Feynman diagrams as fixed points in moduli space

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We want to understand how gauge theories are rewritten as string theories at the pertubative level.

1. How is an integral over the stringy moduli space of a punctured Riemann surface encoded in the gauge theory?
2. How is the discreteness of Feynman diagrams (i.e. finite numbers of vertices and propagators) realised in the string theory?

There have been two answers in the literature:

1. Either Feynman diagrams with an increasing number of interaction vertices join up to form a continuous worldsheet (à la double-scaled matrix model [1, 2, 3]);
2. Or they cover the moduli space, with the gauge theory providing some kind of measure on the moduli space (à la Penner [4] or Kontsevich [5] models).

Here we concentrate on the latter option.

Question: What is the measure? And how does it relate to operator insertions?

In a series of papers in the mid-00's Gopakumar [6, 7, 8] conjectured that for the free theory one could define a measure on the moduli space à la Penner related to momentum on the propagators in the Feynman diagrams.

An alternative possibility raised for Hermitian matrix models by Chekhov and Makeenko [9], Razamat [10, 11], de Mello Koch and Ramgoolam [12], Gopakumar [13] and others is that, at least for these matrix models, the measure might be discrete, with contributions coming only from Riemann surfaces defined over the algebraic numbers $\overline{\mathbb{Q}}$.

It is possible this discrete measure is also there for general free quantum field theories with spacetime dependence. The spacetime dependence of each Feynman diagram is then just a multiplying factor, not related to the measure on moduli space as in the earlier Gopakumar papers [6, 7, 8].

Thus the discreteness of the gauge theory translates into a discreteness on the moduli space.

The following conjecture is for how this discreteness might arise from a continuous string model.

Conjecture: Free gauge theories can be expressed as some kind of A-model-like integration over some moduli space of maps from worldsheets into some space. There is a $U(1)$ action on this moduli space. The integral over moduli space can then be computed using localisation with respect to this $U(1)$ action. The fixed points are discrete and correspond exactly to the Feynman diagrams.

Example: The A-model on $\mathbb{CP}^1$. Kontsevich [14] showed that the $U(1)$ action of rotations of the sphere lift to the moduli space of holomorphic maps into $\mathbb{CP}^1$ form the worldsheet, $\mathcal{M}_{g,n}(\mathbb{CP}^1)$. The fixed points in this space are labelled by bipartite dessin d'enfants, which are exactly the Feynman diagrams of the complex Z-model from [15]. The integral over the moduli space reduces to a weighted sum over these fixed points.

[Note that this localization is independent of the first A model localization of the path integral onto just holomorphic maps.]

For the vanilla AdS/CFT correspondence it would be interesting to pursue this idea for the A model on spaces with $PSU(2,2|4)$ symmetry, such as the coset $U(2,2|4)/U(1) \times U(1,2|4)$, the non-compact cousin of $\mathbb{CP}^{3|4}$. Or the moduli space of flat $PSU(2,2|4)$-bundles over a particular Riemann surface.

For interacting quantum field theories one would expect the discreteness to be washed out with increasing numbers of interaction points, so that one recovered a continuous integration over the moduli space for large λ.

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