Open-open duality with spacetime dependence

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1 Introduction

In a talk in 2010 [1] Gopakumar suggested that there might exist dualities between field theories which correspond to graph dualities of their Feynman diagrams. He called this open-open duality, in contrast to open-closed string duality. As an example he gave the two different matrix models for 2d topological gravity: the double-scaled Hermitian matrix model, where observables appear as vertices, and the Kontsevich matrix model, where observables are associated to the faces of the graph expansion.

This open-open duality was demonstrated for complex matrix models in [2].

In this note we consider the extension of this open-open duality from matrix models to free field theories. In fact it had already been studied by Kazakov in 2000 [3].

The resulting dual theory is strange, in that its fields are bilocal. Considered on a discrete spacetime, it is not dissimilar to the IKKT IIB matrix model [4].

2 Open-open duality for a free complex scalar field

In this section we compute the open-open dual for a free complex scalar field in 4d. It is the field theory version of the complex matrix model duality in [2].

The partition function for a free scalar $N \times N$ matrix field $Z^e_f(x)$ with sources for holomorphic and antiholomorphic observables is

$$\int [dZ(x)] \exp\left\{ -\int d^4x \, \mathrm{tr}_N \left[\partial_\mu Z(x) \partial^\mu Z^\dagger(x) \right] + \int d^4x \sum_{k=1}^{\infty} \left(J_k(x) \, \mathrm{tr}_N [Z(x)^k] + \bar{J}_k(x) \, \mathrm{tr}_N [Z^\dagger(x)^k] \right) \right\} \tag{1}$$

The theory is free, so no interaction with any gauge field. We do however choose gauge-invariant observables with the gauge theory in mind.

Now reduce the spacetime integration to a sum over spacetime points $x \in X$, where we're not really specifying what X is, i.e. discrete or $\mathbb{R}^4$. The path integral is now over an infinite number of complex matrices Z_x

$$\int \prod_{x \in X} [dZ_x] \exp\left\{ -\sum_{x,y \in X} \mathrm{tr}_N \left[Z_x Z^\dagger_y (x-y)^2 \right] + \sum_{x \in X, k} \left(t_{k,x} \, \mathrm{tr}_N [Z^k_x] + \bar{t}_{k,x} \, \mathrm{tr}_N [Z^{\dagger k}_x] \right) \right\} \tag{2}$$

The kinetic term has been modified to give the correct propagator

$$\left\langle Z_{x}{}^{e}_{f} \; Z^{\dagger g}_{yh} \right\rangle = \frac{\delta^e_h \delta^g_f}{(x-y)^2} \tag{3}$$

This toy model is open-open dual to another model with $n|X| \times n|X|$ matrices (i.e. could be infinite-sized) F^{xi}_{yj} where $i, j \in \{1, \ldots n\}$.

$$\int [dF] \exp\left\{ -\sum_{x,y,i,j} F^{xi}_{yj} F^{\dagger yj}_{\;\; xi} (x-y)^2 + N \sum_{k=1}^{\infty} \tfrac{1}{k} \, \mathrm{tr}_{n|X|} \left[(AFBF^\dagger)^k \right] \right\} \tag{4}$$

The F fields are essentially bilocal, which you could see by studying the graph duality directly. The propagators of the F model are orthogonal to those of the Z model, so they have an x on one side and a y on the other.

The matrices A and B encode the sources $t_{k,x}$ and $\bar{t}_{k,x}$. A^{xi}_{yj} is diagonal $A^{xi}_{yj} = \delta^x_y \delta^i_j a_{x,i}$. We can also write each of the $|X|$ $n \times n$ blocks as A_x, i.e. $(A_x)^i_j \equiv A^{xi}_{xj} = \delta^i_j a_{x,i}$.

$$t_{k,x} = \sum_{i=1}^{n} \tfrac{1}{k} a^k_{x,i} = \tfrac{1}{k} \, \mathrm{tr}_n [A^k_x] \tag{5}$$

The propagator is

$$\left\langle F^{xi}_{yj} F^{\dagger zk}_{\;\; ul} \right\rangle = \frac{\delta^{xi}_{ul} \delta^{zk}_{yj}}{(x-y)^2 - N a_{x,i} b_{y,j}} = \frac{\delta^{xi}_{ul} \delta^{zk}_{yj}}{(x-y)^2} \sum_{k=0}^{\infty} \left[\frac{N a_{x,i} b_{y,j}}{(x-y)^2} \right]^k \tag{6}$$

This is not the same as the Eguchi-Kawai model, since there you have d $N \times N$ matrices for $N \to \infty$.

3 Comparison to Kazakov's model

The set-up here is practically the same as Kazakov's matrix model for QFT in [3].

The F model kinetic term can be written like

$$\mathrm{tr}([X^\mu, F][X_\mu, F^\dagger]) \tag{7}$$

with X^μ appropriately defined.

If $X^{xi}_{yj} = \delta^x_y \delta^i_j x$ then

$$\mathrm{tr}([X, F][X, F^\dagger]) = \sum_{x,y,i,j} F^{xi}_{yj} F^{\dagger yj}_{\;\;xi} (2xy - x^2 - y^2) \tag{8}$$

Q: Why is this quenched?

3.1 Translation from Kazakov

His p is my $|X|$ and his q is my n. Thus his n is my $|X|n$.

For him (3.11) as for me get

$$\frac{1}{p}\sum_{i=1}^{p} \cdots \to_{p\to\infty} \int d^4x \cdots \tag{9}$$

He points out that N of original model is a fixed parameter (also true for me).

4 Comparison to Wilson-loop-amplitude duality

For F-model have non-local bits of string with labels x, i at one end and y, j at the other. Looks like Wilson line with cusps specified by points.

A similar graph duality has already been observed in the duality between Wilson line observables with light-like intervals and $\mathcal{N} = 4$ amplitudes.

5 Comparison to IKKT IIB Matrix Model

The F model is not a million miles from the IKKT IIB matrix model [4], derived by a reduction of 10d super-Yang-Mills.

N become number of discrete spacetime points. The eigenvalues of A_μ represent their spacetime coordinates.

How does the shape of spacetime emerge? They claim in [5] you can get a gauge theory by blocking $SU(N) \to SU(n)^{N/m}$ where now there are only m spacetime points. In the review [6] they claim you can see $\mathcal{N} = 4$ SYM from IKKT on page 35 by strongly peaking, etc..

References

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