu}^*_{\ell,t} \right)$$