Open-open duality with multiple matrices and a loop correction

T.W. Brown / nworbmot@gmail.com

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1 Introduction

In a talk in 2010 [1] Gopakumar suggested that there might exist dualities between field theories which correspond to graph dualities of their Feynman diagrams. He called this open-open duality, in contrast to open-closed string duality. As an example he gave the two different matrix models for 2d topological gravity: the double-scaled Hermitian matrix model, where observables appear as vertices, and the Kontsevich matrix model, where observables are associated to the faces of the graph expansion.

This open-open duality was demonstrated for complex matrix models in [2].

Setup here: In this note we consider the extension of [2] from one complex matrix to many complex matrices with holomorphic observables, and then perturb by mixed operators that are effective vertices for loop-corrections in $\mathcal{N} = 4$ SYM. This is equivalent to restricting $\mathcal{N} = 4$ SYM to the $U(2)$ sector and calculating correlation functions order-by-order in the coupling. Then we find the graph-dual matrix model, including g_{YM}^2 corrections.

Note that this sector is closed (only up to two loops or higher?), see [3].

The basic 2-complex-matrix model is

$$\mathcal{Z} = \int e^{\mathrm{tr}\left[-XX^\dagger - YY^\dagger + \sum_{k=1}^\infty \frac{1}{k}(A\circ X + C\circ Y)^k + \sum_{k=1}^\infty \frac{1}{k}(B\circ X^\dagger + D\circ Y^\dagger)^k\right]} \left(1 + g_{YM}^2 : \mathrm{tr}([X,Y][X^\dagger,Y^\dagger]) : + \mathcal{O}(g_{YM}^4)\right) \tag{1}$$

The couplings to all possible (anti-)holomorphic multitrace operators are kept track of with Kontsevich-like variables A, B, C, D.

The effective vertices for each loop order must be calculated by hand, which is complicated, and is only known up to (3 or 4) loop orders.

By inspection the dual graphs in this expansion have arbitrary valence, unlike the free theory which only has vertices of even valency [2]. I think it's true that for planar two-point functions we can restrict to 2- and 3-valent vertices.

Idea: Work out the graph dual theory in this sector.

Motivation: Tractible gauge/gravity dualities are of 'F-type' [1] like the Kontsevich model and Chern-Simons theory. Perhaps the reason we've made no progress with AdS/CFT is that we're starting from the wrong point; we should transform to the graph dual theory. Then holes really correspond to closed string vertex operators. See [4] for more motivation.

Basic (hopeful) conjecture: Perhaps the dual vertices required for the one-loop dual theory are sufficient to determine the all-loop answer. In other words what is hard and horrible in V theory might be simpler in the F theory.

Plan: Find dual theory for free theory (correctly conjectured in [2]). Then extend with 1-loop correction and check result.

Haven't been able to disprove conjecture's extension to higher loops.

2 Duality for multiple complex matrices model in free theory

In this section we show the open-open dual for a model with two complex matrices and $g_{YM} = 0$. This proves a conjecture made in Appendix D of [2].

First we have to explain the notation in equation (1).

$(A \circ X)^I_J = A^i_j X^e_f$ is the Kronecker matrix product of A and X so that $I = e + (i-1)N \in \{1, \dots nN\}$ for $i \in \{1, \dots n\}$ and $e \in \{1, \dots N\}$. In particular the trace has the property

$$\operatorname{tr}(A \circ X) = \operatorname{tr}(A) \operatorname{tr}(X) \tag{2}$$

Secondly we can sample all single-trace operators using

$$\begin{aligned} \frac{1}{k} \operatorname{tr}\left[(A \circ X + C \circ Y)^k\right] &= \sum_{\mu_1, \mu_2, [\alpha]} \tfrac{1}{|\mathrm{Sym}(\alpha) \cap S_{\mu_1} \times S_{\mu_2}|} \operatorname{tr}(\alpha\ (A \circ X)^{\mu_1} (C \circ Y)^{\mu_2}) \\ &= \sum_{\mu_1, \mu_2, [\alpha]} \tfrac{1}{|\mathrm{Sym}(\alpha) \cap S_{\mu_1} \times S_{\mu_2}|} \operatorname{tr}(\alpha\ A^{\mu_1} C^{\mu_2}) \ \ \operatorname{tr}(\alpha\ X^{\mu_1} Y^{\mu_2}) \end{aligned} \tag{3}$$

The sum is over all the holomorphic single-trace operators built out of μ_1 X's and μ_2 Y's. α is a single k-cycle $\alpha \in [k] \subset S_{\mu_1 + \mu_2}$ where $k = \mu_1 + \mu_2$. The trace with a permutation is defined by

$$\operatorname{tr}(\alpha\, X^{\mu_1} Y^{\mu_2}) = X^{i_1}_{i_{\alpha(1)}} \cdots X^{i_{\mu_1}}_{i_{\alpha(\mu_1)}} Y^{i_{\mu_1+1}}_{i_{\alpha(\mu_1+1)}} \cdots Y^{i_{\mu_1+\mu_2}}_{i_{\alpha(\mu_1+\mu_2)}} \tag{4}$$

It is unique up to conjugation $\alpha \sim \rho^{-1} \alpha \rho$ for $\rho \in S_{\mu_1} \times S_{\mu_2}$ so we only sum over conjagacy classes $[\alpha]$ for this relation.

The coefficients of the holomorphic operators in equation (3) can be treated like couplings $t, \bar{t}$ for a generalised Kontsevich-Miwa transformation

$$\begin{aligned} t_{\{\mu_1, \mu_2, [\alpha]\}} &= \tfrac{1}{|\mathrm{Sym}(\alpha) \cap S_{\mu_1} \times S_{\mu_2}|} \operatorname{tr}(\alpha\ A^{\mu_1} C^{\mu_2}) \\ \bar{t}_{\{\mu_1, \mu_2, [\alpha]\}} &= \tfrac{1}{|\mathrm{Sym}(\alpha) \cap S_{\mu_1} \times S_{\mu_2}|} \operatorname{tr}(\alpha\ B^{\mu_1} D^{\mu_2}) \end{aligned} \tag{5}$$

The matrices A, B, C, D do not commute and are not diagonalisable, unlike the single complex matrix case. For a single cycle $\mathrm{Sym}(\alpha) \cong \mathbb{Z}_k$. Some examples:

$$\begin{aligned} t_{\operatorname{tr}(X^k)} &= \tfrac{1}{k} \operatorname{tr}(A^k) \qquad & t_{\operatorname{tr}(X^2 Y^2)} &= \operatorname{tr}(A^2 C^2) \\ t_{\operatorname{tr}(Y^k)} &= \tfrac{1}{k} \operatorname{tr}(C^k) \qquad & t_{\operatorname{tr}(XYXY)} &= \tfrac{1}{2} \operatorname{tr}(ACAC) \end{aligned}$$

2.1 Derivation of dual model

To proceed to the dual model, integrate in and out matrices just like in Section 2 of [2]

$$
\begin{aligned}
\mathcal{Z} &= \int \exp \operatorname{tr}\left[-XX^\dagger - YY^\dagger + \sum_{k=1}^{\infty}\frac{1}{k}(A\circ X + C\circ Y)^k + \sum_{k=1}^{\infty}\frac{1}{k}(B\circ X^\dagger + D\circ Y^\dagger)^k\right] \\
&= \int \exp \operatorname{tr}\left[-XX^\dagger - YY^\dagger\right]\frac{1}{\det\left[1-(A\circ X + C\circ Y)\right]}\frac{1}{\det\left[1-(B\circ X^\dagger + D\circ Y^\dagger)\right]} \\
&= \int \exp -\left[\operatorname{tr}(XX^\dagger + YY^\dagger) + P_{ie}(\mathbb{I} - A^i_j X^e_f - C^i_j Y^e_f)P^{\dagger jf} + Q_{ie}(\mathbb{I} - B^i_j X^{\dagger e}_{\ f} - D^i_j Y^{\dagger e}_{\ f})Q^{\dagger jf}\right] \\
&= \int \exp -\left[(X^e_f - Q^{\dagger je}Q_{if}B^i_j)(X^{\dagger f}_{\ e} - P^{\dagger lf}P_{ke}A^k_l) + (Y^e_f - Q^{\dagger je}Q_{if}D^i_j)(Y^{\dagger f}_{\ e} - P^{\dagger lf}P_{ke}C^k_l)\right. \\
&\qquad\qquad \left. + P_{ie}P^{\dagger ie} + Q_{ie}Q^{\dagger ie} - Q^{\dagger je}Q_{if}B^i_j P^{\dagger lf}P_{ke}A^k_l - Q^{\dagger je}Q_{if}D^i_j P^{\dagger lf}P_{ke}C^k_l\right] \\
&= \int \exp -\left[P_{ie}P^{\dagger ie} + Q_{ie}Q^{\dagger ie} - Q^{\dagger je}Q_{if}B^i_j P^{\dagger lf}P_{ke}A^k_l - Q^{\dagger je}Q_{if}D^i_j P^{\dagger lf}P_{ke}C^k_l\right] \\
&= \int \exp -\left[(F^l_j - Q_{if}B^i_j P^{\dagger lf})(F^{\dagger j}_{\ l} - Q^{\dagger je}P_{ke}A^k_l) + (G^l_j - Q_{if}D^i_j P^{\dagger lf})(G^{\dagger j}_{\ l} - Q^{\dagger je}P_{ke}C^k_l)\right. \\
&\qquad\qquad \left. + P_{ie}P^{\dagger ie} + Q_{ie}Q^{\dagger ie} - Q^{\dagger je}Q_{if}B^i_j P^{\dagger lf}P_{ke}A^k_l - Q^{\dagger je}Q_{if}D^i_j P^{\dagger lf}P_{ke}C^k_l\right] \\
&= \int \exp -\left[\operatorname{tr}(FF^\dagger + GG^\dagger) + P_{ie}P^{\dagger ie} + Q_{ie}Q^{\dagger ie} - P_{ke}(A^k_l F^l_j + C^k_l G^l_j)Q^{\dagger je} - Q_{if}(B^i_j F^{\dagger j}_{\ l} + D^i_j G^{\dagger j}_{\ l})P^{\dagger lf}\right] \\
&= \int \exp \operatorname{tr}\left[-FF^\dagger - GG^\dagger\right]\frac{1}{\det\left[1 - \begin{pmatrix} 0 & AF+CG \\ BF^\dagger + DG^\dagger & 0 \end{pmatrix}\right]^N} \\
&= \int \exp \operatorname{tr}\left[-FF^\dagger - GG^\dagger + N\sum_{k=1}^{\infty}\frac{1}{k}\begin{pmatrix} 0 & AF+CG \\ BF^\dagger + DG^\dagger & 0 \end{pmatrix}^k\right] \\
&= \int \exp \operatorname{tr}\left[-FF^\dagger - GG^\dagger + N\sum_{k=1}^{\infty}\frac{1}{k}\left((AF+CG)(BF^\dagger + DG^\dagger)\right)^k\right] \qquad (6)
\end{aligned}
$$

This model was predicted in Appendix D of [2].

3 Dual model with loop correction

Now consider inserting into the above derivation an effective vertex $\operatorname{tr}(XYY^\dagger X^\dagger)$ with coefficient θL^4 where $\theta^2 = 0$ and L is a matrix with $\operatorname{tr}(L^4) = g^2_{YM}$.

NB: haven't normal ordered $\operatorname{tr}(XYY^\dagger X^\dagger)$.

We proceed with one eye on the dynamical graph duality

$$\begin{aligned}
\mathcal{Z} &= \int \exp \operatorname{tr}\left[-XX^\dagger - YY^\dagger + \sum_{k=1}^{\infty} \frac{1}{k}(A\circ X + C\circ Y)^k + \sum_{k=1}^{\infty} \frac{1}{k}(B\circ X^\dagger + D\circ Y^\dagger)^k + \theta L^4 \circ XYY^\dagger X^\dagger\right] \\
&= \int \exp \operatorname{tr}\left[-XX^\dagger - YY^\dagger + \theta L^4 \circ XYY^\dagger X^\dagger\right] \frac{1}{\det\left[1-(A\circ X + C\circ Y)\right]} \frac{1}{\det\left[1-(B\circ X^\dagger + D\circ Y^\dagger)\right]} \\
&= \int \exp -\left[\operatorname{tr}(XX^\dagger + YY^\dagger) + P_{ie}(\mathbb{I} - A^i_j X^e_f - C^i_j Y^e_f)P^{\dagger jf} + Q_{ie}(\mathbb{I} - B^i_j X^{\dagger e}_{\ f} - D^i_j Y^{\dagger e}_{\ f})Q^{\dagger jf}\right. \\
&\qquad \left. + \operatorname{tr}(RR^\dagger + SS^\dagger + TT^\dagger + UU^\dagger) - \theta_1 U_{ie} L^i_j Y^{\dagger e}_{\ f} R^{\dagger jf} - \theta_2 RLX^\dagger S^\dagger - \theta_3 SLXT^\dagger - \theta_4 TLYU^\dagger\right] \\
&= \int \exp -\left[(X^e_f - Q^{\dagger je}Q_{if}B^i_j - \theta_2 S^{\dagger je}R_{if}L^i_j)(X^{\dagger f}_{\ e} - P^{\dagger lf}P_{ke}A^k_l - \theta_3 T^{\dagger lf}S_{ke}L^k_l)\right. \\
&\qquad + (Y^e_f - Q^{\dagger je}Q_{if}D^i_j - \theta_1 R^{\dagger je}U_{if}L^i_j)(Y^{\dagger f}_{\ e} - P^{\dagger lf}P_{ke}C^k_l - \theta_4 U^{\dagger lf}T_{ke}L^k_l) \\
&\qquad + P_{ie}P^{\dagger ie} + Q_{ie}Q^{\dagger ie} - Q^{\dagger je}Q_{if}B^i_j P^{\dagger lf}P_{ke}A^k_l - Q^{\dagger je}Q_{if}D^i_j P^{\dagger lf}P_{ke}C^k_l \\
&\qquad + \operatorname{tr}(RR^\dagger + SS^\dagger + TT^\dagger + UU^\dagger) \\
&\qquad - Q^{\dagger je}Q_{if}B^i_j\theta_3 T^{\dagger lf}S_{ke}L^k_l - \theta_2 S^{\dagger je}R_{if}L^i_j P^{\dagger lf}P_{ke}A^k_l - \theta_2 S^{\dagger je}R_{if}L^i_j\theta_3 T^{\dagger lf}S_{ke}L^k_l \\
&\qquad \left. - Q^{\dagger je}Q_{if}D^i_j\theta_4 U^{\dagger lf}T_{ke}L^k_l - \theta_1 R^{\dagger je}U_{if}L^i_j P^{\dagger lf}P_{ke}C^k_l - \theta_1 R^{\dagger je}U_{if}L^i_j\theta_4 U^{\dagger lf}T_{ke}L^k_l\right]
\end{aligned} \tag{7}$$

Not really sure whether we want to allow these $\theta_1\theta_4$ and $\theta_2\theta_3$ terms. Do they correspond to self-contraction?

$$\begin{aligned}
&= \int \exp -\left[(F^l_j - Q_{if}B^i_j P^{\dagger lf} - \theta_2' RLP^\dagger - \theta_3' QBT^\dagger)(F^{\dagger j}_{\ l} - Q^{\dagger je}P_{ke}A^k_l - \theta_2' S^\dagger PA - \theta_3' QSL)\right. \\
&\qquad + (G^l_j - Q_{if}D^i_j P^{\dagger lf} - \theta_4' QDU^\dagger - \theta_1' ULP^\dagger)(G^{\dagger j}_{\ l} - Q^{\dagger je}P_{ke}C^k_l - \theta_4' Q^\dagger TL - \theta_1' R^\dagger PC) \\
&\qquad + P_{ie}P^{\dagger ie} + Q_{ie}Q^{\dagger ie} - Q^{\dagger je}Q_{if}B^i_j P^{\dagger lf}P_{ke}A^k_l - Q^{\dagger je}Q_{if}D^i_j P^{\dagger lf}P_{ke}C^k_l \\
&\qquad + \operatorname{tr}(RR^\dagger + SS^\dagger + TT^\dagger + UU^\dagger) \\
&\qquad - Q^{\dagger je}Q_{if}B^i_j\theta_3 T^{\dagger lf}S_{ke}L^k_l - \theta_2 S^{\dagger je}R_{if}L^i_j P^{\dagger lf}P_{ke}A^k_l \\
&\qquad \left. - Q^{\dagger je}Q_{if}D^i_j\theta_4 U^{\dagger lf}T_{ke}L^k_l - \theta_1 R^{\dagger je}U_{if}L^i_j P^{\dagger lf}P_{ke}C^k_l\right]
\end{aligned} \tag{8}$$

Vaguely $\theta_i'^2 = \theta_i$ is only nonvanishing. Also have $\theta_1'\theta_4' = 0$?

$$\begin{aligned}
&= \int \exp -\left[\operatorname{tr}(FF^\dagger + GG^\dagger) + (P,Q,R,S,T,U)(1-M)(P^\dagger, Q^\dagger, R^\dagger, S^\dagger, T^\dagger, U^\dagger)\right] \\
&= \int \exp \operatorname{tr}\left[-FF^\dagger - GG^\dagger\right] \frac{1}{\det\left[1-M\right]^N} \\
&= \int \exp \operatorname{tr}\left[-FF^\dagger - GG^\dagger + N\sum_{k=1}^{\infty}\frac{1}{k}(M)^k\right]
\end{aligned} \tag{9}$$

$$M = \begin{pmatrix}
0 & AF+CG & \theta_1' CG & \theta_2' AF & 0 & 0 \\
BF^\dagger + DG^\dagger & 0 & 0 & 0 & \theta_3' BF^\dagger & \theta_4' DG^\dagger \\
\theta_2' LF^\dagger & 0 & 0 & 0 & 0 & 0 \\
0 & \theta_3' LF & 0 & 0 & 0 & 0 \\
0 & \theta_4' LG & 0 & 0 & 0 & 0 \\
\theta_1' LG^\dagger & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{10}$$

Note that M is linear in the fields and $\operatorname{tr}(M) = 0$.

$$\frac{1}{2}\operatorname{tr}(M^2) = \operatorname{tr}\left[(AF+CG)(BF^\dagger + DG^\dagger) + \theta_1'\theta_2' CGLF^\dagger + \theta_3'\theta_4' LGBF^\dagger\right] \tag{11}$$

These are correct quadratic terms for (18).

$$\frac{1}{3}\,\mathrm{tr}(M^3) = \mathrm{tr}\left[(AF+CF)\theta_4' DG^\dagger \theta_1' LG^\dagger + \theta_2' AF\theta_3' LF(BF^\dagger + DG^\dagger)\right] \tag{12}$$

These are correct cubic terms for (18).

4 Checks

$$\begin{aligned} &Ntr\left[(A+L)F(B+L)F^\dagger + (A+L)F(D+L)G^\dagger + (C+L)G(B+L)F^\dagger + (C+L)G(D+L)G^\dagger \right.\\ &\left. +(AF+CG)(DG^\dagger + iBF^\dagger)(LF^\dagger + iLG^\dagger) + (BF^\dagger + DG^\dagger)(CG + iAF)(LF + iLG)\right] \end{aligned} \tag{13}$$

Adding cubic terms to $2\mathbb{C}$ MM captures the tree and 1-loop partition function. Hope: perhaps it's correct at higher loops too, given how different it looks to dual model.

Consider the planar term of the 1-loop correction

$$\left\langle XXYY, :[X,Y][X^\dagger, Y^\dagger]:, X^\dagger X^\dagger Y^\dagger Y^\dagger \right\rangle = -2N^5 + 2N^3 \tag{14}$$

The two planar contributions come from

$$\left\langle XXYY :, XYY^\dagger X^\dagger :, X^\dagger X^\dagger Y^\dagger Y^\dagger \right\rangle = N^5 + \mathcal{O}(N^3) \tag{15}$$

and

$$\left\langle XXYY, : YXX^\dagger Y^\dagger :, X^\dagger X^\dagger Y^\dagger Y^\dagger \right\rangle = N^5 + \mathcal{O}(N^3) \tag{16}$$

NB coupling for X^2Y^2 is A^2C^2 and for $XYXY$ is $\frac{1}{2}ACAC$.

Another approach: given that we've added $g_{YM}^2 tr(:[X,Y][X^\dagger, Y^\dagger]:)$ as a vertex in original model, we want it to be a face now in dual model, i.e. perhaps introduce matrix L with $\mathrm{tr}(L^4) = g_{YM}^2$.

This would also require even vertices like $tr(AFLG^\dagger)$. Would need mixed $tr(AACCL) = 0$ and $tr(L^4) = g_{YM}^2$, which could be achieved with $L = diag(0^n, \sqrt{g_{YM}})$.

For example for (15) the following contains the correct diagram

$$-LFDg, AFLg, CGBf, AFBfLf, DgCGLG = -AACC, BBDD, LLLL \tag{17}$$

and for (16) the following contains the correct diagram

$$-AFDg, LGBf, CGLf, CGDgLg, BfAFLF = -AACC, LLLL, BBDD \tag{18}$$

The factor two comes from the symmetric factors.

Similarly we find

$$\frac{1}{2}CGBf, CGBf, AFDg, AFDgdf, DgCGcF = \cdots 2\frac{1}{2}CdDc, BDBD, CAAC \tag{19}$$

and

$$\frac{1}{2}CGBf, AFDg, AFDg, CGBfbg, BfAFaG = \cdots 2\frac{1}{2}CCAA, BaAb, BDBD \tag{20}$$

Together these give the planar result

$$\left\langle XYXY, :[X,Y][X^\dagger, Y^\dagger]:, X^\dagger X^\dagger Y^\dagger Y^\dagger \right\rangle = 4N^5 - 4N^3 \tag{21}$$

Similarly

$$\frac{1}{2}CGBf, CGBf, AFDg, AFBfbg, DgAFaG = \cdots 2\frac{1}{2}CACA, BBDD, AbBa \tag{22}$$

and

$$\frac{1}{2}CGBf, AFDg, AFDg, CGDgdf, BfCGcF = \cdots 2\frac{1}{2}CdDc, BBDD, ACAC \tag{23}$$

For

$$\langle XYXY, :[X,Y][X^\dagger,Y^\dagger]:, X^\dagger Y^\dagger X^\dagger Y^\dagger \rangle = -8N^5 + 8N^3 \tag{24}$$

we must have:

$$-AFDg, AFLg, LFDg, CGBfLf, BfCGLG = -\frac{4}{2\times 2}ACAC, LLLL, BDBD, \tag{25}$$

and

$$-CGBf, LGBf, CGLf, AFDgLg, DgAFLF = -\frac{4}{2\times 2}CACA, BDBD, LLLL, \tag{26}$$

5 Philosophy

3-valent is the rule, like Kontsevich etc. Cover all with 3-valent from 1-loop vertices, everything else is filler for pathological free theory, to get operators.

6 All vertices of dual model in permutations

In old complex MM, have only 2-valent faces for planar 2 point function. Add single 4-valent for planar 3pt function and two 4-valent or one 6-valent for extremal planar 4pt function or torus 2pt function.

Switch to Hermitian way of looking at things, i.e. 2n indices and 2-cycle for each contraction. Now 1-loop should appear as two 3-cycles?

7 Onwards to higher loops

Try restricting to $SU(2)_R$ sector. This sector should be closed under dilatation (need three complex scalars to get mixing with fermions).

0303060, see equation (1.14) for higher loop effective vertices.

Also Bellucci et al 0505106 equation (6)

$$\begin{aligned}
&: tr([[X,Y],\tilde{Y}][[\tilde{X},\tilde{Y}],Y]) : + : tr([[X,Y],\tilde{X}][[\tilde{X},\tilde{Y}],X]) : + : tr([[X,Y],T^a][[\tilde{X},\tilde{Y}],T^a]) : \\
&=: tr([[X,Y],\tilde{Y}][[\tilde{X},\tilde{Y}],Y]) : + : tr([[X,Y],\tilde{X}][[\tilde{X},\tilde{Y}],X]) : +2N : tr([X,Y][\tilde{X},\tilde{Y}]) :
\end{aligned} \tag{27}$$

For $U(N)$ on latter line have $(T^a)^i_j(T^a)^k_l = \delta^i_l\delta^k_j$; for $SU(N)$ it is $(T^a)^i_j(T^a)^k_l = \delta^i_l\delta^k_j - \frac{1}{N}\delta^i_j\delta^k_l$.

The first two terms come from genuine 3-legged interaction terms. The last is from a two-leg term with an internal loop.

NB higher-order vertices in dual model account for non-planarity, NOT for higher loops.

Probably will need all corelation functions with overall factor g_{YM}^{-2} and then $tr(L^{2p}) = g_{YM}^{2p}$.

See Appendix C of 0303060 for two disconnected copies of 1-loop vertiex, something to do with $(log)^2$ for conformal invariance.

this isn't quite what we want. Should be face i.e. closed string op for every 3-valent vertex with coefficient g_{YM}? We get these higher-order vertices from elaborate SUSY cancellations don't we?

References

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